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素数分布规律的发现(II)以及黎曼猜想的直接证明

——基础理论和素数分布规律原理的发现(1)

The Discovery of the Law of the Distribution of Prime Numbers (II) and the Direct Proof of the Riemann Conjecture

--- Discovery of Basic Theory and Principle of Prime Number Distribution (1)

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摘要: 黎曼猜想有二层含义: 一是指黎曼对素数在自然数的分布规律的有关假设和推测。用语言文字描述, 即素数规律所包含的信息都在实部 $x=1/2$ 的这条直线上。不限具体的数学表示形式。二是指黎曼猜想的具体的数学表达形式, 给出了黎曼 ζ 函数。也就是说, 存在与素数分布有关的复数 $s = \sigma + it$, 使得所有非平凡零点都在实部 $\sigma = 1/2$ 的直线上。为此, 本文作者基于素数通项公式的首先发现, 并利用该公式直接、巧妙地证明了黎曼假设的素数所有信息都在实部等于 $1/2$ 的直线上。因此, 推断黎曼猜想成立。

Abstract: Riemann Conjecture has two meanings: First, it refers to Riemann Hypothesis and speculation about the distribution law of prime numbers in natural numbers. To put it into words, the law of prime numbers contains all the information on this line with the real part $x = 1/2$. Not limited to specific mathematical representations. The second refers to the concrete mathematical

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expression of Riemann Conjecture, and gives the Riemann zeta function. That is, there are complex numbers $s = \sigma + it$, related to the distribution of prime numbers such that all nontrivial zeros are on a line with the real part $\sigma = 1/2$. Therefore, based on the first discovery of the general term formula of prime numbers, the author of this paper directly and skillfully proves that all the information of Riemann's assumption of prime numbers is on the line with the real part equal to $1/2$. Therefore, we conclude that the Riemann Conjecture is true.

关键词:素数分布规律; 素数通项公式; 黎曼猜想; 直接初等证明。

Key words: Distribution of prime numbers; Prime number general term formula; Riemann Conjecture; Direct elementary proof.

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第一部分 基础理论和素数分布规律原理的发现(1)

Part I Discovery of Basic Theory and principle of prime Number distribution (1)

§ 1 引言

§ 1 Introduction

1.1 正整数以及素数和合数的概念

1.1 Concepts of positive integers, prime numbers and composite numbers

我们把 1, 2, 3, 4, 5, ..., n, ..., 这些数叫做正整数, 也称非零自然数。通常用 N^* 表示。其中: 2, 4, 6, 8, ..., 能够被正整数 2 整除的, 叫做偶数。其中: 1, 3, 5, 7, ..., 不能被正整数 2 整除的, 叫做奇数。若按除 1 和它本身外, 是否能被其他正整数整除, 正整数又可分为素数和合数。如 1, 2, 3, 5, 7, 11, ..., 这些正整数除 1 和它本身外, 不能被其他正整数整除, 则为素数。否则就叫做合数, 如 4, 6, 8, 9, 10, 12, 14, 15, ..., 这些正整数, 则为合数。很明显, 合数当中既有偶数, 也有奇数。而除偶素数 2 之外, 其他素数必为奇数, 也称为奇素数。但是奇数则不一定是素数。正整数的分类, 以及素数和正整数的关系, 如图 1 所示。

We put 1,2,3,4,5,... n... These numbers are called positive integers, also known as non-zero natural numbers. It's usually denoted by N^* . Where: 2,4,6,8,... Those that are divisible by the positive integer 2 are called even numbers. Where: 1,3,5,7,... Those that are not divisible by the positive integer 2 are called odd. If in addition to 1 and itself, can be divisible by other positive integers, positive integers can be divided into prime numbers and composite numbers. 1,2,3,5,7,11,... These positive integers, except for 1 and itself, are not divisible by other positive integers and are prime numbers. Otherwise it is called composite numbers, such as 4,6,8,9,10,12,14,15,... These positive integers are composite numbers. Obviously, composite numbers have both even and odd numbers. Except for the even prime number 2, the other prime numbers must be odd, also known as odd prime numbers. But odd numbers are not necessarily prime numbers. The classification of positive integers and the relationship between primes and positive integers are shown in Figure 1.

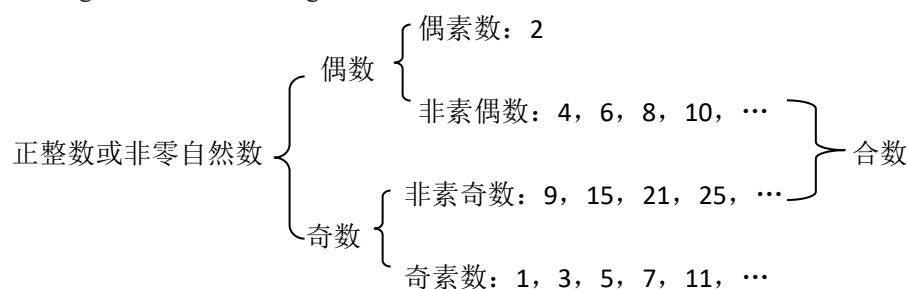


图 1. 正整数的分类

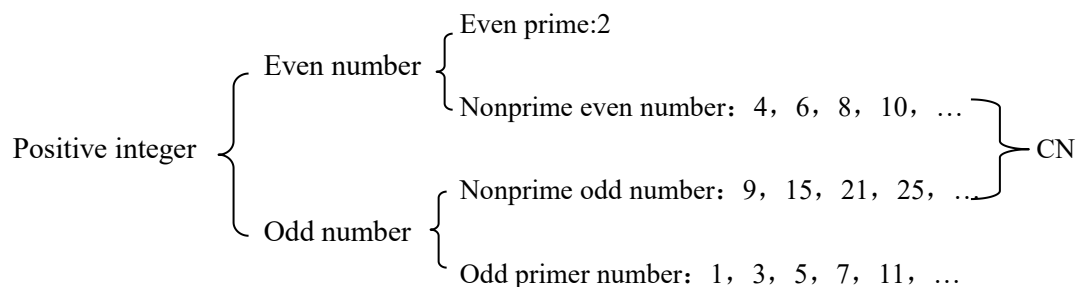


Figure 1. Classification of positive integers

(The composite number is abbreviated as CN)

由于自然单位数 **1**，符合素数定义，为素数最小单位元，并且排序在所有素数之首。为更加全面、正确和科学地反映以及揭示素数的客观规律。所以，作者将它定义为素数。更多相关论述详见下文。

Since the natural unit number 1 meets the definition of a prime number, it is the smallest unit of a prime number and ranks first among all prime numbers. In order to reflect and reveal the objective law of prime numbers more comprehensively, correctly and scientifically. Therefore, the author defines it as prime. More on this can be found below.

根据以上所述，在整数范围之内，我们还可以推得：

According to the above, within the integer range, we can also deduce:

整数 $\times$ 整数=整数；

整数 $\pm$ 整数=整数；

偶数+偶数=偶数；

奇数+奇数=偶数；

奇数+偶数=奇数。

Integer $\times$ integer = integer;

Integer $\pm$ integer = integer;

Even + even = even;

Odd + odd = even;

Odd + even = odd.

1.2 有关的素数猜数

1.2 Prime number conjectures

素数是指除 **1** 和它本身以外，不能再被其它整数整除的正整数，或者非零自然数。我们称之为素数，或者是质数。除了 **1** 和它本身以外，还能再被其它整数整除的正整数，称为合数。早在二千三百多年前的古希腊时期，古希腊数学家，几何学的鼻祖，欧几里得(约公元前 325—公元前 265 年)，就在其名著《几何原本》中给出了质数和合数两个概念的明确定义[1]。随后，他又用反证法证明了素数在自然数中有无穷多个。2000 多年来，古今中外全世界无数的数学家、数学工作者以及数学爱好者，对于素数规律的研究和探讨，都非常执迷。并且前赴后继，孜孜不倦的探索。但是，由于数学家们一直苦于没有找到一个素数通项表达式，导致有关素数问题的各种各样猜想依旧存在。成为世界数学难题，至今没有解决。

A prime number is a positive integer other than 1 and itself that is no longer divisible by other integers, or a non-zero natural number. We call them prime numbers, or prime numbers. A positive integer that, in addition to 1 and itself, can be divided by other integers is called a composite number. As early as 2,300 years ago in ancient Greece, Euclid (about 325-265 BC), the ancient Greek mathematician and the originator of geometry, gave a clear definition of the two concepts of prime and composite numbers in his masterpiece *Elements*[1]. Later, he proved that there are infinitely many prime numbers in the natural numbers by proof by contradiction. For more than 2000 years, countless mathematicians, mathematicians and mathematics lovers all over the world have been very obsessed with the study and discussion of the law of prime numbers. And continue to explore tirelessly. However, because mathematicians have been struggling to find a prime number general term expression formula, leading to all kinds of conjectures about the prime number problem still exist. Became the world's mathematical problem, has not been solved.

大约在公元 1800 年，高斯(Friedrich Gauss)与勒让德(Adrien-Marie Legendre)独立地提出了素数定理的命题猜想[2]。几乎一个世纪过去，还是没有人能够证明。1896 年阿达玛(Jacques Hadamard)与瓦莱·泊桑(Ch.de la valle Poussn)各自采用了复变分析的方法证明了素数定理成立[2]。成为 19 世纪数论领域取得的最高成就之一。到目前为止，人类对素数的认识和研究，主要成就是：

Around 1800 AD, Friedrich Gauss and Adrien-Marie Legendre independently proposed the propositional conjecture of the Prime Number Theorem[2]. Almost a century later, no one has been able to prove it. In 1896, Jacques Hadamard and Ch.de la vallée Poussn proved the prime number theorem by using the method of complex analysis[2]. It became one of the highest achievements in the field of number theory in the 19th century. So far, the main achievements of human understanding and research on prime numbers are:

素数定理[2]，是指当 x 很大时，小于 x 的素数个数近似等于 $x/\ln(x)$ 。用数学式子表示，即为：

The Prime Number Theorem states that when x is large[2], the number of primes less than x is approximately equal to $x/\ln(x)$. Expressed in mathematical formula, that is:

$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\ln(x)} = 1 \quad (1.1)$$

其中： $\ln(x)$ 是以 $e=2.718\ 281\ 8\cdots$ 为底的自然对数。在 1948 年埃德斯(Paul Erdős)与塞尔伯格(Atle Selberg)发现了素数定理的初等证明[2]。

Where: $\ln(x)$ is written as $e = 2.718\ 281\ 8\ldots$ The natural logarithm of the base. In 1948, Paul Erdős and Atle Selberg discovered an elementary proof of the Prime Number Theorem[2].

素数定理对素数深邃分布规律的描述，仅仅限于大概、粗略、近似的描述。也就是说，素数定理，仅限于当 x 趋于无穷大时成立。比如，当素数分别为 1, 2, 3, 5 等时，就不成立，并且两者相差甚大。这不能准确地揭示任意一个素数客观、真实、准确分布密度比率规律。下面是素数实际个数 $\pi(x)$ 与 $x/\ln(x)$ 近似值的比较[3]，如表 1 所示。

The description of the Prime Number Theorem on the deep distribution of prime numbers is only limited to the general, rough and approximate description. In other words, the Prime Number Theorem only holds if x goes to infinity. For example, when the prime numbers are 1,2,3,5, etc., it is not true, and the two are very different. This can not accurately reveal the objective, true and accurate distribution density ratio rule of any prime number. Here is the actual number of prime numbers $\pi(x)$ comparison with the approximate value of $x/\ln(x)$ is shown in Table 1 [3].

表 1 素数实际个数 $\pi(x)$ 与 $x/\ln(x)$ 近似值的比较表

Table 1. Table comparing the actual number of primes $\pi(x)$ with the approximate value of $x/\ln(x)$

Natural number x	$\pi(x)$	$x/\ln(x)$	$\frac{\pi(x)}{x/\ln(x)}$
10	4	4.34	0.921
100	25	21.71	1.151
1000	168	144.76	1.160
10^4	1229	1085.74	1.132
10^5	9592	8686	1.104
10^6	78 498	72382.41	1.085
10^7	664579	620417	1.071
10^8	5761455	5428681	1.061
10^9	50 847 534	48 254 942.43	1.054
10^{10}	455 052 512	434 294 481.90	1.048
...	...	...	...
∞	∞		1

特别提示：表中实际素数个数 $\pi(x)$ 不包括单位素数 1 在内。

Note: The actual prime number $\pi(x)$ in the table does not include the unit prime number 1.

其次是，素数的判别和埃拉托斯尼筛法。这是公元前 230 年，由古希腊数学家埃拉托斯尼(Eratosthenes)提出的一种寻找素数的筛选法。是非常传统古老的方法。其方法是：若 $n \leq N$ 为正整数，它不能被不超过 $\sqrt{N}$ 的所有素数整除，则 n 必为素数。这种判别正整数是否为素数的方法，称为埃拉托斯尼筛法。现我们经常查阅的素数表，就是依照此法建立起来的。

Secondly, the identification of prime numbers and Eratosny sieve method. This is a screening method for finding prime numbers proposed by the ancient Greek mathematician Eratosthenes in 230 BC. It's a very old fashioned way. The method is: if $n \leq N$ is a positive integer that cannot be evenly divided by all prime numbers up to $\sqrt{N}$, then n must be prime. This method of determining whether positive integers are prime numbers is called Eratosny's sieve method. The table of prime numbers, which we now often consult, was built up in this way.

第三是，素数的个数是无限的。

The third is that the number of prime numbers is infinite.

早在 2300 多年前，欧几里得时期就给出了一种巧妙的证明方法[4]，其证明方法如下。

As early as 2,300 years ago, a clever proof method was given in the time of Euclid [4], which is as follows.

假设素数集合 P 是由有限个素数 P_m 构成，即 $P_1, P_2, P_3, \dots, P_m$ ，有任意一个整数

Suppose that the set of prime numbers P consists of a finite number of primes P_m , that is, $P_1, P_2, P_3, \dots, P_m$, with any integer:

$$m = P_1 P_2 P_3 \cdots P_m + 1$$

并存在一个素因数 q ，由于假设素数是有限的，则 $q \in P$ 。那么可推得：

And there is a prime factor q , since the prime number is assumed to be finite, then $q \in P$. then we can deduce:

$$q \mid (m - P_1 P_2 P_3 \cdots P_m)$$

也就是说， $q \mid 1$ ，这是不可能的。因为 1 是任何正整数的最基本的最小数单位，不可能再被任何整数再整数，推出结论与假设矛盾。所以说，素数有无穷多。

In other words, $q \mid 1$, which is impossible. Since 1 is the most basic and smallest unit of any positive integer, it cannot be re-integer by any integer to derive conclusions and hypotheses to the shield. So, there are infinitely many prime numbers.

但是，到目前为此，人们对素数的认识还十分有限。绝大多数数学家认为素数的变化是不规则的。无规律可循。因此，千百年来，人们对涉及素数问题的认识提出了许许多多猜想。其中最为著名的涉及素数问题的猜想主要是：

However, so far, people's understanding of prime numbers is very limited. Most mathematicians believe that prime numbers change irregularly. There are no rules to follow. For thousands of years, therefore, many conjectures have been put forward regarding the understanding of problems involving prime numbers. The most famous conjectures involving prime numbers are:

猜想 1.1. 哥德巴赫猜想

Conjecture 1.1. Goldbach's Conjecture

所谓哥德巴赫猜想就是指任何一个大于 2 的偶数，都可表示成两个素数之和。哥德巴赫在 1742 年 6 月 7 日与欧拉的一封书信来往中提出这个猜想[2]。当前主流数学界对于哥德巴赫猜想的研究仅仅限于大偶数和大奇数。如果定义自然单位数 1，也属于素数。其实，不难验证哥德巴赫猜想对于任意偶数，或者是任意奇数都是成立的。例如，对于偶数德巴赫猜想有： $2=1+1$ ， $4=1+3$ ， $4=2+2$ ， $6=1+5$ ， $8=3+5$ ， $10=3+7$ ， $\cdots$ ；或者对于奇数德巴赫猜想有： $3=1+1+1$ ， $5=1+2+2$ ， $7=1+3+3$ ， $\cdots$ 。如图 2 所示。

Goldbach's Conjecture means that any even number greater than 2 can be expressed as the sum of two prime numbers. Goldbach proposed this conjecture in a letter to Euler dated 7 June 1742[2]. At present, the study of Goldbach's Conjecture is limited to large even numbers and large odd numbers. If the natural unit number 1 is defined, it is also a prime number. In fact, it is not difficult to verify that Goldbach's Conjecture holds for any even number, or for any odd number. For example, for even numbers, the Debach's conjecture is: $2 = 1+1$, $4 = 1+3$, $4 = 2+2$, $6 = 1+5$, $8 = 3+5$, $10 = 3+7$, $\cdots$; Or for odd numbers, the Debach conjecture is: $3 = 1+1+1$, $5 = 1+2+2$, $7 = 1+3+3$, $\cdots$. As shown in Figure 2.

其实，在哥德巴赫猜想的最初书信中，自然单位数 1 是被定义为素数的。这里有哥德巴赫的最初书信图片资料为证。

In fact, in the original letter of Goldbach's conjecture, the natural unit number 1 is defined as prime. There are pictures of Goldbach's first letters to prove it.

因此，对于哥德巴赫猜想研究不仅仅局限于大偶数或者大奇数，而且应当包括所有偶数或者奇数。

Therefore, the study of Goldbach's Conjecture is not limited to large even numbers or large odd numbers, but should include all even numbers or odd numbers.

Figure 2. This is the original original letter of Goldbach's conjecture.

In this letter, the natural unit number 1 is identified as prime.

猜想 1.2. 相邻素数对以及 Polignac 猜想(1849)和孪生素数猜想

Conjecture 1.2. Adjacent Prime Pairs and Polignac Conjecture (1849) and Twin Prime Conjecture

Polignac 猜想[4]是指存在无穷多对相邻素数 P_n 和 P_{n+1} , 使得

The Polignac Conjecture[4] states that there are infinitely many pairs of adjacent prime numbers P_n and P_{n+1} , make:

$$P_{n+1} - P_n = 2k \quad (1,2)$$

其中: $k \geq 1$ 为正整数。Where: $k \geq 1$ is a positive integer.

注意这里 k 取值未反映出与素数的序数相关或者相互之间的联系。

Note that the value of k here does not reflect any ordinal correlation with or relation to prime numbers.

特别是 $k=1$, Polignac 猜想, 也就是说有无穷多个素数 P , 使得 $P+2$ 也是素数。这就是著名的孪生素数猜想。到目前为止, 数学家仍然认为素数很不规则。相邻的两个素数间常常会有很大的间隙。似乎有相当多的场合使得素数 P 紧接着又存在另一个素数 $P+2$ 。这样的素数对称为孪生素数。在素数序列中, 前面的一些孪生素数是: 第一对孪生素数 3 和 5。其他依次为 5 和 7, 11 和 13, 17 和 19, 29 和 31, 41 和 43, 59 和 61, ...。使用计算机, 我们可以找到一些很大的孪生素数。例如,

In particular, $k=1$, Polignac Conjecture, means that there are infinitely many prime numbers P such that $P+2$ is also prime. This is the famous Twin Prime Conjecture. So far, mathematicians have considered prime numbers to be very irregular. There is often a large gap between two adjacent prime numbers. There seem to be quite a few cases where a prime P is followed by another prime $P+2$. Such pairs of prime numbers are called twin primes. In a sequence of prime numbers, some of the first twin primes are: the first twin primes 3 and 5. The others are 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, in order. Using a computer, we can find some very large twin prime numbers. For example,

$$242\,206\,083 \cdot 2^{38\,880} - 1 \text{ 与 } 242\,206\,083 \cdot 2^{38\,880} + 1$$

都是作为支持孪生素数猜想成立的证据之一[2]。

Both are used as evidence to support the hypothesis of Twin Prime Numbers[2].

在素数序列中, 任意相邻的 2 个素数, 本文作者称为相邻素数对。素数序列就是由彼此有序的 2 个相邻素数对相互连接以至无穷构成的。用数学式表示为:

In the sequence of prime numbers, any two adjacent prime numbers are called Adjacent Prime Pairs by the authors of this paper. A sequence of prime numbers is composed of two adjacent pairs of prime numbers that are ordered to each other and connected to infinity. It can be expressed mathematically as:

$$P_m - P_{m-1} = K_m + 1 \quad (1.3)$$

上式(1.3)作者称为相邻素数对猜想。

The above formula (1.3) is called the adjacent prime pair conjecture by the authors.

式中: $\{K_m\} = 0, 0, 0, 1, 1, 3, 1, 3, 1, 3, 5, 1, \dots, AAK_m, \dots, \infty$; 是与素数数列

的序数 m 依次取值相关的一个对应变量。作者称为素数的特征无穷有序数列。因此，本文作者这种表达方式(1.3)与上式(1.2)有本质的区别。

Where: $\{K_m\} = 0, 0, 1, 1, 3, 1, 3, 1, 3, 3, 5, 1, \dots, K_m, \dots, \infty$; Is a corresponding variable related to the sequential value of the ordinal m of the prime number series. A series of characteristic infinite ordered numbers called prime numbers by the authors. Therefore, this way of expression (1.3) is fundamentally different from the above formula (1.2).

猜想 1.3. 黎曼猜想

Conjecture 1.3. Riemann Conjecture

1859 年 8 月，当时年纪仅为 32 岁的伯恩哈德·黎曼(Bernhard Riemann)成为柏林科学院的院士。对于一个年轻数学家黎曼来说，这是一个非常崇高的荣誉。依照惯例，黎曼需要向科学院提交一篇论文，对他正在从事进行的研究作一个陈述。黎曼提交的论文题目是：“论小于给定值的素数个数”。全文仅仅有 8 页。正是这篇短短几页论文，包含了黎曼对素数规律研究的许多重要信息和推测。因此，黎曼猜想[3]：一是指黎曼有关对素数在自然数的分布规律的假设和推测。用语言文字描述和表达，即素数规律所包含的信息都在实部 $x=1/2$ 的这条垂直直线上。不限具体的数学表示形式。二是指黎曼猜想的具体的明确的数学表达形式，给出了黎曼 ζ 函数。并且提出满足黎曼 ζ 函数所有非凡零点都分布在实部等于 $1/2$ 的直线上。也就是说，黎曼 ζ 函数的非平凡零点分布与素数的分布有关。存在复数 $s = \sigma + it$ ，使得所有非平凡零点都在 $\sigma = 1/2$ 的直线上。即

In August 1859, Bernhard Riemann, then only 32 years old, became a member of the Berlin Academy of Sciences. For a young mathematician named Riemann, this was a very high honor. By convention, Riemann was required to submit a paper to the Academy of Sciences, making a statement about the research he was working on. Riemann submitted a paper entitled "On the Number of primes less than a given value". The full text is only eight pages long. It is this paper, which is only a few pages long, that contains much important information and speculation about Riemann's study of the law of prime numbers. Therefore, Riemann Conjecture[3]: First, it refers to Riemann's hypothesis and speculation about the distribution law of prime numbers in natural numbers. To describe and express in words, that is, the information contained in the law of prime numbers is on this vertical line with the real part $x = 1/2$. Not limited to specific mathematical representations. The second refers to the specific and explicit mathematical expression of the Riemann Conjecture, and gives the Riemann zeta function. It is also proposed that all extraordinary zeros satisfying the Riemann zeta function are distributed on a line with real part equal to $1/2$. In other words, the distribution of nontrivial zeros of the Riemann zeta function is related to the distribution of prime numbers. There are complex numbers $s = \sigma + it$, such that all nontrivial zeros are on the line of $\sigma = 1/2$. That is,

$$\zeta(s) = \sum_n \frac{1}{n^s} = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1} \quad (1.4)$$

其中： n 为所有正整数， P 为所有素数(这里 $p \neq 1$)。复数 $s = \sigma + it$ 。 σ 、 t 为任意实数。 i 为虚数单位。

Where: n is all positive integers and P is all prime numbers (where $p \neq 1$). Plural $s = \sigma + it$. σ 、 t is any real number. i is an imaginary unit.

本论文研究与主流数学研究有所不同的，是依据作者首次发现的素数分布规律和据此推导出的素数通项表达公式，以及所有有序素数的原始定义，即所有素数都分布在实轴 x 等于

1, 或者复数 Z_m 的实部(Rez)等于 1 的直线上。因此, 在复平面内, 任意一个以 1 为实部, 以有序素数 P_m 为虚部的高斯整数 Z_m , 可以表示为:

This research is different from the mainstream mathematical research, based on the distribution law of prime numbers first discovered by the author and the formula of prime number general terms derived from it, as well as the original definition of all ordered prime numbers, that is, all prime numbers are distributed on the line where the real axis x is equal to 1, or the real part (Rez) of the complex number Z_m is equal to 1. Therefore, in the complex plane, any Gaussian integer Z_m with 1 as its real part and P_m as its imaginary part can be expressed as:

$$Z_m = 1 + iP_m \quad (1.5)$$

式中: 复数 Z_m 的实部为实数 1, 虚部为普通素数 P_m 。然后再依据有序素数通项表达式和任意一个素数的平行四边形分解定理, 可以得到黎曼 ζ 函数的非平凡零点分布与素数的分布有关的复数为:

Where: The real part of the complex number Z_m is the real number 1, and the imaginary part is the ordinary prime number P_m . Then the expression is based on ordered prime numbers Using the parallelogram decomposition theorem for any prime number, it is obtained that the distribution of nontrivial zeros of the Riemann zeta function is the complex number related to the distribution of prime numbers:

$$s = z' = \frac{1}{2} + i \frac{1}{2} P_m \quad (1.6)$$

然后再将 $s = z'$ 直接代入原始黎曼 ζ 函数, 则可直接巧妙地证明黎曼猜想成立。

Then $s = z'$ is substituted directly into the original Riemann zeta function, and the Riemann conjecture is proved directly and ingeniously.

除以上所述三个著名的有关素数猜想外, 还有:

In addition to the three well-known conjectures about prime numbers mentioned above, there are:

费马数猜想[4]: 数列 $F(n) = 2^{2^n} + 1$, $n = 0, 1, 2, 3, 4, \dots$ 。其中的自然数称为费马数。证明费马数中只有有限多个素数。当 $n = 0, 1, 2, 3, 4$ 时, 费马数 $F(n)$ 是素数。1732 年欧拉发现 $F(5)$ 是合数。此后没有再发现其它费马数 $F(n)$ 是素数。

Fermat Number Conjecture [4]: Series $F(n) = 2^{2^n} + 1$, $n = 0, 1, 2, 3, 4, \dots$. The natural numbers are called Fermat numbers. Prove that there are only a finite number of prime numbers in Fermat numbers. When $n = 0, 1, 2, 3, 4$, the Fermat number $F(n)$ is prime. In 1732, Euler discovered that $F(5)$ is composite. No other Fermat number $F(n)$ has ever been found to be prime.

梅森素数猜想[2]: 形如 $2^n - 1$ 的正整数中, 有无穷多个素数。这个猜想大约在 1639 年提出, 已经经过 380 多年了。

Mersenne's Prime Conjecture[2]: form $2^n - 1$ There are infinitely many primes in the positive integers of phi. It's been more than 380 years since the conjecture was first proposed around 1639.

$N^2 + 1$ 素数猜想[2]: 存在无穷多个自然数 N , 使得 $N^2 + 1$ 是素数。

$N^2 + 1$ Prime Conjecture [2]: There are infinitely many natural numbers N , such that $N^2 + 1$ is prime.

勒让德猜想[3]: 任意两个完全平方数之间, 都存在至少一个素数。即对于任意正整数 n , 存在素数 P , 满足不等式 $n^2 < P < (n + 1)^2$ 。等等, 这里就不一个一个叙述。

Legendre's Conjecture[3]: states that there is at least one prime number between any two perfect square numbers. That is, for any positive integer n , there is a prime P that satisfies the inequality $n^2 < P < (n+1)^2$. Wait a minute. This is not a one-by-one.

本文笔者通过多年对素数的研究和探讨,首次发现了在非零自然数集合中任意有序素数序列的分布原理规律和任意有序素数序列的通项表达公式。以及素数通项公式有三种推导方法。并依据此素数的通项表达公式,直接、巧妙地证明了哥德巴赫猜想、黎曼猜想、孪生素数猜想、Polignac conjecture (1849), 费马数猜想、梅森素数猜想、勒让德猜想、 $N^2 + 1$ 素数猜想等有关素数猜想的成立。本文限于篇幅,笔者仅就黎曼猜想的研究成果论述如下。

In this paper, through many years of research and discussion on prime numbers, the author first found the distribution principle of any ordered prime number sequence in the set of non-zero natural numbers and the general term expression formula of any ordered prime number sequence. There are three derivation methods for the formula of prime number general term. According to the general term expression formula of the prime number, it is proved directly and skillfully that Goldbach's conjecture, Riemann Conjecture, twin prime conjecture, Polignac conjecture (1849), Fermat's conjecture, Mersenne's prime conjecture, Luddre's conjecture and $N^2 + 1$ prime conjecture are valid. In this paper, the author only discusses the research results of Riemann Conjecture as follows.

§ 2 自然数秩序公理以及集合和复数的概念

§ 2 The axioms of natural number order and the concepts of sets and complex numbers

2.1 自然数的秩序公理

2.1. The order axiom of natural numbers

定义 1. 满足下列条件的集合 N , 称为自然数集[5]。

Definition 1.

The set N that satisfies the following conditions is called the set of natural numbers[5].

- (1) $0 \in N$;
 - (2) 对任意 $X \in N$, 有其后继 $X^+ \in N$;
 - (3) 对任意 $X \in N$, 有其后继 $X^+ \neq 0$;
 - (4) 对任意 $X, Y \in N$, 如果 $X \neq Y$, 则 $X^+ \neq Y^+$;
 - (5) 对任意 $A \subseteq N$, 如果 A 满足如下两个条件: ① $0 \in A$; ② 对任意 $X \in A$, 有 $X^+ \in A$, 则有 $A = N$.
- (1) $0 \in N$;
 - (2) For any $X \in N$, there is a successor $X^+ \in N$;
 - (3) For any $X \in N$, there is a successor $X^+ \neq 0$;
 - (4) For any $X, Y \in N$, if $X \neq Y$, then $X^+ \neq Y^+$;
 - (5) For any $A \subseteq N$, if A meets the following two conditions: ① $0 \in A$; ② For any $X \in A$, if there is $X^+ \in A$, there is $A = N$.

因此, 根据以上定义, 所有非零自然数或正整数 $1, 2, 3, 4, 5, \dots$, 所组成的正整数集合, 用 N^* 表示。其最本质的数学属性可用数学语言描述如下:

Therefore, according to the above definition, all non-zero natural numbers or positive integers $1, 2, 3, 4, 5, \dots$. The set of positive integers, represented by N^* . Its most essential mathematical properties can be described in mathematical language as follows:

归纳公理. 假设 $A \subseteq N^*$, 若 A 满足下述条件:

Axiom of induction. Suppose $A \subseteq N^*$, if A satisfies the following conditions:

- (1) $1 \in A$;
 (1) $1 \in A$;
 (2) 如果 $n \in A$, 那么 $n+1 \in A$, 则 $A = N^*$.
 (2) If $n \in A$, then $n+1 \in A$, $A = N^*$.

根据以上自然数集的定义和归纳公理可知, 自然数是有无限多的有秩序集合。0 是最小的自然数, 没有比它再小的, 其所有表示为空集 $\emptyset$, 即 $0 = \emptyset$; 对于自然数 1, 只有 0 比它小, 所以表示为 $1 = \{0\} = \{\emptyset\}$; 对于自然数 2, 自然数 0、1 都比它小, 应该表示为 $2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}$; 等等。

According to the above definition of the set of natural numbers and the axioms of induction, we can see that there are infinitely many ordered sets of natural numbers. 0 is the smallest natural number, there is no smaller, all of which is expressed as the empty set $\emptyset$, i.e. $0 = \emptyset$; For the natural number 1, only 0 is smaller, so it is expressed as $1 = \{0\} = \{\emptyset\}$; For the natural number 2, the natural numbers 0 and 1 are smaller than it and should be expressed as $2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}$; Let's wait.

为了更加清楚, 我们将前面几个自然数的集合表示一起列出其表示如下[5]。

To be more clear, we list together the set representations of the previous natural numbers whose representations are as follows [5].

$0 = \emptyset$;
 $1 = \{0\} = \{\emptyset\}$;
 $2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}$;
 $3 = \{0, 1, 2\} = \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$;

上述这种自然数的集合表示方法是由冯·诺依曼(John von Neumann)提出的。因此, 根据前面所给出的自然数的集合表示方法, 这里我们也可以“直观”地将自然数集 N 表示为:

The set representation of the natural numbers mentioned above was proposed by John von Neumann. Therefore, according to the set representation of natural numbers given earlier, here we can also "intuitively" represent the set of natural numbers N as:

$$N = \{\emptyset, \emptyset^+, \emptyset^{++}, \emptyset^{+++}, \dots\}$$

对于所有正整数集或者非零自然数集 N^* , 我们也可以直观地表示为:

For the set of all positive integers or the set of non-zero natural numbers N^* , we can also intuitively express it as:

$$N^* = \{\emptyset^+, \emptyset^{++}, \emptyset^{+++}, \dots\}$$

数学归纳法. 设 $P(n)$ 是关于正整数 n 的一个命题, 如果

Mathematical induction. Let $P(n)$ be a statement about the positive integer n , if

- (1) 当 $n=1$ 时, $P(1)$ 成立;
 (1) When $n = 1$, $P(1)$ is true;
 (2) 由命题 $P(n)$ 成立, 可以推出 $P(n+1)$ 成立, 则对任意 $n \in N^*$, 命题 $P(n)$ 都成立。
 (2) As the proposition $P(n)$ is true, it can be deduced that $P(n+1)$ is true, then for any $n \in N^*$, the proposition $P(n)$ is true.

第二数学归纳法. 设 $P(n)$ 是关于正整数 n 的一个命题, 如果

Second mathematical induction. Let $P(n)$ be a statement about the positive integer n , if

- (1) 当 $n=1$ 时, $P(1)$ 成立;

(1) When $n = 1$, $P(1)$ is true;

(2) 由命题 $P(1)$, $P(2)$, $P(3)$, $\dots$, $P(n)$ 都成立, 可以推出 $P(n+1)$ 成立, 那么, 对任意 $n \in N^*$, 命题 $P(n)$ 都成立。

(2) By the propositions $P(1)$, $P(2)$, $P(3)$, $\dots$, $P(n)$ is true, it can be deduced that $P(n+1)$ is true, then, for any $n \in N^*$, the statement $P(n)$ is true.

最小数原理. 正整数集的任意一个非空子集都有一个最小元。

Principle of minimum numbers. Any nonempty subset of a positive set of integers has a minimum element.

2.2 集合的概念

2.2 Concept of set

为揭示客观世界的规律, 我们往往把具有某些共同特征的事物看成一个整体来加以研究。一般地, 我们把研究整体构成的对象统称为元素。把由这些具有共同特性的元素组成的整体叫做集合, 简称集。例如, 由 $0, 1, 2, 3, \dots$ 全体非负整数组成的集合称为自然数集(或非负整数集), 记作 N ; 由 $1, 2, 3, 4, 5, \dots$ 所有正整数组成的集合称为正整数集, 或者非零自然数集, 记作 N^* ; 由 $1, 2, 3, 5, 7, \dots$ 所有素数组成的集合称为素数集, 记作 P , 等等。对于给定的任意集合, 它的元素总是确定的。也就是说, 给定一个集合, 那么任何一个元素在不在这个集合中就确定了, 并非可有可无。比如, 自然单位素数 1 , 它是素数集合中固有的元素之一。因此, 对于素数集合 P , 自然单位素数必然存在其中。否则, 素数集合就存在欠缺, 不能科学准确的反映素数的客观规律。

In order to reveal the laws of the objective world, we often study things with certain common characteristics as a whole. In general, we study the overall composition of the object collectively referred to as the element. The whole composed of these elements with common properties is called a set, referred to as a set. For example, by $0, 1, 2, 3, \dots$ The set of all non-negative integers is called the set of natural numbers (or the set of non-negative integers) and is denoted as N ; By $1, 2, 3, 4, 5, \dots$ The set of all positive integers is called the set of positive integers, or the set of non-zero natural numbers, referred to as N^* ; By $1, 2, 3, 5, 7, \dots$ The set of all prime numbers is called the set of prime numbers, denoted P , and so on. For any given set, its elements are always definite. That is, given a set, the absence of any element in that set is determined, not dispensable. For example, the natural unit, prime 1 , is one of the elements inherent in the set of prime numbers. Therefore, for the set of prime numbers P , the natural unit prime must exist in it. Otherwise, the set of prime numbers is deficient and cannot reflect the objective law of prime numbers scientifically and accurately.

一个给定集合中的元素是互不相同的。也就是说, 集合中的元素是不能重复出现的。只要构成两个集合的元素是一样的, 我们就称这两个集合是相等的。我们通常用大写字母 $A, B, C, \dots$ 表示集合, 用小写字母 $a, b, c, \dots$ 表示集合中的元素。如果 a 是集合 A 的元素, 就说 a 属于集合 A , 记作 $a \in A$; 如果 a 不是集合 A 的元素, 就说 a 不属于集合 A , 记作 $a \notin A$ 。

The elements of a given set are not the same. That is, the elements of a set cannot be repeated. Two sets are said to be equal as long as the elements that make up them are the same.

We usually use capital letters $A, B, C, \dots$ To represent a collection, use lowercase letters $a, b, c, \dots$ Represents an element in a collection. If a is a member of set A , it is said that A belongs to set a and is denoted as $a \in A$; If a is not a member of set A , it is said that A is not a member of set A , called $a \notin A$.

我们可以用自然语言描述一个集合。除此之外，还可以用以下 2 种方式表示集合：

1.列举法。把集合的元素一一列举出来，并用大括号“{ }”括起来表示集合的方法叫做列举法。例如，组成数的十进制数码集合为 A，那么

We can describe a set in natural language. In addition, collections can be represented in the following two ways:

1.Enumeration. The method of listing the elements of a set one by one and enclosing the set with braces "{ }" is called enumeration. For example, if the set of decimal digits that make up the number is A, then

$$A=\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

由 1~20 以内的所有素数组成的集合为 B，那么

The set of all prime numbers from 1 to 20 is B, then

$$B=\{1, 2, 3, 5, 7, 11, 13, 17, 19\}.$$

在所有素数由小到大依次排列的序列中，相邻素数之间没有合数间隔的三个素数组成的集合为 C，那么

In the sequence of all prime numbers from smallest to largest, the set of three primes without composite intervals between adjacent primes is C, then

$$C=\{1, 2, 3\}$$

在所有个位数含有数码 5 的正整数中，是素数的集合为 D，那么

In all positive integers with the digit 5, the set of prime numbers is D, then

$$D=\{5\}$$

2.描述法。当我们不能将集合的元素列举完时，我们可以用这个集合中元素所具有的共同特征来描述。这种用集合所含元素的共同特征表示集合的方法称为描述法。

2. Descriptive methods. When we cannot list all the elements of a set, we can describe them in terms of the common characteristics of the elements in the set. This method of representing a set in terms of the common characteristics of its elements is called a description method.

例如，任何一个奇数可以表示为 $x=2k+1$ ($k \in \mathbb{N}$) 的形式。所以，我们可以把所有奇数的集合表示为：

For example, any odd number can be expressed as $x=2k+1$ ($k \in \mathbb{N}$). So, we can represent the set of all odd numbers as:

$$E=\{x \mid x=2k+1, x \in \mathbb{Z}, k \in \mathbb{N}\}$$

式中： $\mathbb{Z}$ 表示为整数集， $\mathbb{N}$ 表示自然数集。

In the formula, $\mathbb{Z}$ represents the set of integers and $\mathbb{N}$ represents the set of natural numbers.

又例如，除 1 和它本身外，不能被整除的所有正整数，称为素数。我们可以用集合表示为：

For example, all positive integers that are not divisible except 1 and itself are called prime numbers. We can express the set as:

$$P=\{p \in \mathbb{N}^* \mid \text{除 1 和它本身外，不能被其他整数整除的正整数}\}$$

$$P=\{p \in \mathbb{N}^* \mid \text{positive integer not divisible by any other integer except 1 and itself}\}$$

式中： $\mathbb{N}^*$ 表示正整数集，或非零自然数集。

Where: $\mathbb{N}^*$ represents the set of positive integers, or the set of non-zero natural numbers.

有时，素数集合还可以表示为：

Sometimes, the set of prime numbers can also be expressed as:

$$P=\{1, 2, 3, 5, 7, 9, 11, 13, \cdots, \infty\}$$

2.2.1 集合之间的基本关系

2.2.1 Basic relationships between sets

一般地，对于两个集合 A 和 B，如果集合 A 中任意一个元素都是集合 B 中的元素，我们就说这两个集合有包含关系，称集合 A 为集合 B 的子集，记作

In general, for two sets A and B, if any element of set A is an element of set B, we say that the two sets have an inclusion relation, and call set A a subset of set B, denoted

$$A \subseteq B \text{ (or } B \supseteq A)$$

读作“A 包含于 B”(或“B 包含 A”)

Read as "A contains B" (or "B contains A")

如果集合 A 是集合 B 的子集($A \subseteq B$)，且集合 B 是集合 A 的子集($B \subseteq A$)，此时，集合 A 与集合 B 中的元素是一样的，因此，集合 A 与集合 B 相等，记作

If set A is A subset ($A \subseteq B$) of set B, and set B is A subset ($B \subseteq A$) of set A, then set A is the same as the elements in set B, so set A is equal to set B

$$A = B$$

如果集合 $A \subseteq B$ ，但存在元素 $x \in B$ ，且 $x \notin A$ ，我们称集合 A 是集合 B 的真子集，记作

If the set $A \subseteq B$, but there are elements $x \in B$, and $x \notin A$, we say set A is a proper subset of set B, denoted

$$A \subsetneq B \text{ (or } B \supsetneq A)$$

我们把不含任何元素的集合叫做空集，记作 $\emptyset$ ，并规定：空集 $\emptyset$ 是任何集合 A 的子集。

We call the set that does not contain any elements the empty set, denoting $\emptyset$, and specify that the empty set $\emptyset$ is A subset of any set A. namely

$$\emptyset \subseteq A \text{ or } A \supseteq \emptyset.$$

由上述集合之间的基本关系，我们可以得到下列结论：

From the basic relationship between the above sets, we can draw the following conclusions:

1、任何一个集合是它本身的子集。即

1. Any set is a subset of itself. namely

$$A \subseteq A \text{ or } A \supseteq A;$$

2、对于集合 A，B，C，如果 $A \subseteq B$ ，且 $B \subseteq C$ ，那么 $A \subseteq C$ 。

2. For sets A, B, C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

2.2.2 集合的基本运算

2.2.2 Basic operations of sets

1、并集。一般地，由所有属于集合 A 或者属于集合 B 的元素组成的集合，称为集合 A 与 B 的并集，记作 $A \cup B$ 。即

1. Union. Generally, A set consisting of all the elements belonging to set A or to set B is called the union of sets A and B and is denoted $A \cup B$. namely

$$A \cup B = \{x | x \in A, \text{ or } x \in B\}.$$

2、交集。一般地，由属于集合 A 且属于集合 B 的所有元素组成的集合，称为 A 与 B 的交集，记作 $A \cap B$ 。即

2. Intersection. In general, the set consisting of all elements belonging to set A and belonging to set B is called the intersection of A and B and is denoted $A \cap B$. namely

$$A \cap B = \{x | x \in A, \text{ and } x \in B\}.$$

3、全集和补集。一般地，如果一个集合含有我们所有研究问题中涉及的所有元素，那么就称这个集合为全集，通常记作 U。

3. Complete and complementary works. In general, if a set contains all of the elements involved in all of our research problems, then the set is called the complete set, usually referred to as U.

对于一个集合 A ，由全集 U 中不属于集合 A 的所有元素组成的集合 A 称为集合 A 相对于全集 U 的补集，简称集合 A 的补集，记作 $C_U A$ 。即

For A set A, the set A composed of all the elements of the total set U that do not belong to the set A is called the complement of the set A relative to the total set U, and is referred to as the complement of the set $C_U A$. namely

$$C_U A = \{x | x \in U, \text{ and } x \notin A\}.$$

2.2.3 集合中元素的个数

2.2.3 Number of elements in the collection

在研究集合时，我们会经常遇到有关集合中元素的个数问题。我们把含有限个元素的集合叫做有限集，用 card (是英文 *cardinal* 基数的缩写) 来表示有限集合 A 元素的个数。

一般地，对任意两个有限集合 A, B ，有。

When studying sets, we often encounter problems about the number of elements in a set. We call A set of finite elements a finite set, and use card (short for *cardinal*) to represent the number of elements of the finite set A. In general, for any two finite sets A, B, there is

$$\text{card}(A \cup B) = \text{card}(A) + \text{card}(B) - \text{card}(A \cap B).$$

有限集合中元素，我们可以一一数出来。而对于元素个数无限的集合，如

We can count the elements of a finite set. For a set with an infinite number of elements, such as

$$A = \{1, 2, 3, 4, 5, \dots, n, \dots\};$$

$$B = \{2, 4, 6, 8, 10, \dots, 2n, \dots\};$$

$$C = \{1, 3, 5, 7, 9, 11, \dots, 2n-1, \dots\}.$$

我们无法数出集合中元素的个数，但可以比较这些集合中元素个数的多少。

We can't count the number of elements in a set, but we can compare the number of elements in those sets.

2.3 实数和复数的分类

2.3 Classification of real and complex numbers

在数学里，我们将所有的数分类如下。如图 3 所示。

In mathematics, we classify all numbers as follows. As shown in Figure 3.

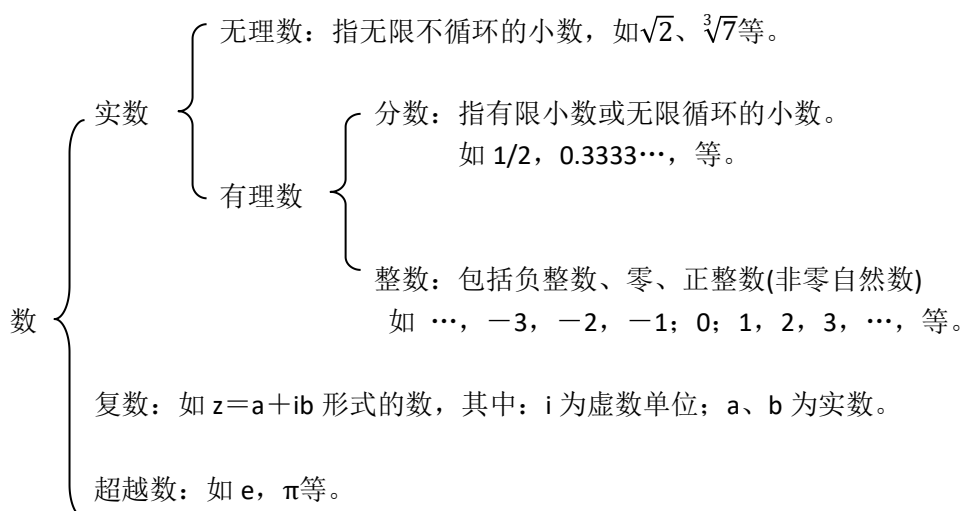


图 3：数的分类示意图

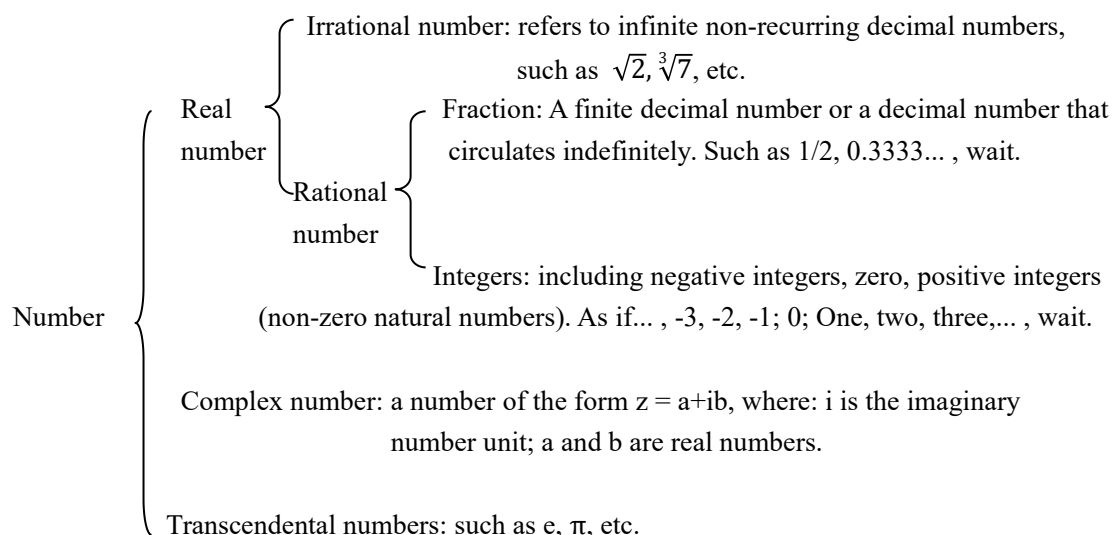


Figure 3: Schematic diagram of number classification

下面我们将进一步把数的概念拓展，在复数的范围之内研究有关素数的整数数论问题。

Next, we will further expand the concept of number, and study the integer number theory of prime numbers in the range of complex numbers.

2.4 复数

2.4 Complex Numbers

2.4.1 复数的概念

2.4.1 Concept of complex numbers

我们知道方程：We know the equation:

$$x^2 = -1$$

在实数范围之内无解。如果我们假设一个特殊的新数“i”满足 $i^2 = -1$ ，并将扩展到实数集当中，我们就可以得到另一类新数，称之为复数，记作：

There is no solution within the range of real numbers. If we assume that a special new number "i" satisfies $i^2 = -1$, and extend it to the set of real numbers, we can get another class of new numbers, called complex numbers, written as

$$z = a + bi$$

其中：a, b ∈ R 实数。a, b 分别叫做复数 z 的实部和虚部，记作：

Where: a, b ∈ R real numbers. a and b are called the real and imaginary parts of the complex number z, and are denoted as

$$a = \text{Re}z, b = \text{Im}z$$

由复数全体构成的集合，记作 $\mathbb{C}$ 。

The set of all complex numbers, called $\mathbb{C}$.

对于复数 $z = a + bi$ (a, b ∈ R)，当 b = 0 时，z 表示一个实数；当 b ≠ 0 时，称 z 为虚数；当 a = 0，且 b ≠ 0 时，称 z 为纯虚数。

For the complex number $z = a + bi$ (a, b ∈ R), when b = 0, z represents a real number; When b ≠ 0, z is called imaginary number; When a = 0 and b ≠ 0, z is called a pure imaginary number.

每一个复数 $z = a + bi$ (a, b ∈ R)，在平面直角坐标系内都存在有一个唯一点 Z(a, b) 对应，我们称这个平面为复平面。实数在复平面上所对应的点都在 x 轴上，所以也称 x 轴为实轴(Re)。

纯虚数在复平面上所对应的点都在 y 轴上，所以也称 y 轴为虚轴(Im)。

For every complex number $z = a+bi$ ($a, b \in \mathbb{R}$), there exists a unique point $Z(a, b)$ in the rectangular coordinate system of the plane, which is called the complex plane. The points corresponding to the real numbers in the complex plane are on the X-axis, so the X-axis is also called the real axis (Re). The points corresponding to pure imaginary numbers in the complex plane are all on the Y-axis, so the Y-axis is also called the imaginary axis (Im).

两个复数 $z_1 = a+bi$, $z_2 = c+di$ ($a, b, c, d \in \mathbb{R}$)相等的充分必要条件是：

The necessary and sufficient conditions for two complex numbers $z_1 = a+bi$, $z_2 = c+di$ ($a, b, c, d \in \mathbb{R}$) to be equal are:

$$a=c, b=d$$

也就是它们的实部与虚部对应相等，并且复数相等当且仅当它们在复平面上的对应的点重合。两个复数能够比较大小当且仅当它们均为实数。

That is, their real and imaginary parts correspond to each other, and their complex numbers are equal if and only if they coincide at their corresponding points on the complex plane. Two complex numbers can compare magnitudes if and only if they are both real numbers.

复数有如下 4 种表示形式：代数形式，几何形式，三角形式和指数形式。

Complex numbers can be expressed in four forms: algebraic, geometric, trigonometric, and exponential.

我们熟悉的 $z = a+bi$ ($a, b \in \mathbb{R}$)，称为复数 z 的代数形式。

The familiar $z = a+bi$ ($a, b \in \mathbb{R}$) is called the algebraic form of the complex number z .

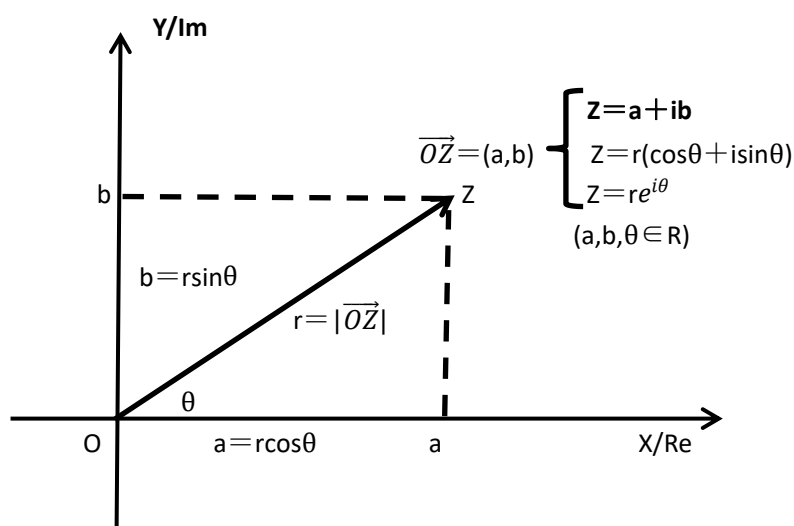


图 4：复数的几种表示形式与几何意义

Figure 4: Several representations and geometric meanings of complex numbers

复数的代数形式 $z=a+bi$ 可与复平面上的位置向量 $\overrightarrow{OZ}=(a, b)$ 一一对应, 这称为复数的几何形式。复数的几何形式将复数 $z=a+bi$ 对应于复平面上的一点 $Z(a, b)$ 。

The algebraic form $z=a+bi$ of complex numbers corresponds one-to-one to the position vector $\overrightarrow{OZ}=(a, b)$ on the complex plane, which is called the geometric form of complex numbers. The geometric form of a complex number corresponds to a point $Z(a, b)$ on the complex plane.

现设点 Z 的极坐标为 (r, θ) , 那么就有

Let's say the polar coordinate of point Z is (r, θ) , then we have

$$a=r\cos\theta, \quad b=r\sin\theta,$$

因而复数 $Z(a, b)$ 也可以表示为:

Therefore, the complex number $Z(a, b)$ can also be expressed as:

$$Z=r(\cos\theta+isin\theta)$$

其中 $r=|z|=\sqrt{a^2+b^2}$, 称复数 z 的模, θ 称为复数 z 的辐角, 并记作 $\theta=\text{Arg}z$ 。这称为复数 z 的三角形式。

Where $r=|z|=\sqrt{a^2+b^2}$ is called the module of the complex number z , and θ is called the argument of the complex number z and denoted $\theta=\text{Arg}z$. This is called the triangular form of the complex number z .

由基本的三角函数知识可知, 如果 θ 为复数 z 的一个辐角, 那么 $\text{Arg}z=\theta+2k\pi$ ($k\in\mathbb{Z}$ 为整数) 也是 z 的一个辐角。因此复数 z 的辐角有无穷多个。在复数 z 的辐角中, 仅有一个满足 $0\leq\theta<2\pi$, 这时 θ 称为复数 z 的轴角主值, 记作 $\theta=\text{arg}z$ 。

From basic trigonometry we know that if θ is an argument of z , then $\text{Arg}z=\theta+2k\pi$ ($k\in\mathbb{Z}$ is an integer) is also an argument of z . So the argument of the complex number z is infinite. In the argument of the complex number z , only one satisfies

$0\leq\theta<2\pi$, then θ is called the principal value of the axial Angle of the complex number z , denoted $\theta=\text{arg}z$.

如果我们记 $e^{i\theta}=\exp(i\theta)=\cos\theta+isin\theta$ ($\theta\in\mathbb{R}$), 则复数 z 的三角形式还可以表示为:

If we remember $e^{i\theta}=\exp(i\theta)=\cos\theta+isin\theta$ ($\theta\in\mathbb{R}$), then the triangular form of the complex number z can also be expressed as:

$$z=re^{i\theta},$$

这称为复数 z 的指数形式。复数的几种表示形式与几何意义, 如图 4 所示。

This is called the exponential form of the complex number z . Several expressions and geometric meanings of complex numbers. As shown in Figure 4.

2.4.2 复数的运算

2.4.2 Operation of complex numbers

(一)复数的四则运算

(1) The four operations of complex numbers

对于复数 $z_1=a+bi$, $z_2=c+di$ ($a, b, c, d\in\mathbb{R}$),

For the complex number $z_1=a+bi$, $z_2=c+di$ ($a, b, c, d\in\mathbb{R}$),

复数 z_1, z_2 的加法运算 z_1+z_2 定义为:

The addition operation z_1+z_2 for the complex numbers z_1, z_2 is defined as:

$$z_1+z_2=(a+bi)+(c+di)=(a+c)+(b+d)i;$$

减法运算 z_1-z_2 定义为:

The subtraction operation z_1-z_2 is defined as:

$$z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i;$$

乘法运算 $z_1 \cdot z_2$ 定义为:

Multiplication $z_1 \cdot z_2$ is defined as:

$$z_1 \cdot z_2 = (a + bi)(c + di) = (ac - bd) + (ad + bc)i;$$

除法运算 $z_1 \div z_2$ 定义为:

Division operation $z_1 \div z_2$ is defined as:

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{a + bi}{c + di} \left(= \frac{(a + bi)(c - di)}{(c + di)(c - di)} \right) \\ &= \frac{(ac + bd) + (-ad + bc)i}{c^2 + d^2} \end{aligned}$$

这里 c, d 不全为 0, 即 $z_2 \neq 0$ 。

Here c and d are not all 0, that is, $z_2 \neq 0$.

如果我们将复数 z_1, z_2 表示为复数 z 的指数形式, 即

If we represent the complex number z_1, z_2 as the exponential form of the complex number z .

Namely.

$$z_1 = r_1 e^{i\theta_1}, \quad z_2 = r_2 e^{i\theta_2}.$$

其中 $r_1, r_2 \geq 0, \theta_1, \theta_2 \in \mathbb{R}$.

Where $r_1, r_2 \geq 0, \theta_1, \theta_2 \in \mathbb{R}$.

依据两角和的正弦、余弦公式, 有

According to the sine and cosine formula of the sum of two angles, there is

$$\begin{aligned} z_1 \cdot z_2 &= r_1 e^{i\theta_1} \cdot r_2 e^{i\theta_2} \\ &= r_1 (\cos\theta_1 + i\sin\theta_1) \cdot r_2 (\cos\theta_2 + i\sin\theta_2) \\ &= r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)] \\ &= r_1 r_2 e^{i(\theta_1 + \theta_2)} \end{aligned}$$

即 That is,

$$r_1 e^{i\theta_1} \cdot r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1 + \theta_2)}.$$

据此, 我们也可以得出复数乘方运算的指数形式:

From this, we can also derive the exponential form of the complex multiplication operation:

$$(re^{i\theta})^n = r^n e^{in\theta}.$$

这里 $n \in \mathbb{Z}$ 为整数。

Here $n \in \mathbb{Z}$ is an integer.

(二)复数四则运算的运算规律

(2) Operation rules of complex four operations

复数的加法运算满足交换律与结合律成立, 即对任意复数 z_1, z_2, z_3

The addition operation of complex numbers satisfies the commutative law and associative law, that is, for any complex number z_1, z_2, z_3

$$\begin{aligned} z_1 + z_2 &= z_2 + z_1 \\ (z_1 + z_2) + z_3 &= z_1 + (z_2 + z_3) \end{aligned}$$

成立。Established.

复数的乘法运算满足交换律与结合律成立，即对任意复数 z_1, z_2, z_3

Multiplication of complex numbers satisfies the commutative law and associative law, that is, for any complex number z_1, z_2, z_3

$$\begin{aligned} z_1 \cdot z_2 &= z_2 \cdot z_1 \\ (z_1 \cdot z_2) \cdot z_3 &= z_1 \cdot (z_2 \cdot z_3) \end{aligned}$$

成立。Established.

复数的乘法对加法运算分配律成立，即对任意复数 z_1, z_2, z_3

Multiplication of complex numbers holds for the distributive law of addition, that is, for any complex number z_1, z_2, z_3

$$z_1(z_2 + z_3) = z_1 \cdot z_2 + z_1 \cdot z_3$$

(三)复数的乘方与开方运算

(3) Power and root operations of complex numbers

我们假设 Let's assume

$$z_0 = r_0 e^{i\theta_0}$$

这里 $r_0 > 0$ 是复数 z_0 的模， $\theta_0 \in \mathbb{R}$ 是复数 z_0 的辐角。设含有未知变量复数

Here $r_0 > 0$ is the magnitude of the complex z_0 and $\theta_0 \in \mathbb{R}$ is the argument of the complex z_0 .

Set to contain unknown variable complex

$$z = r e^{i\theta}$$

则方程可化为：

Then the equation can be reduced to:

$$(r e^{i\theta})^n = r^n e^{in\theta} = r_0 e^{i\theta_0}$$

即 That is,

$$r^n e^{in\theta} = r_0 e^{i\theta_0}$$

比较等式两边的模，就有 $r^n = r_0$ ，所以

Comparing the modules on both sides of the equation, we get $r^n = r_0$, so

$$r = \sqrt[n]{r_0}$$

比较等式两边的辐角，就有

If you compare the arguments on both sides of the equation, you get

$$n\theta = \theta_0 + 2k\pi.$$

这里 $k \in \mathbb{Z}$ 为整数。所以

Here $k \in \mathbb{Z}$ is an integer. So

$$\theta = \frac{\theta_0 + 2k\pi}{n}$$

那么 So

$$Z = \sqrt[n]{r_0} e^{\frac{\theta_0 + 2k\pi}{n} i}.$$

其中， $k \in \{0, 1, 2, 3, \dots, n-1\}$

Where $k \in \{0, 1, 2, 3, \dots, n-1\}$

(四)共轭复数及运算性质

(4) Conjugate complex numbers and operation properties

对复数 $z = a + bi$ ($a, b \in \mathbb{R}$)，我们定义共轭复数为 $z = a - bi$ ($a, b \in \mathbb{R}$)。其代数运算性质如下：

For the complex number $z = a+bi$ ($a, b \in \mathbb{R}$), we define the conjugate complex number $\bar{z} = a-bi$ ($a, b \in \mathbb{R}$). Its basic properties are as follows.

- 1、 $\overline{\bar{z}} = z$;
- 2、 $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$;
- 3、 $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$;
- 4、 $\overline{z^n} = (\bar{z})^n$, n is any integer; n 为任意整数;
- 5、 $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$ ($z_2 \neq 0$)
- 6、 $z + \bar{z} = 2\text{Re}z$, $z - \bar{z} = 2i\text{Im}z$;
- 7、 $z \in \mathbb{R} \Leftrightarrow \bar{z} = z$, $z \in \{bi | b \in \mathbb{R}\} \Leftrightarrow \bar{z} = -z$.

(The necessary and sufficient condition for z to be a real number is $\bar{z} = z$; The necessary and sufficient condition for z to be imaginary is $\bar{z} = -z$ and $z \neq 0$.)

(五)复数的模及运算

(5) Modulus r and operations of complex numbers

从几何上说, 复数 z 的模指复平面上 z 的对应点 Z 与原点 O 的距离, 若 $z = a+bi$ ($a, b \in \mathbb{R}$), 则:

Geometrically, the module r of the complex number z refers to the distance between the corresponding point z of Z in the complex plane and the origin O . If $z = a+bi$ ($a, b \in \mathbb{R}$), then:

$$r = |z| = |\vec{OZ}| = \sqrt{a^2 + b^2}.$$

关于复数的模有以下的代数运算性质。

The module r with respect to complex numbers has the following algebraic properties.

1. $|\bar{z}| = |z|$, $z \cdot \bar{z} = |z|^2$;
2. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
3. $|z^n| = |z|^n$, n is any integer; n 为任意整数;
4. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ ($z_2 \neq 0$);

$$5. |z| \geq |\text{Re}z|, |z| \geq |\text{Im}z|;$$

6. The Triangle inequality. 三角不等式

$$||z_1| - |z_2|| \leq |z_1 \pm z_2| \leq |z_1| + |z_2|$$

其中左侧等号成立 $\Leftrightarrow \vec{OZ_1}, \vec{OZ_2}$ 反向; 右侧等号成立 $\Leftrightarrow \vec{OZ_1}, \vec{OZ_2}$ 同向。这里 Z_1, Z_2 指复数 z_1, z_2 在复平面上的对应点。

“ $\Leftrightarrow$ ” 表示命题成立。

Among them, the left equal sign is $\Leftrightarrow \vec{OZ_1}$ and $\vec{OZ_2}$ is reversed; Right side equals $\Leftrightarrow \vec{OZ_1}, \vec{OZ_2}$ in the same direction. Here Z_1, Z_2 refers to the complex number z_1, z_2 the corresponding point on the complex plane.

2.4.3 复数运算的几何意义。

2.4.3 Geometric meaning of complex operation.

1、复数加减法的几何意义

1. Geometric meaning of complex addition and subtraction

设复数 z_1, z_2 分别对应于复平面上的位置向量为: $\vec{OZ_1} = (a, b)$, $\vec{OZ_2} = (c, d)$ 。由于

Let the complex number z_1, z_2 correspond to the position vectors on the complex plane as: $\vec{OZ_1} = (a, b)$, $\vec{OZ_2} = (c, d)$. On account of

$$z_1 \pm z_2 = (a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i;$$

可知, $z_1 \pm z_2$ 在复平面上的对应点 Z 的坐标是 $(a \pm c, b \pm d)$ 。即有如下向量等式:

It can be seen that the coordinate of the corresponding point Z of $z_1 \pm z_2$ on the complex plane is $(a \pm c, b \pm d)$. We have the following vector equation:

$$\overrightarrow{OZ} = \overrightarrow{OZ_1} \pm \overrightarrow{OZ_2}$$

如图 5 所示。

As shown in Figure 5.

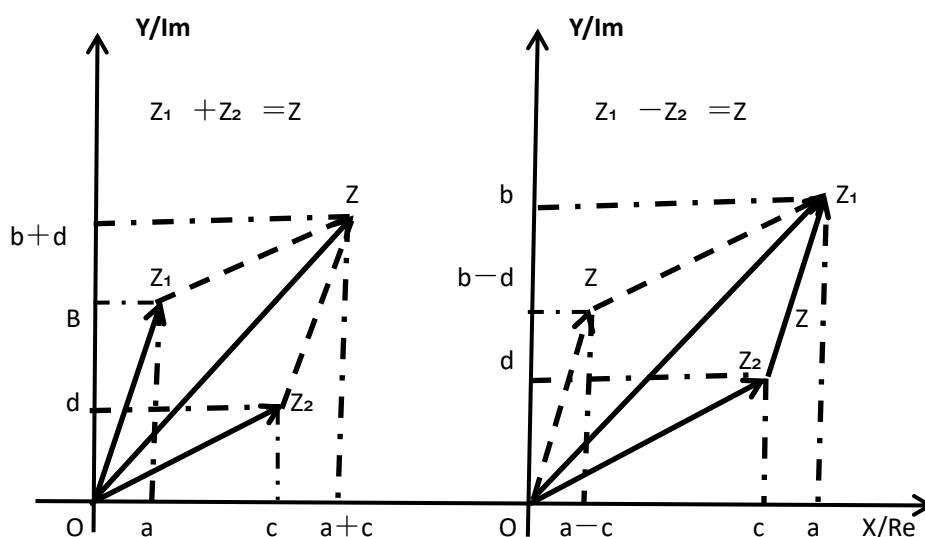


Figure 5: Geometric meaning of addition and subtraction of complex numbers

图 5: 复数加减法的几何意义

2、复数乘除法的几何意义

2. Geometric meaning of multiplication and division of complex numbers

将复数 z_1, z_2 表示为指数形式, $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$, 则有

If we express the complex number z_1, z_2 in exponential form, $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$, we have

$$z_1 z_2 = r_1 e^{i\theta_1} \cdot r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

由此, 复数 $z_1 z_2$ 对应的向量就是把 $\overrightarrow{OZ_1}$ 绕点 O 顺时针旋转角 θ_2 (这里当 $\theta_2 < 0$ 时, 逆时针旋转角 θ_2 应当理解为顺时针旋转角 $|\theta_2|$) 在将它的长度变为原来的 r_2 倍。

Thus, the vector corresponding to the complex number $z_1 z_2$ is the rotation of the Angle θ_2 by $\overrightarrow{OZ_1}$ clockwise about the point O (here, the counterclockwise Angle θ_2 should be understood as the clockwise Angle $|\theta_2|$ when $\theta_2 < 0$), changing its length by r_2 times its original length.

根据复数的除法运算是乘法运算的逆运算，当 $z_2 \neq 0$ 时，可以得到

According to the division operation of complex numbers is the inverse of the multiplication operation,

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \left(\frac{r_1}{r_2}\right) e^{i(\theta_1 - \theta_2)}$$

当 $z_2 \neq 0$ 时，when $z_2 \neq 0$, we get:

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

故当 $z_2 \neq 0$ 时， z_1/z_2 对应的向量就是把 $\overrightarrow{OZ_2}$ 绕点 O 顺时针旋转角 θ_2 ，再将它的模变为原来的 $1/r_2$ 倍。

Therefore, when $z_2 \neq 0$, the corresponding vector of z_1/z_2 is to rotate the Angle θ_2 clockwise by $\overrightarrow{OZ_2}$ around the point O, and then change its modulus to $1/r_2$ times the original.

3、共轭复数的几何意义

3、Geometric meaning of conjugate complex numbers

对复数 $z = a + bi$ ($a, b \in \mathbb{R}$)，我们定义共轭复数为 $\bar{z} = a - bi$ ($a, b \in \mathbb{R}$)。可得到对于指数形式的 $z = re^{i\theta}$ ($r \geq 0, \theta \in \mathbb{R}$)，其共轭复数 $\bar{z} = re^{-i\theta}$ 。共轭复数的几何意义是：

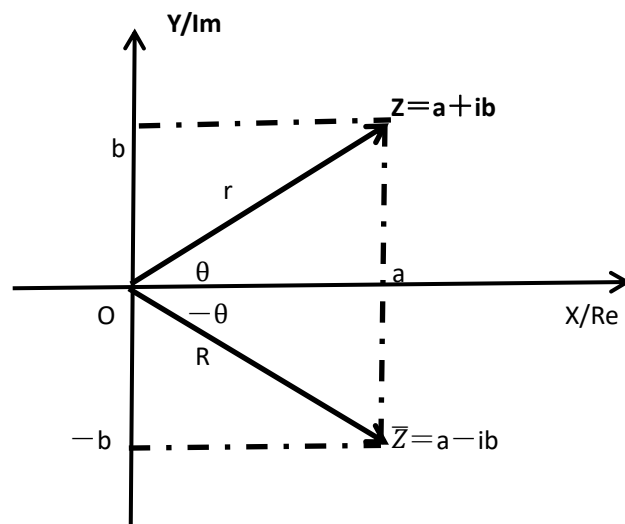


图 6：共轭复数的几何意义

Figure 6: Geometric meaning of conjugate complex numbers

For the complex number $z = a+bi$ ($a, b \in \mathbb{R}$), we define the conjugate complex number $\bar{z} = a-bi$ ($a, b \in \mathbb{R}$). For the exponential form $z = re^{i\theta}$ ($r \geq 0, \theta \in \mathbb{R}$), the conjugate complex number $\bar{z} = re^{-i\theta}$ can be obtained. The geometric meaning of conjugate complex numbers is:

复数 $\bar{z}$ 的对应点关于实轴对称，它们的模等长，辐角互为相反数，如图 6 所示。

The corresponding points of complex number $\bar{z}$ are symmetric about the real axis, their modules are equal in length, and their arguments are negative to each other. As shown in Figure 6.

2.5 高斯整数

2.5 Gauss Integers

定义 1. 我们将形如

Definition 1. We will look like

$$\alpha = a+bi \quad (a, b \in \mathbb{N}^*)$$

复数。我们称之为高斯整数，或者复数整数。通常我们用希腊字母来表示高斯整数，例如 α, β, γ 和 δ 等。

Plural. We call them Gaussian integers, or complex integers. Usually we use Greek letters to represent Gaussian integers, such as α, β, γ and δ .

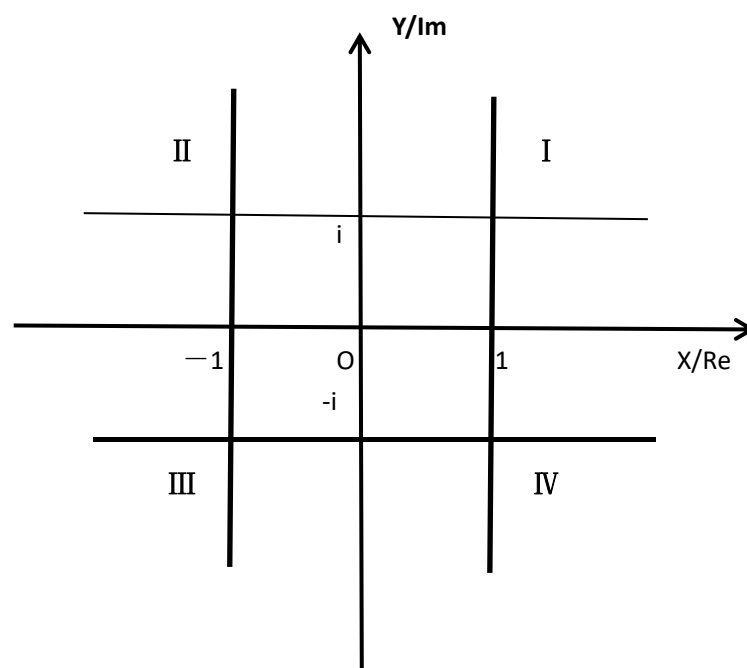


图 7：高斯整数分布示意图

Figure 7: Gaussian integer distribution diagram

若 $\gamma = a+bi$ 是高斯整数，则它满足如下方程的代数整数。

If $\gamma = a+bi$ is a Gaussian integer, then it satisfies the algebraic integer of the following equation.

$$\gamma^2 - 2a\gamma + (a^2 + b^2) = 0$$

高斯整数与普通的整数有许多共同的性质。任意两个高斯整数的和、差、积仍然是高斯整数。因此，高斯整数集是复数集的一个子集。

Gaussian integers share many properties with ordinary integers. The sum, difference, and product of any two Gaussian integers are still Gaussian integers. Therefore, the set of Gaussian integers is a subset of the set of complex numbers.

如同一般整数，我们称 1 和 -1 是整数仅有的两个单位数。在高斯整数集，我们则称 1、-1、i 和 -i 为单位高斯整数。如果高斯整数 α 只能被它自身和单位高斯整数整除，则称 α 为高斯素数，或者复数素数。所以，高斯整数总是分布在 $\text{Re}z \geq 1$ 和 $\text{Re}z \leq -1$ 之外的复平面内(不包括 $-1 < \text{Re}z < 1$ 的范围)，或者在 $\text{Im}z \geq 1$ 和 $\text{Im}z \leq -1$ 之外的复平面内(不包括 $-1 < \text{Im}z < 1$ 的范围)。如图 7 所示。

As with integers, we say that 1 and -1 are the only two units of an integer. In the set of Gaussian integers, we call 1, -1, i, and -i the unit Gaussian integers. If the Gaussian integer α is divisible only by itself and the unit Gaussian integer, α is called a Gaussian prime, or complex prime. Therefore, Gaussian integers are always distributed in the complex plane outside $\text{Re}z \geq 1$ and $\text{Re}z \leq -1$ (excluding the range of $-1 < \text{Re}z < 1$), or in the complex plane outside $\text{Im}z \geq 1$ and $\text{Im}z \leq -1$ (excluding the range of $-1 < \text{Im}z < 1$). As shown in Figure 7.

定义 2. 设 α 和 β 是高斯整数。我们称 α 整除 β 是指存在一个高斯整数 γ 使得 $\beta = \alpha\gamma$ 。若 α 整除 β ，则记作 $\alpha|\beta$ ；若 α 不整除 β ，则记作 $\alpha \nmid \beta$ 。

Definition 2. Let α and β be Gaussian integers. We say α divisible β when there is a Gaussian integer γ that makes $\beta = \alpha\gamma$. If α divides β , it is called $\alpha|\beta$ and if α does not divide β , it is called $\alpha \nmid \beta$.

对于任意高斯整数 $a+bi$ ，均有一 $i|(a+bi)$ 。这是因为只要 a, b 为整数，就有 $a+bi = -i(-b+ai)$ 。除了一 i 之外，能够整除任意一个高斯整数的还有 1，-1 和 i 。因此，高斯整数的单位分别为 1，-1， i 和 $-i$ 四个。所以对于任意一个高斯整数 β 来说，就有 4 个与它相伴的高斯整数，它的全部相伴是高斯整数 $\beta, -\beta, i\beta$ 和 $-i\beta$ 。

For any Gaussian integer $a+bi$, there is $-i|(a+bi)$. This is because as long as a and b are integers, there is $a+bi = -i(-b+ai)$. In addition to $-i$, there are 1, -1, and i that divide any Gaussian integer. Therefore, the units of Gaussian integers are 1, -1, i , and $-i$ respectively. So for any Gaussian integer, there are four associated Gaussian integers, all of which are associated with the Gaussian integers $\beta, -\beta, i\beta$ and $-i\beta$.

定义 3. 若复数 $z=x+yi$ ，则有复数的模 $r=\sqrt{x^2+y^2}$ 。因此，我们定义 $x+yi$ 的范数为：

Definition 3. If the complex number $z = x+yi$, then there is a complex number of modules $r = \sqrt{x^2+y^2}$. Therefore, we define the norm of $x+yi$ as:

$$N(x+yi) = x^2 + y^2$$

我们将每个复数 $x+yi$ 与平面上的点坐标 (x, y) 等同起来，特别地，从 $x+yi$ 到原点 o 的距离为 $\sqrt{x^2+y^2}$ 。因此，从这个意义上看，复数 α 的范数是复数的一种几何度量，它反映了复数 α 的大小。范数具有非常重要的代数性质，即积的范数等于范数的积。为此，使得范数在研究高斯整数时，成为是一种非常重要的有用工具。

We equate each complex number $x+yi$ with a point coordinate (x, y) on the plane, and in particular,

the distance from $x+yi$ to the origin o is $\sqrt{x^2 + y^2}$. Therefore, in this sense, the norm of the complex number α is a geometric metric of the complex number, which reflects the magnitude of the complex number α . The norm has a very important algebraic property, that is, the norm of the product is equal to the product of the norm. For this reason, norm becomes a very important and useful tool in the study of Gaussian integers.

范数的几个重要性质如下所述。

Several important properties of norms are described below.

把复数映成非负实数的范数函数 N 满足下列性质：

- 1、对任意复数 z , $N(z)$ 是非负实数。
- 2、对任意复数 z 和 w , $N(zw) = N(z)N(w)$ 。
- 3、 $N(zw) = 0$ 当且仅当 $z = 0$ 。

The norm function N that maps complex numbers to non-negative real numbers satisfies the following properties:

1. For any complex number z , $N(z)$ is non-negative real.
2. For any complex numbers z and w , $N(zw) = N(z)N(w)$.
3. $N(zw) = 0$ if and only if $z = 0$.

2.6 高斯素数

2.6 Gauss prime numbers

整数集合中的素数我们称之为有理素数。或者普通素数。

The primes in the set of integers are called rational primes. Or ordinary prime numbers.

定义 4. 若非零高斯整数 π 不是单位，而且只能够被单位和它的相伴整除，则称之为高斯素数。

Definition 4. A non-zero Gaussian integer π is called a Gaussian prime if it is not a unit and is divisible only by the unit and its associates.

根据以上定义，可知一个高斯整数是素数的当且仅当它正好有 8 个因子——4 个单位和它的 4 个相伴，即 $1, -1, i, -i, \pi, -\pi, i\pi$ 和 $-i\pi$ 。既不是单位也不是素数的高斯整数必有多于 8 个相异因子。因此，有些有理素数仍然是高斯素数，但是有些就不再是高斯素数。

From the above definition, we know that A Gaussian integer is prime if and only if it has exactly eight factors - four units and its four companions, namely $1, -1, i, -i, \pi, -\pi, i\pi$, and $-i\pi$. A Gaussian integer that is neither a unit nor a prime number must have more than eight distinct factors. Thus, some rational primes are still Gaussian primes, but some are no longer.

那么我们如何判断一个高斯整数是否为素数。

So how do we determine whether a Gaussian integer is prime or not.

若 π 高斯整数，而且 $N(\pi) = p$ ，其中 p 是有理素数，则 π 和 $\bar{\pi}$ 是高斯素数，而 p 不是高斯素数。

If the integers are Gaussian and $N(\pi) = p$, where p is a rational prime, then π and $\bar{\pi}$ are Gaussian primes and p is not.

这里假设 $\pi = \alpha\beta$ ，其中 α, β 是高斯整数，则 $N(\pi) = N(\alpha\beta) = N(\alpha)N(\beta)$ ，因此 $P = N(\alpha)N(\beta)$ 。由于 $N(\alpha)$ 和 $N(\beta)$ 是正整数，所以 $N(\alpha) = 1$ 且 $N(\beta) = p$ ，或者 $N(\alpha) = p$ 且 $N(\beta) = 1$ 。由于 α 是单位，或 β 是单位。这就意味着 π 不能分解成两个非单位的高斯整数的乘积，因此它必然是一个高斯素数。根据有关高斯素数的性质，我们可以分类如下。

$\pi = \alpha\beta$ is assumed here, where α, β is a Gaussian integer, then $N(\pi) = N(\alpha\beta) = N(\alpha)N(\beta)$, and therefore $P = N(\alpha)N(\beta)$. Since $N(\alpha)$ and $N(\beta)$ are positive integers, $N(\alpha) = 1$ and $N(\beta) = p$, or $N(\alpha) = p$

and $N(\beta) = 1$. Because α units, or β units. This means that π cannot be decomposed into the product of two non-unit Gaussian integers, so it must be a Gaussian prime number. According to the properties of Gaussian primes, we can classify them as follows.

(1) $1+i$ 为高斯素数;

(1) $1+i$ is a Gaussian prime number;

(2) 设 P 是普通素数且 $P \equiv 3 \pmod{4}$, 则 P 是高斯素数。

(2) Let P be an ordinary prime and $P \equiv 3 \pmod{4}$, then P is a Gaussian prime.

(3) 设 P 是普通素数且 $P \equiv 1 \pmod{4}$, 将 P 表成两平方数之和 $P = u^2 + v^2$, 则 $u+vi$ 是高斯素数。以上性质分类证明如下。

(3) Let P be an ordinary prime and $P \equiv 1 \pmod{4}$, and table P as the sum of two square numbers $P = u^2 + v^2$, then $u+vi$ is a Gaussian prime. The above property classification is proved as follows.

因为每个高斯素数等于一个单位数(± 1 或者 $\pm i$)乘以形式(1), 或(2), 或(3)中的一个高斯素数。如果 $N(\alpha)$ 是普通素数, 则 α 必为高斯素数。类别(1)中的数 $1+i$, 具有范数 2, 所以它是高斯素数。类似地, 类别(3)中的数 $u^2 + v^2$ 具有范数 $u^2 + v^2 = p$, 所以它们都是高斯素数。

Because each Gaussian prime is equal to a unit number (± 1 or $\pm i$) multiplied by a Gaussian prime of the form (1), or (2), or (3). If $N(\alpha)$ is an ordinary prime, then α must be a Gaussian prime. The number $1+i$ in the class (1) has the norm 2, so it is a Gaussian prime. Similarly, the numbers $u^2 + v^2$ in category (3) have the norm $u^2 + v^2 = p$, so they are all Gaussian prime numbers.

下面证明类别(2), 设 $\alpha = p$ 为普通素数且 $P \equiv 3 \pmod{4}$ 。如果 α 能表成两个高斯整数的乘积, 比如 $(a+bi)(c+di) = \alpha$, 则通过取范数可得:

The following proves class (2), let $\alpha = p$ be an ordinary prime and $P \equiv 3 \pmod{4}$. If α can be expressed as the product of two Gaussian integers, such as $(a+bi)(c+di) = \alpha$, then by taking the norm:

To prove category (2), let's say $\alpha = p$. If it can be expressed as the product of two Gaussian integers, such as $(a+bi)(c+di) = B$, then by taking the norm:

$$(a^2 + b^2)(c^2 + d^2) = N(\alpha) = P^2$$

为得到非平凡的因式分解, 我们需要解方程。

To get a nontrivial factorization, we need to solve the equation.

$$a^2 + b^2 = P, \quad c^2 + d^2 = P$$

根据两平方数之和定理, 由于 $P \equiv 3 \pmod{4}$, 所以 P 不能表成两平方数之和。因此上面两个方程无解。因为每个高斯素数必属于三类中的一类。故 P 不能分解, 从而 P 为高斯素数。

□

According to the sum of two square numbers theorem, since $P \equiv 3 \pmod{4}$, P cannot be represented as the sum of two square numbers. Therefore, the above two equations have no solution. Because every Gaussian prime must belong to one of three categories. Therefore, P cannot be decomposed and thus P is a Gaussian prime number. □

至此, 有关素数扩展到复数范围内的研究, 这里作者将不做长篇论述。这里仅就高斯整数和高斯素数作简单的概念性描述, 借此研究普通素数规律。因此, 根据以上关于高斯整数和高斯素数的定义, 我们可知高斯素数在复平面内的分布是关于实轴上下对称分布, 或者是关于虚轴左右对称分布的。为此, 本文仅就正整数素数, 在复数平面第一象限的对应情形进行论述。

So far, the study on the extension of prime numbers to the range of complex numbers, here

the author will not do a long discussion. This paper only gives a simple conceptual description of Gaussian integers and Gaussian primes to study the law of ordinary prime numbers. Therefore, according to the above definition of Gaussian integers and Gaussian primes, we know that the distribution of Gaussian primes in the complex plane is symmetric about the real axis, or symmetric about the imaginary axis. Therefore, this paper only discusses the corresponding cases of positive integer primes in the first quadrant of the complex plane.

§ 3 有序素数分布规律和原理的发现

§ 3. The discovery of the distribution law and principle of ordered prime numbers

3.1 定义

3.1 Definition

定义 1. 素数集合

Definition 1. Set of prime numbers

除非零自然单位数 1 和它本身以外,不能再被其他任何正整数整除的正整数构成的集合,即称为素数集合,记作 P , 或者 $\{P_m\}$ 。

The set of positive integers that can no longer be divisible by any other positive integers except the zero natural unit number 1 and itself is called the Set of Prime Numbers, denoted P , or $\{P_m\}$.

根据集合的概念,对于给定的任意集合,其集合的属性是固有的,构成集合的元素总是确定的。也就是说,给定一个集合,那么任何一个元素在不在这个集合中是已经确定了,并非可有可无。它是不以人的主观意志为转移的,否则该素数集合就不能客观地反映集合的固有属性和规律。比如,任意正整数的单位数 1,它是素数集合中固有的元素之一。并且是 3 个非常特殊的重要素数当中的其中一个,是所有非零自然数之源和最小单元。它与偶素数 2 是所有偶数的最小单元;奇素数 3 是所有奇数的最小单元,三者与自然数集和素数集合中都是缺一不可的。因此,对于素数集合 P ,单位素数 1 必然存在在素数集合之中,否则素数集合元素就存在欠缺,不能客观准确科学地反映素数存在的客观规律。这方面与当今主流数论理论观点不同。其理由详见下文相关论述。

According to the concept of sets, for a given arbitrary set, the properties of its set are inherent, and the elements that make up the set are always determined. That is, given a set, the absence of any element in the set is determined, not dispensable. It is independent of human subjective will, otherwise the set of prime numbers cannot objectively reflect the inherent properties and laws of the set. For example, the unit number 1 of any positive integer is one of the elements inherent in the set of prime numbers. It is one of three very special important prime numbers, the source and smallest unit of all non-zero natural numbers. It, along with the even prime number 2, is the smallest element of all even numbers; The odd prime number 3 is the smallest element of all odd numbers, and all three are indispensable in the set of natural numbers and the set of prime numbers. Therefore, for the prime number set P , the unit prime number 1 must exist in the prime number set, otherwise there is a lack of prime number set elements, can not objectively and accurately reflect the objective law of the existence of prime numbers. This aspect is different from the current mainstream number theory. The reasons for this are discussed below.

根据以上定义,本文笔者所称素数集合是:

According to the above definition, the set of prime numbers called by the author in this paper is:

$$P=\{1, 2, 3, 5, 7, 11, 13, 17, \cdots, P_m, \cdots, \infty\}$$

其中，包含自然单位数 1 在内。

It includes the natural unit number 1.

也可更加一般地表示为：

It can also be expressed more generally as:

$$P=\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, \cdots, P_m, \cdots, \infty\}$$

即有，If $P_1=1; P_2=2; P_3=3; P_4=5; P_5=7; \cdots; P_m=P; \cdots; \infty$

定义 2. 有序素数集合序列，或者有序素数数列集合。

Definition 2. Sequence of ordered primes, or sequence of ordered primes.

本论文包括自然单位数 1 在内，将所有素数按照在非零自然数中出现的先后秩序，从单位素数 1 开始由小到大进行依次排序，直至无穷大，所得到的素数数列，则称为有序素数序列，或者有序素数数列，即

In this paper, including the natural unit number 1, all prime numbers are sorted according to the order of occurrence in non-zero natural numbers, starting from the unit prime number 1 from small to large, to infinity, and the obtained prime number sequence is called a prime sequence, or a sequence of prime numbers. namely

$$P=1, 2, 3, 5, 7, 11, 13, 17, \cdots, p_m, \cdots$$

其中： m 为所有素数由小到大的排列序数， m 为非零自然数，即 $m \in N^*$ 。

Where: m is the ordinal number of all prime numbers from small to large, and m is a non-zero natural number, that is $m \in N^*$.

由于素数序列当中的每一项，都与素数序数相关。因此，我们也可以将素数序数称之为素数数列。

Since each item in the prime sequence is related to the ordinal number of the prime number. Therefore, we can also call prime ordinals prime numbers series.

除非另有说明，本文所描述的素数集包含自然单位数 1。这是本文的研究与许多学者的研究的不同之处。这也可能是数学界许多学者没有发现素数分布规律的根本原因。详见下文相关论述。

Unless otherwise noted, the set of prime numbers described in this article contains the natural unit number 1. This is the difference between the research of this paper and that of many other scholars. This may also be the fundamental reason why many scholars in the mathematical field have not found the distribution law of prime numbers. See further discussion below.

定义 3. 狭义孪生素数

Definition 3. Narrow Twin Prime Numbers

在有序非零自然数序列中，两个相邻素数间隔的合数个数仅为 1 个的两个相邻素数，称为狭义孪生素数，简称孪生素数。

In a sequence of prime numbers, two adjacent primes whose composite numbers are separated by only 1 in the sequence of non-zero natural numbers are called Narrow Twin Primes, referred to as twin primes.

定义 4. 相邻素数对，或称广义孪生素数

Definition 4. Pairs of Adjacent Prime Numbers, or Generalized Twin Prime Numbers

在非零自然数序列当中，两个相邻素数间隔的合数个数为 1 个或者以上奇数的两个素数，称为相邻素数对，或广义孪生素数。从此意义上讲，除素数 1、2、3 三个连续素数外，其他所有素数均为孪生素数。本定义所指的相邻素数对包含孪生素数，其含义比孪生素数更为广泛。因此，所有非零自然数序列，都可以看成是由相邻的两个素数其中间再插入奇数个数的构成链节，并由无穷的这种链节连接起来的。

In a sequence of non-zero natural numbers, two primes separated by a composite number of one or more odd numbers are called adjacent prime pairs, or generalized twin primes. In this sense, all prime numbers are twin primes, except for the three consecutive primes: 1, 2, and 3. This definition refers to pairs of adjacent prime numbers that contain twin primes and have a broader meaning than twin primes. Therefore, all sequences of non-zero natural numbers can be regarded as consisting of two adjacent prime numbers with an odd number inserted between them, and connected by an infinite number of such chains.

定义 5. 有序素数序列间隔合数的个数级数。

Definition 5. Number series of ordered prime sequence separated by composite numbers.

根据有序素数序列的定义，最初 3 个素数 1、2 和 3 没有合数间隔，是空集 $\emptyset$ ，为此我们定义其合数个数为 0。因此，在有序素数序列中相邻两个素数间隔合数的个数依次分别是：

According to the definition of prime sequence, the first three prime numbers 1, 2, and 3 have no composite interval and are the empty set $\emptyset$. So we define its composite number to be 0. Therefore, in a sequence of ordered prime numbers, the number of composite numbers separated by two adjacent prime numbers is, in order:

$$\{K_m\}=0, 0, 0, 1, 1, 3, 1, 3, 1, 3, 5, 1, 5, 3, \dots, \infty$$

若将所有素数间隔的合数个数，全部用“+”联接起来，并记作：

If the above composite number sequence of all prime numbers spacing, all with "+" joined, and recorded as:

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \dots + K_m + \dots + \infty \quad (3.1)$$

式中： K_m 表示两个相邻素数对所间隔的合数个数，或者在有序素数集合中，两个相邻素数对所间隔的合数元素个数。 $n>m$ ； $n, m \in \mathbb{N}^*$ 。

In the formula, K_m represents the number of composite numbers separated by two adjacent prime pairs, or the number of composite elements separated by two adjacent prime pairs in an ordered set of prime numbers. $n>m$; $n, m \in \mathbb{N}^*$.

$S_m(P_m)$ 则称为有序素数序列之间的合数个数级数，是一个为无穷发散级数。文中有时为了书写方便，则简写为 S_m 。

$S_m(P_m)$ is called a series of composite numbers between ordered prime numbers, and is an infinite divergent series. The text is sometimes shortened to S_m for ease of writing.

由于素数 1, 2 和 3 间隔合数个数为零，因此从第 4 个素数开始，上式(3.1)也可表示为：

Since prime numbers 1, 2 and 3 have zero composite numbers, starting with the fourth prime number, the above equation (3.1) can also be expressed as:

$$S_m(P_m) = \sum_{m=4}^{\infty} K_m = 1 + 1 + 3 + 1 + 3 + 1 + 3 + 5 + \dots + K_m + \dots + \infty$$

式中: Where: $n > m$; $n, m \in \mathbb{N}^*$.

因为素数个数有无穷多个,任意相邻的两个素数之间的合数个数也有无穷之多。所以,显然,以上素数间隔合数个数级数为无穷发散级数。

Since there are infinitely many prime numbers, there are also infinitely many composite numbers between any two adjacent prime numbers. As a result, Obviously, the above series of prime interval composite numbers is an infinite divergent series.

式中: K_m 为第 m 个素数间隔合数的个数,其中前 3 个素数间隔合数为零。从第 4 个素数 5 开始,任意相邻的两个素数间隔合数的个数均为奇数,即 K_m 为 1, 或 3, 或 5, 或 7, $\dots$, 或 $2n-1$ 。 m, n 为正整数。

Where: K_m is the number of the composite number of the interval of the m th prime number, where the composite number of the interval of the first 3 prime numbers is zero. Starting with the fourth prime number 5, any two adjacent primes have an odd number of composite numbers, i.e., K_m is 1, or 3, or 5, or 7, ... Or $2n-1$. n, m is a positive integer. $n, m \in \mathbb{N}^*$.

又根据以上定义,若将所有素数间隔的合数个数级数(3.1)

According to the above definition, if all prime numbers are separated by a composite number series (3.1):

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \dots + K_m + \dots + \infty$$

式中: $n > m$; $n, m \in \mathbb{N}^*$

取前 m 项和(包含第 m 个素数间隔的合数个数在内),并记作:

Where: $n > m$; $n, m \in \mathbb{N}^*$.

The first m terms and (including the number of composite numbers between the m th prime numbers), and are denoted as:

$$S_m(P_m) = \sum_{m=1}^m b_m = b_1 + b_2 + b_3 + \dots + b_m$$

笔者称 $S_m(P_m)$ 为素数间隔合数个数的前 m 项和(包含第 m 个素数间隔的合数个数在内)。即, $S_m(P_m)$ is called the partial sum of the first m terms of the infinite series. If m is 1, 2, 3, ... $S_m(P_m)$ constitutes a new sequence. That is,

$$S_1(P_1), S_2(P_2), S_3(P_3), \dots, S_m(P_m)$$

3.2 在有序素数序列中两个相邻素数之间的合数个数按奇数分布规律原理的发现。

3.2 Discovery of the principle of odd distribution of composite numbers between two adjacent primes in an ordered prime number sequence.

本文笔者将自然数为 10000 以内的所有素数,均按 1, 2, 3, 5, 7, 11, 13, 17, $\dots$, p_m , $\dots$ 由小到大依次排序,便形成一个有序素数序列集合,其中 m 为所有素数由小到大的排列序号。由此可见,每两个相邻素数之间的合数个数,在自然数序列当中间隔的合数的个数总是 1, 或 3, 或 5, 或 7, $\dots$, $(2n-1)$ 。即从素数 5 开始,任意两个相邻素数之间的合数个数在自然数序列中总是按奇数个数规律出现的。当然这一规律不仅局限于自然数 10000 以内的素数,它适合于所有非零自然数当中的素数。这就是笔者首次发现的素数分布规律原理。如图 8 所示。

In this paper, all prime numbers within 10000 natural numbers are classified as

$$1, 2, 3, 5, 7, 11, 13, 17, \dots, P_m, \dots$$

Ordering from the smallest to the largest forms a set of ordered prime sequences, where m is the ordering ordinal of all prime numbers from the smallest to the largest. It follows that for every number of composite numbers between two adjacent prime numbers, the number of composite numbers separated in the sequence of natural numbers is always 1, or 3, or 5, or 7, ..., $(2n-1)$. That is, starting with the prime number 5, the number of composite numbers between any two adjacent prime numbers always appears as odd numbers in the sequence of natural numbers. Of course, this law is not limited to prime numbers within 10000 natural numbers, it applies to all non-zero natural numbers among the primes. This is the first time that the author discovered the principle of distribution of prime numbers. As shown in Figure 8.

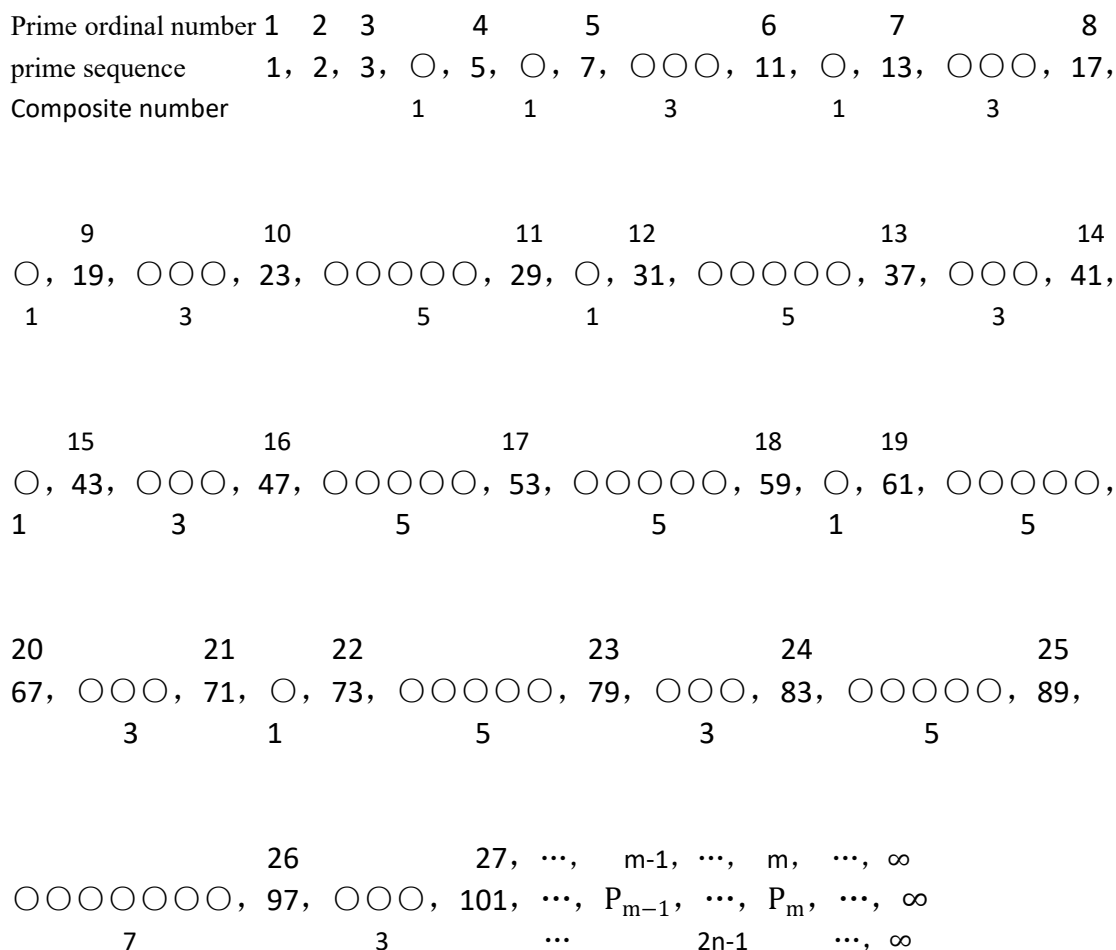


图 8: 素数分布原理和规律示意图

Figure 8: Prime number distribution principle and law diagram

从素数 5 开始, 任意两个相邻素数之间在非零自然数序列中总是间隔奇数个数的合数。图中“○”表示合数。

Starting with the prime number 5, between any Two Adjacent Prime Numbers in the sequence of non-zero natural numbers, A composite number always separated by odd numbers. The "○" in the figure represents composite numbers.

由上图 8 可见, 在素数序列中最初 3 个素数 1、2 和 3, 两个素数之间没有间隔任何合

数，为空集 $\emptyset$ 。因此我们定义，它们两个素数之间间隔的合数个数为零。从素数 5 开始，即

As can be seen from Figure 8, the first three primes 1, 2, and 3 in the prime sequence have no composite number between them, which is the empty set $\emptyset$. Therefore, we define that the number of composite numbers separated between them is zero. Starting with the prime number 5, i.e

素数 3 和 5 之间间隔的合数是 4，合数个数为 1；

素数 5 和 7 之间间隔的合数是 6，合数个数为 1；

素数 7 和 11 之间间隔的合数是 8、9 和 10，合数个数为 3；

素数 11 和 13 之间间隔的合数是 12，合数个数为 1；

素数 13 和 17 之间间隔的合数是 14、15 和 16，合数个数为 3；

素数 17 和 19 之间间隔的合数是 18，合数个数为 1；

素数 19 和 23 之间间隔的合数是 20、21 和 22，合数个数为 3；

素数 23 和 29 之间间隔的合数是 24、25、26、27 和 28，合数个数为 5；

素数 29 和 31 之间间隔的合数是 30，合数个数为 1；

素数 31 和 37 之间间隔的合数是 32、33、34、35 和 36，合数个数为 5；

素数 37 和 41 之间间隔的合数是 38、39 和 40，合数个数为 3；

素数 41 和 43 之间间隔的合数是 42，合数个数为 1；

素数 43 和 47 之间间隔的合数是 44、45 和 46，合数个数为 3；

素数 47 和 53 之间间隔的合数是 48、49、50、51 和 52，合数个数为 5；

素数 53 和 59 之间间隔的合数是 54、55、56、57 和 58；合数个数为 5；

素数 59 和 61 之间间隔的合数是 60，合数个数为 1；

素数 61 和 67 之间间隔的合数是 62、63、64、65 和 66，合数个数为 5；

.....

The composite number between the prime numbers 3 and 5 is 4 and the composite number is 1;

The composite number between the prime numbers 5 and 7 is 6 and the composite number is 1;

The composite numbers between the prime numbers 7 and 11 are 8, 9, and 10, and the composite number is 3;

The composite number between the prime numbers 11 and 13 is 12, and the composite number is 1;

The composite numbers between the prime numbers 13 and 17 are 14, 15 and 16, and the composite number is 3;

The composite number between the prime numbers 17 and 19 is 18, and the composite number is 1;

The composite numbers between the prime numbers 19 and 23 are 20, 21 and 22, and the composite number is 3;

The composite numbers between the prime numbers 23 and 29 are 24, 25, 26, 27 and 28, and the composite number is 5;

The composite number between the prime numbers 29 and 31 is 30, and the composite number is 1;

The composite numbers between the prime numbers 31 and 37 are 32, 33, 34, 35, and 36, and the composite number is 5;

The composite numbers between the prime numbers 37 and 41 are 38, 39, and 40, and the composite number is 3;

The composite number between the prime numbers 41 and 43 is 42 and the composite number is 1;

The composite numbers between the prime numbers 43 and 47 are 44, 45, and 46, and the composite number is 3;

The composite numbers between the prime numbers 47 and 53 are 48, 49, 50, 51, and 52, and the composite number is 5;

The composite numbers separated by the prime numbers 53 and 59 are 54, 55, 56, 57, and 58; The composite number is 5;

The composite number between the prime numbers 59 and 61 is 60 and the composite number is 1;

The composite numbers separated by the prime numbers 61 and 67 are 62, 63, 64, 65 and 66, and the composite number is 5;

...

由此可见，前一个素数与后一个素数两者之间间隔合数的个数均为奇数 $(2n-1)$ 。例如，间隔合数个数是 1 个，为孪生素数对(5, 7)、(11, 13)、(17, 19)、(29, 31)、 $\cdots$ 、 (p_m, p_{m+1}) ；连续间隔 3 个合数的素数对，分别为(7, 11)、(13, 17)、(19, 23)、(37, 41)、 $\cdots$ ；连续间隔 5 个合数的素数对，分别是(23, 29)、(31, 37)、(47, 53)、 $\cdots$ ；连续间隔 7 个合数的素数对，分别是(89, 97)、(359, 367)、(389, 397)、 $\cdots$ ；连续间隔 9 个合数的素数对，分别是(139, 149)、(181, 191)、(241, 251)、 $\cdots$ ；

.....

连续间隔 17 个合数的素数对，分别是(523, 541)、(1069, 1087)、(1259, 1277)、 $\cdots$ ；连续间隔 19 个合数的素数对，分别是(887, 907)、(1637, 1657) $\cdots$ 。

Thus, the number of composite numbers between the previous prime and the next prime is odd $(2n-1)$. For example,

The interval composite number is 1, which is a pair of twin prime numbers (5,7), (11, 13), (17, 19), (29,31), $\cdots$, (p_m, p_{m+1}) ;

A pair of primes with three consecutive composite numbers, respectively (7,11), (13,17), (19,23), (37, 41), $\cdots$;

A pair of prime numbers separated by 5 composite numbers in a row, namely (23,29), (31,37), (47,53), $\cdots$;

The prime pairs with 7 consecutive composite numbers are (89,97), (359,367), (389,397), $\cdots$;

A pair of prime numbers with 9 consecutive composite numbers, respectively (139,149), (181,191), (241,251), $\cdots$;

...

The prime pairs with 17 consecutive composite numbers are (523,541), (1069,1087), (1259,1277), $\cdots$;

The prime pairs with 19 consecutive composite numbers are (887,907), (1637,1657) $\cdots$.

在非零自然数 10000 内，最多连续间隔合数个数是 35 个，素数对是(9951, 9987)，仅为一对。由此可见，在有限的非零自然数序列中，连续间隔合数个数越多的越来越少。由于篇幅限制，下面仅列出非零自然数 10000 以内，连续 2 个素数间隔合数的个数。如表 2 所示。

In the non-zero natural number 10000, the maximum number of consecutive interval composite number is 35, and the prime number pair is (9951,9987), which is only a pair. It can be seen that in the finite sequence of non-zero natural numbers, the more consecutive interval composite numbers, the fewer and fewer. Due to space limitations, the following only lists the number of non-zero natural numbers within 10000, the number of consecutive prime numbers separated by composite numbers. As shown in Table 2.

表 2. 非零自然数 10000 以内任意相邻两个素数之间间隔的合数个数出现次数表

Table 2. The number of occurrences of composite numbers spaced between any Two Adjacent Prime Numbers within 10000 of non-zero natural numbers

Composite number (K_m)	Two Pairs of Adjacent Prime Numbers that first appear in sequence (p_m and p_{m-1})	frequency (1227)	Conjec- -ture
1	3 and 5, 5 and 7, 11 and 13, 17 and 19, ...	205	∞
3	7 and 11, 13 and 17, 19 and 23, ...	202	∞
5	23 and 29, 31 and 37, 47 and 53, ...	299	∞
7	89 and 97, 359 and 367, 389 and 397, ...	101	∞
9	139 and 149, 181 and 191, 241 and 251, ...	119	∞
11	199 and 211, 211 and 223, 467 and 479, ...	105	∞
13	113 and 127, 293 and 307, 317 and 331, ...	54	∞
15	1831 and 1847, 1933 and 1949, 2113 and 2129, ...	33	∞
17	523 and 541, 1069 and 1087, 1259 and 1277, ...	40	∞
19	887 and 907, 1637 and 1657, 3089 and 3109, ...	15	∞
21	1129 and 1151, 1951 and 1973, 2311 and 2333, ...	16	∞
23	1669 and 1693, 2179 and 2203, 4523 and 4547, ...	15	∞
25	2477 and 2503, 3137 and 3163, 5531 and 5557, ...	3	∞
27	2971 and 2999, 3271 and 3299, 5119 and 5147, ...	5	∞
29	4297 and 4327, 4831 and 4861, 5351 and 5381, ...	11	∞
31	5591 and 5623, ...	1	∞
33	1327 and 1361, 8467 and 8501, ...	2	∞
35	9551 and 9587, ...	1	∞
37	...	...	∞

...	...	...	∞
∞	...	...	∞

备注：本表原始素数数据来源参考美国克莱数学研究所网站。网址：<https://www.claymath.org>

Note: The original prime data source in this table refer to the Clay Mathematics Institute website.

Website: <https://www.claymath.org>

由上表 2 可见，除最初素数 1, 2, 3 外，任意两个相邻素数对之间间隔的合数个数总是奇数 1，或是 3，或者 5，或者 7，…。同时，作者据此猜测每个奇数在素数集中出现的次数都是无穷多的。

As can be seen from the above table, except for the initial prime numbers 1,2,3, the composite number between any two adjacent pairs of prime numbers is always an odd number 1, or 3, or 5, or 7,... . At the same time, the author guesses that the number of times each odd number appears in the set of prime numbers is infinite.

在素数序列中，任意相邻的 2 个素数，本文作者称为相邻素数对。素数序列就是由彼此有序的 2 个相邻素数对相互连接以至无穷构成的。用数学式表示为：

In the sequence of prime numbers, any two adjacent prime numbers are called Adjacent Prime Pairs by the authors of this paper. A sequence of prime numbers is composed of two adjacent pairs of prime numbers that are ordered to each other and connected to infinity. It can be expressed mathematically as:

$$P_{m+1} - P_m = K_m + 1 \quad (3.2)$$

上式(3.2)作者称为相邻素数对猜想。The above formula (3.2) is called the adjacent prime pair conjecture by the authors.

式中： $\{K_m\}=0, 0, 0, 1, 1, 3, 1, 3, 1, 3, 5, 1, \dots, K_m, \dots, \infty$ ；当 $m \geq 4$ 时，任意两个相邻素数对之间间隔的合数个数总是奇数 1，或是 3，或者 5，或者 7，…。是与素数数列的序数 m 依次取值相关的一个对应变量。作者称为素数的特征无穷有序数列。因此，本文作者这种表达方式(3.2)与上式(1.2)有本质的区别。

Where: $\{K_m\}=0, 0, 1, 1, 3, 1, 3, 1, 3, 3, 3, 5, 1, 5, 3, \dots, \infty$; When m is greater than or equal to 4, the number of composite numbers between any two adjacent prime pairs is always odd: 1, or 3, or 5, or 7,... . Is a corresponding variable related to the sequential value of the ordinal m of the prime number series. A series of characteristic infinite ordered numbers called prime numbers by the authors. Therefore, the author's expression (3.2) is fundamentally different from the above formula (1.2).

式(3.2)中，等式的左边为任意相邻的两个素数之差，等式右边为两个素数之间所间隔的合数个数与两个相邻素数的序数之差为 1 相加。其几何特征和意义详见下文相关论述。

In formula (3.2), the left side of the equation is the difference between any two adjacent prime numbers, and the right side of the equation is the difference between the number of composite numbers separated between the two prime numbers and the Ordinal Numbers of two adjacent prime numbers equal to 1. Its geometric features and significance are discussed below.

更多两个相邻素数对间隔合数个数及其间隔合数个数级数之和，详见表 3 所示。

More two adjacent prime pairs of interval composite numbers and the sum of interval composite number series are shown in Table 3.

(未完，待续 Unfinished, to be continued)

表 3. 非零自然数 10 万以内的素数间隔合数个数级数表 (素数 9593 个)

Table 3. Composite number of primes spacing within 100,000 of non-zero natural numbers (9593 primes)

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	1	1	3	1	3	1	
	0	0	0	1	2	5	6	9	10	
1	3	5	1	5	3	1	3	5	5	1
	13	18	19	24	27	28	31	36	41	42
2	5	3	1	5	3	5	7	3	1	3
	47	50	51	56	59	64	71	74	75	78
3	1	3	13	3	5	1	9	1	5	5
	79	82	95	98	103	104	113	114	119	124
4	3	5	5	1	9	1	3	1	11	11
	127	132	137	138	147	148	151	152	163	174
5	3	1	3	5	1	9	5	5	5	1
	177	178	181	186	187	196	201	206	211	212
6	5	3	1	9	13	3	1	3	13	5
	217	220	221	230	243	246	247	250	263	268
7	9	1	3	5	7	5	5	3	5	7
	277	278	281	286	293	298	303	306	311	318
8	3	7	9	1	9	1	5	3	5	7
	321	328	337	338	347	348	353	356	361	368
9	3	1	3	11	7	3	7	3	5	11
	371	372	375	386	393	396	403	406	411	422
10	1	17	5	9	5	5	1	5	9	5
	423	440	445	454	459	464	465	470	479	484
11	5	1	5	5	3	1	11	9	1	3
	489	490	495	500	503	504	515	524	525	528
12	5	5	1	11	3	5	7	9	7	9
	533	538	539	550	553	558	565	574	581	590
13	7	5	5	3	7	5	3	7	3	13
	597	602	607	610	617	622	625	632	635	648
14	9	11	1	9	1	3	1	9	13	3
	657	668	669	678	679	682	683	692	705	708
15	1	3	13	3	1	3	19	3	7	9
	709	712	725	728	729	732	751	754	761	770
16	7	3	5	5	13	3	5	5	7	5
	777	780	785	790	803	806	811	816	823	828
17	11	3	5	1	9	1	5	9	1	9
	839	842	847	848	857	858	863	872	873	882
18	1	5	17	3	1	3	5	5	7	5
	883	888	905	908	909	912	917	922	929	934

	0	1	2	3	4	5	6	7	8	9
19	5	21	1	9	7	9	5	5	7	11
	939	960	961	970	977	986	991	996	1003	1014
20	3	5	5	1	5	11	9	17	1	3
	1017	1022	1027	1028	1033	1044	1053	1070	1071	1074
21	5	1	5	3	1	3	11	1	5	33
	1079	1080	1085	1088	1089	1092	1103	1104	1109	1142
22	5	5	7	17	9	13	3	1	3	5
	1147	1152	1159	1176	1185	1198	1201	1202	1205	1210
23	7	3	1	5	11	9	1	3	1	3
	1217	1220	1221	1226	1237	1246	1247	1250	1251	1254
24	5	11	11	7	11	5	3	5	7	3
	1259	1270	1281	1288	1299	1304	1307	1312	1319	1322
25	7	3	13	3	5	1	3	5	1	5
	1329	1332	1345	1348	1353	1354	1357	1362	1363	1368
26	9	19	5	3	1	23	3	1	9	11
	1377	1396	1401	1404	1405	1428	1431	1432	1441	1452
27	1	9	7	5	5	5	17	5	3	1
	1453	1462	1469	1474	1479	1484	1501	1506	1509	1510
28	11	9	11	7	15	13	5	3	1	3
	1521	1530	1541	1548	1563	1576	1581	1584	1585	1588
29	1	9	11	5	5	17	1	15	1	21
	1589	1598	1609	1614	1619	1636	1637	1652	1653	1674
30	5	7	5	3	1	3	7	5	9	1
	1679	1686	1691	1694	1695	1698	1705	1710	1719	1720
31	9	13	9	5	11	1	3	1	9	11
	1729	1742	1751	1756	1767	1768	1771	1772	1781	1792
32	1	15	1	5	3	1	9	7	17	23
	1793	1808	1809	1814	1817	1818	1827	1834	1851	1874
33	3	5	7	15	1	3	7	15	1	3
	1877	1882	1889	1904	1905	1908	1915	1930	1931	1934
34	7	5	5	3	11	1	21	5	1	5
	1941	1946	1951	1954	1965	1966	1987	1992	1993	1998
35	3	5	13	5	3	1	5	3	5	11
	2001	2006	2019	2024	2027	2028	2033	2036	2041	2052
36	5	5	13	3	5	11	7	5	3	25
	2057	2062	2075	2078	2083	2094	2101	2106	2109	2134
37	17	9	7	3	5	1	5	21	11	1
	2151	2160	2167	2170	2175	2176	2181	2202	2213	2214
38	15	7	3	11	13	9	1	3	7	5
	2229	2236	2239	2250	2263	2272	2273	2276	2283	2288
39	5	3	1	3	5	7	3	1	5	9
	2293	2296	2297	2300	2305	2312	2315	2316	2321	2330

	0	1	2	3	4	5	6	7	8	9
40	1	9	7	3	13	9	11	1	5	3
	2331	2340	2347	2350	2363	2377	2383	2384	2389	2392
41	1	15	13	3	5	7	5	3	17	7
	2393	2408	2421	2424	2429	2436	2441	2444	2461	2468
42	9	5	5	7	9	11	13	3	5	5
	2477	2482	2487	2494	2503	2514	2527	2530	2535	2540
43	1	27	1	9	7	3	13	3	7	11
	2541	2568	2569	2578	2585	2588	2601	2604	2611	2622
44	5	11	3	5	19	9	1	15	25	3
	2627	2638	2641	2646	2665	2674	2675	2690	2715	2718
45	1	11	5	3	11	5	7	3	7	21
	2719	2730	2735	2738	2749	2754	2761	2764	2771	2792
46	1	3	1	11	27	1	5	5	5	3
	2793	2796	2797	2808	2835	2836	2841	2846	2851	2854
47	5	1	11	3	11	1	9	1	15	1
	2859	2860	2871	2874	2885	2886	2895	2896	2911	2912
48	15	5	19	15	7	3	1	3	1	21
	2927	2932	2951	2966	2973	2976	2977	2980	2981	3002
49	7	11	5	9	1	3	5	1	5	9
	3009	3020	3025	3034	3035	3038	3043	3044	3049	3058
50	1	11	9	1	9	13	5	3	5	7
	3059	3070	3079	3080	3089	3102	3107	3110	3115	3122
51	5	5	15	11	1	3	13	5	3	7
	3127	3132	3147	3158	3159	3162	3175	3180	3183	3190
52	9	7	5	5	21	5	1	9	13	3
	3199	3206	3211	3216	3237	3242	3243	3252	3265	3268
53	5	17	1	9	13	3	1	9	13	3
	3273	3290	3291	3300	3313	3316	3317	3326	3339	3342
54	7	17	3	5	1	3	5	1	11	3
	3349	3366	3369	3374	3375	3378	3383	3384	3395	3398
55	19	21	11	1	3	5	5	1	5	21
	3417	3438	3449	3450	3453	3458	3463	3464	3469	3490
56	1	5	15	5	11	1	5	11	15	1
	3491	3496	3511	3516	3527	3528	3533	3544	3559	3560
57	3	5	13	3	1	17	23	9	5	1
	3563	3568	3581	3584	3585	3602	3625	3634	3639	3640
58	9	1	9	1	9	5	1	9	1	9
	3649	3650	3659	3660	3669	3674	3675	3684	3685	3694
59	5	7	29	9	1	9	7	5	9	17
	3699	3706	3735	3744	3745	3754	3761	3766	3775	3792
60	5	11	11	1	17	5	3	5	5	17
	3797	3808	3819	3820	3837	3842	3845	3850	3855	3872

	0	1	2	3	4	5	6	7	8	9
61	1	9	13	5	3	1	3	23	1	11
	3873	3882	3895	3900	3903	3904	3907	3930	3931	3942
62	5	15	7	5	5	17	15	1	3	5
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	86409	86412	86429	86478	86483	86488	86491	86496	86503	86508
923	9	25	9	5	1	9	1	9	5	37
	86517	86542	86551	86556	86557	86566	86567	86576	86581	86618
924	11	3	7	9	19	5	5	5	17	9
	86629	86632	86639	86648	86667	86672	86677	86682	86699	86708
925	1	11	15	1	11	11	3	25	9	5
	86709	86720	86735	86736	86747	86758	86761	86786	86795	86800
926	19	17	39	11	7	9	11	1	17	11
	86819	86836	86875	86886	86893	86902	86913	86914	86931	86942
927	9	1	9	25	3	5	11	7	3	29
	86951	86952	86961	86986	86989	86994	87005	87012	87015	87044
928	5	1	5	15	23	23	17	11	11	7
	87049	87050	87055	87070	87093	87116	87133	87144	87155	87162
929	5	3	7	9	7	5	3	19	9	25
	87167	87170	87177	87186	87193	87198	87201	87220	87229	87254
930	3	23	5	1	11	41	17	5	3	25
	87257	87280	87285	87286	87297	87338	87355	87360	87363	87388
931	5	27	5	1	9	7	5	5	9	7
	87393	87420	87425	87426	87435	87442	87447	87452	87461	87468
932	9	1	21	1	3	19	3	5	35	13
	87477	87478	87499	87500	87503	87522	87525	87530	87565	87578
933	3	19	21	5	13	5	9	7	3	1
	87581	87600	87621	87626	87639	87644	87653	87660	87663	87664
934	3	13	17	33	7	21	13	9	23	5
	87667	87680	87697	87730	87737	87758	87771	87780	87803	87808
935	1	9	1	5	9	25	17	9	17	23
	87809	87818	87819	87824	87833	87858	87875	87884	87901	87924
936	17	1	23	39	1	3	5	1	5	9
	87941	87942	87965	88004	88005	88008	88013	88014	88019	88028
937	25	5	11	11	5	3	35	1	9	11
	88053	88058	88069	88080	88085	88088	88123	88124	88133	88144
938	23	1	3	7	9	5	1	3	23	1
	88167	88168	88171	88178	88187	88192	88193	88196	88219	88220
939	3	35	1	21	13	23	17	41	5	3
	88223	88258	88259	88280	88293	88316	88333	88374	88379	88382
940	7	23	15	11	1	3	1	9	1	9
	88389	88412	88427	88438	88439	88442	88443	88452	88453	88462
941	7	3	35	7	3	11	17	5	5	13
	88469	88472	88507	88514	88517	88528	88545	88550	88555	88568
942	21	1	5	23	5	9	23	19	21	5
	88589	88590	88595	88618	88623	88632	88655	88674	88695	88700

	0	1	2	3	4	5	6	7	8	9
943	13	35	27	5	7	5	23	5	11	27
	88713	88748	88775	88780	88787	88792	88815	88820	88831	88858
944	1	17	3	1	3	19	21	7	9	1
	88859	88876	88879	88880	88883	88902	88923	88930	88939	88940
945	17	3	7	9	13	9	5	7	5	5
	88957	88960	88967	88976	88989	88998	89003	89010	89015	89020
946	11	15	11	13	9	17	1	9	23	23
	89031	89046	89057	89070	89079	89096	89097	89106	89129	89152
947	5	11	1	21	5	19	21	1	3	11
	89157	89168	89169	89190	89195	89214	89235	89236	89239	89250
948	1	5	35	5	21	5	1	27	11	17
	89251	89256	89291	89296	89317	89322	89323	89350	89361	89378
949	1	3	13	5	3	1	9	1	15	1
	89379	89382	89395	89400	89403	89404	89413	89414	89429	89430
950	9	7	5	9	17	11	5	13	3	5
	89439	89446	89451	89460	89477	89488	89493	89506	89509	89514
951	17	11	25	3	5	13	5	9	11	1
	89531	89542	89567	89570	89575	89588	89593	89602	89613	89614
952	3	1	9	23	7	9	31	9	7	9
	89617	89618	89627	89650	89657	89666	89697	89706	89713	89722
953	5	1	17	11	27	29	1	17	3	5
	89727	89728	89745	89756	89783	89812	89813	89830	89833	89838
954	13	5	3	7	21	7	29	17	9	25
	89851	89856	89859	89866	89887	89894	89923	89940	89949	89974
955	3	1	21	7	3	7	5	3	25	3
	89977	89978	89999	90006	90009	90016	90021	90024	90049	90052
956	11	19	17	5	1	9	17	1	3	5
	90063	90082	90099	90104	90105	90114	90131	90142	90145	90150
957	1	11	27	5	19	5	15	7	5	5
	90151	90162	90189	90194	90213	90218	90233	90240	90245	90250
958	3	5	19	11	5	3	19	5	15	5
	90253	90258	90277	90288	90293	90296	90315	90320	90335	90340
959	31	9	17	1						
	90371	90380	90397	90398						

备注：表中素数包括自然数 1。本表原始素数数据来源参考美国克莱数学研究所网站。
网址：<https://www.claymath.org> 数据链接 Data link: <https://t5k.org/lists/small/millions/>
Note: The prime numbers in the table include the natural number 1. The original prime data source in this table refer to the Clay Mathematics Institute website. Website: <https://www.claymath.org>
[7] Prime data link: <https://t5k.org/lists/small/millions/>

说明： 依据表 3 所列数据和以上定理 1 给出的素数通项公式(4.1)，我们就可以写出任意相关的素数。其方法是，首先将表中某行数字和某列数字组合，得到某一个素数序数。比如，第 0 行与第 4 列两数字组合，即 04，表示第 4 个素数。第 59 行与第 8 列两数字组合，即 598，表示第 598 个素数。又如，第 127 行与第 0 列两数字组合，即 1270，表示第 1270 个素数。依此类推。其次是，每一行又分为上栏数据和下栏数据。上栏数据是两个相邻素数对间隔合数的个数；下栏是从素数 1 开始，到第 m 个素数为止，所有两个相邻素数对间隔合数个数的前 m 项级数之和。然后将素数序数与某一个素数之前的级数和相加，即得某个素数。

Description: According to the data listed in Table 3 and the prime number general term formula (4.1) given by the above theorem 1, we can write any related prime number. The method is to first combine a row of numbers and a column of numbers in the table to obtain a prime ordinal number. For example, a two-digit combination of row 0 and column 4, i.e. 04, represents the fourth prime number. The combination of row 59 and column 8, which is 598, represents the 598th prime number. For example, the combination of row 127 and column 0, that is, 1270, represents the 1270 prime number. And so on. Secondly, each row is divided into upper column data and lower column data. The data in the upper column is the number of separated composite numbers between two adjacent prime numbers; The bottom column is the sum of the first m -term series of all two adjacent primes, starting with prime number 1 and ending with prime number m . Then add the prime-ordinal number to the sum of the series preceding a prime number to get a prime number.

例如 1，查表 3，由于第 1，2，3 个素数没有合数间隔，因此合数个数均为 0；根据定理 1 素数通项公式(4.1)：

For example, 1, look at Table 3, because the first, two, and three prime numbers have no composite interval, so the composite number is 0; According to theorem 1 prime number general term formula (4.1) :

$$P_m = m + \sum S_m(P_m)$$

则有：There are: $P_1 = 1 + 0 = 1$; $P_2 = 2 + 0 = 2$; $P_3 = 3 + 0 = 3$.

例如 2，查表 3，第 4 个素数间隔合数个数是 1，由于前 3 个素数间隔合数个数为 0，则其合数个数之和为 1。所以，第 4 个素数：

For example, 2, look at Table 3, the number of the fourth prime interval is 1, because the number of the first three prime interval is 0, then the sum of the number of the composite number is 1. So, the fourth prime number:

$$P_4 = 4 + 1 = 5$$

例如 3，查表 3，第 61 个素数的素数间隔合数个数之和为 220；根据定理 1 素数通项公式(4.1)，则有：

For example, 3, look in Table 3, the sum of the number of prime interval composite numbers of the 61st prime number is 220; According to theorem 1 prime number general term formula (4.1), then:

$$P_{61} = 61 + 220 = 281$$

例如 4，第 127 行与第 0 列两数字组合，即 1270，表示第 1270 个素数。其上栏表示间隔合数个数是 3，从第 1 个素数到第 1270 个素数的所有间隔合数个数之和为 9067，根据素数通项公式(4.1)，则得第 1270 个素数是：

For example, 4, the combination of row 127 and column 0, that is, 1270, represents the 1270 prime number. The upper column indicates that the number of interval composite numbers is 3,

and the sum of all interval composite numbers from the first prime number to the 1270th prime number is 9067. According to the prime number general term formula, the 1270th prime number is:

$$P_{1270} = 1270 + 9067 = 10337$$

所有素数均可依此类推，计算出所有两个相邻的素数所间隔合数的个数之和后，按此素数通项公式推算出来。

All prime numbers can be deduced in this way, and the sum of the composite numbers separated by all two adjacent prime numbers can be calculated according to the general term formula of this prime number.