



Network Calculus Theory and Application: A New Graduate Course for Engineering Students

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Abstract: With the development of modern networks, the quality of service guarantee has become even more important than before. However, various new types of traffic and heterogeneous network architecture make it quite challenging to evaluate the quality of service, such as delay and reliability. Therefore, a novel analysis method is needed to solve this situation. Network calculus is a theory used to analyze the queueing problems in networks, which is the main content to be taught in this graduate course. Before network calculus theory is introduced, the classical queueing theory will be taught. Then, the mathematical fundamentals of network calculus, such as min-plus algebra and cumulative arrival/service process, will be introduced. Next, two branches of network calculus, i.e., deterministic network calculus and stochastic network calculus, will be introduced separately, including the definition of deterministic/stochastic arrival curve and service curve, as well as important theorems, such as delay theorem, flow aggregation theorem, server concatenation theorem. After the theoretical knowledge, practical content will be taught in two parts. First, students are encouraged to establish system models based on their graduate research topics, analyze the service guarantee of their models, and present their work in class. Second, a traffic data collection platform will be taught, and students will collect various traffic data and analyze traffic characteristics based

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on network calculus theory. Based on the feedback from students during the past three years, students are satisfied with this new course because it focuses on frontier technology and the teaching activities are well-organized.

Keywords: *Network Calculus, Delay Analysis, Graduate Engineering Course*

1 INTRODUCTION

With the development of the Internet and wireless communication networks, many new types of traffic have emerged in recent years. Unlike traditional traffic, some newly emerged types of traffic, such as telemedicine, remote driving, online games and industrial Internet, require strict delay and high reliability. In addition, the total amount of such new traffic and links increases dramatically, contributing significantly to network congestion and end-to-end delay.

Along with the revolution of traffic, the network architecture and key techniques deployed in networks also developed a lot, such as the proposal of 5G network [1], delay-sensitive network [2] and deterministic network [3]. How to effectively control the network delay has been an important consideration. Various scheduling policies have been used in one network node and routing schemes used in multiple network nodes to optimize the end-to-end delay. Due to network deployment's complexity and unpredictable traffic flows' impact, the delay is quite difficult to model and analyze. Therefore, it is quite important to use suitable methods to establish models and conduct analysis.

Network calculus, proposed by R. L. Cruz [4][5], is a queueing theory for QoS analysis, which has been widely used for performance evaluation in various networks. Compared with classical queueing theory [6], network calculus can be applied in wider and more general scenarios. It has been relied on for various networks, such as time-sensitive network, deterministic network, and wireless network. However, no course, either a bachelor's course or graduate course, about network calculus theory has been established in universities in China. Therefore, a new course focusing on network calculus is greatly needed.

Apart from the mentioned necessity, initiating a graduate-level network calculus course is viable. Network calculus has attracted increasing attention and research in recent years, especially in the context of the development and application of fifth-generation (5G) and sixth-generation (6G) mobile communication technologies. Network calculus has been widely used, such as ultra-reliable low latency communication (uRLLC) [7][8], time-sensitive networks (TSN) [9][10], software-defined networking (SDN) [11][12] and others. Moreover, the number of scientific papers and literature related to network calculus continues to grow year after year, as shown in Figure 1.

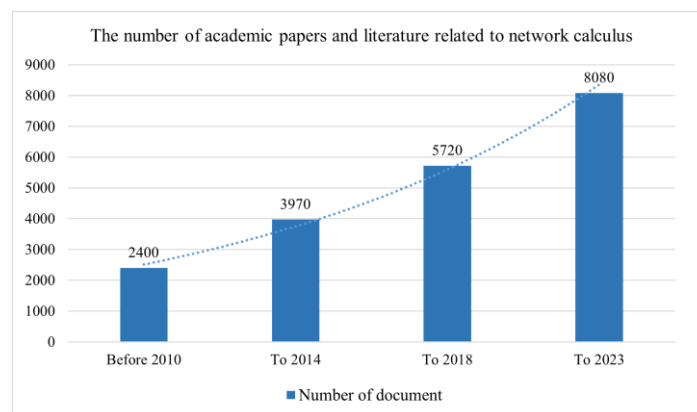


FIG.1 THE NUMBER OF DOCUMENTS RELATED TO NETWORK CALCULUS UNTIL 2023

The authors set up this course in 2021, and the number of students taking this course has increased each year, reflecting the importance of this course in the graduate curriculum. According to the results of the course survey, provided in Section 5, 98% of the students found the course helpful in practicing and analyzing their other projects.

The course is composed of two parts, i.e., the theoretical part and the practical part. In the theoretical part, the classical queueing theory will be taught first. Then, the mathematical fundamentals of network calculus, such as min-plus algebra and cumulative arrival/service process, will be introduced. Next, two branches of network calculus, i.e. deterministic network calculus^[13] and stochastic network calculus^{[14][15]}, will be introduced separately, including the definition of deterministic/ stochastic arrival curve and service curve, as well as important theorems, such as delay theorem, flow aggregation theorem, server concatenation theorem. In the practical part, students are encouraged to establish system models based on their graduate research topics, analyze the service guarantee of their models by applying network calculus theory^{[16][17]} and present their work in the class. Then, a traffic data collection platform will be taught, and students will collect various traffic data and analyze traffic characteristics based on network calculus theory.

After joining this course, graduate students will master the modeling and analysis method of network calculus and will be able to solve practical problems by using network calculus theory.

This paper is organized as follows. The general course information is presented first, including the course syllabus. Then, the teaching contents in the theoretical part will be introduced, followed by the contents in the practical part. The next part is the analysis of the survey questionnaire on the course conducted among the students. Lastly, a summary is given.

2 COURSE DESCRIPTION

This course is a 2-credit course with 32 teaching slots, each lasting 45 minutes. Table 1 lists the syllabus of the whole contents as well as the teaching slots arrangement. The course will be open to master students and PhD students in the majors of information and communication engineering and electronic information engineering as well as other related majors.

The students are required to have studied basic knowledge of communication networks and be familiar with typical traffic and network transmission mechanisms. They are also predicted to have learned the course of “Probability Theory and Stochastic Process.”

By joining this course, the students will be trained to know how to establish the model for one or multiple traffic flows as well as how to find the service model for typical networks. Then, they are qualified to make delay analysis under different configurations.

The main contents of this course can be divided into two parts as indicated by the course name, i.e., theoretical principles and practical applications, which are described in detail as follows.

TABLE 1 COURSE SYLLABUS AND TEACHING SLOTS ARRANGEMENT

| Part | Content | Number of slots |
|------------------------|--|-----------------|
| Part I: Theoretical | Chapter 0: Introduction of this course | 2 |
| | Chapter 1: Queueing theory | 4 |
| | Chapter 2: Deterministic network calculus and min-plus algebra | 6 |
| | Chapter 3: Stochastic network calculus | 4 |
| | Chapter 4: Multiple-flow multiple-server model and end-to-end | 4 |

| | | |
|-----------|---|---|
| | delay analysis | |
| Part II: | Chapter 5: An example: model and analysis | 2 |
| | Group work: model and analysis | 4 |
| Practical | Group work: presentation and discussion | 4 |
| | Summary | 2 |

3 THEORETICAL PART

In the theoretical part, a brief introduction will be given to all students at the beginning. Then, queueing theory, deterministic network calculus and stochastic network calculus will be taught. Lastly, examples will be relied on for the delay analysis by using network calculus theory.

3.1 Introduction of This Course

In this chapter, basic information about this course will be presented. First, various networks' background and development timelines are introduced, which helps students understand that ultra-low delay and ultra-high reliability has become vital for future applications. Then, possible delay analysis methods will be briefly summarized and compared, where special interest will be put on queueing theory and network calculus. Lastly, the arrangement of this course, the purpose, references, and other necessary information will be introduced to students, followed by a Question & Answer (Q&A) session to provide interaction time between the instructor and students.

3.2 Queueing Theory

In Chapter 1, the basic knowledge of queueing theory will be introduced. Queueing theory is a very classical method to analyze queueing problems in various networks. There have been some useful theorems for delay analysis. Ref. ^[6] is an open-access material and will be recommended to students for offline self-study. In this part, the typical single queue problem will be presented by explaining the Kendall Notation, which contains four parts as follows:

- arrival process
- service distribution
- number of servers
- buffer size-queue discipline

Commonly used options include:

- D: Deterministic
- M: Markovian process, Poisson process when used in arrival process and exponential distribution when used in service distribution;
- G: General process
- FIFO: First-In-First-Out
- LIFO: Last-In-First-Out

By combining the four parts, it is possible to describe a system in a very efficient way. For example, M/M/1/ ∞ -FIFO means a Poisson traffic going into a single server with exponential distribution service time and infinite buffer size working in the FIFO manner. Note that the “buffer size-queue discipline” part may be omitted if it is “/ ∞ -FIFO” for further clarification.

Then, the $M/D/1/\infty$ -FIFO model will be used to derive average queueing length, average waiting time, and steady-state probabilities, where Little's Law and Erlang-B formulas will be described and presented. The analysis can be extended to other cases, such as $M/M/1/\infty$ and $M/M/k/m$. Lastly, the analysis is further extended to a network, such as two-node tandem network and Jackson network.

3.3 Deterministic Network Calculus

In Chapters 2 and 3, the network calculus theory, a queueing theory for QoS analysis^[18] and widely used for performance evaluation in various networks, will be taught. It has been developed into two branches, i.e., deterministic network calculus (DNC) and stochastic network calculus (SNC). Deterministic network calculus is easy for students to understand and will be introduced first in Chapter 2. At the beginning of this part, its mathematical basic, a new algebra called min-plus algebra and the properties, will be taught. In addition, the concept of cumulative process will be illustrated. Then, two fundamental concepts, the deterministic arrival curve (dac) and deterministic service curve (dsc), can be defined. Based on these two definitions, the maximum delay and maximum backlog can be derived with the help of min-plus algebra. This course will introduce the understanding of why and how the deterministic arrival and service curves are needed and defined in detail, since these contents are essential for network calculus theory. In addition, several widely used traffic models and their deterministic arrival curves, such as periodic traffic and token-bucket traffic, will be given. At the same time, some typical server models and their deterministic service curve will also be given, such as constant rate server. Then, the delay theorem and backlog theorem are derived as a summarization for DNC.

3.4 Stochastic Network Calculus

After DNC is introduced, the stochastic network calculus will be discussed. In this part, the definition for DAC and SAC will be extended to stochastic arrival curve (SAC) and stochastic service curve (SSC). Then, the upper bounds for delay and backlog distribution functions can be derived. Some examples of typical traffic models with sac and typical server models with SSC will be discussed, such as the Poisson traffic model and discrete-time two-state Gilbert-Elliott channel model.

3.5 Multiple-flow Multiple-server Model and End-to-end Delay Analysis

The previous contents mainly focus on the fundamental content of network calculus theory by considering single-flow and single-server scenario. However, multiple flows exist in a network, and a flow usually goes through a network with multiple nodes. Therefore, the considered scenario will be extended to the multiple-flow, multiple-server model in this part. By applying network calculus theory, multiple flows can be treated as an aggregated flow, and the equivalent sac of the aggregated flow can be derived. Similarly, multiple concatenated servers can be considered equivalent single servers, and the equivalent SSC will be derived. Based on these contents, the end-to-end delay analysis of a flow (or several flows) going through a network will be available.

Based on aggregated flow and equivalent server, the end-to-end delay of a flow going through a link with multiple nodes can be made by applying the delay bound theorem in network calculus. The analysis method will be taught in this part.

4 PRACTICAL PART

In the practical part, a specific example will be taught first. In the example, a system with two aggregated flows and two concatenation servers will be established, and teacher will guide students to analyze the end-to-end delay together. In this process, students will get to know how to convert practical network scenarios

to theoretical models used in network calculus. They will also master how to apply theoretical theorems to the practical scenario.

Then, students are organized in the form of study groups, with 2 or 3 students in each group. Each group can establish an interesting system model based on their research topics, such as a wireless communication system, a delay-sensitive network model, or a fiber communication network. Students can make reasonable assumptions for traffic flows and server models for their considered scenarios, and based on these assumptions, delay analysis and numerical results are obtained. Each group (or some groups) can present their models, assumptions, and delay analysis results to the whole class for comments and discussions.

Next, students are introduced to a traffic data collection platform, including both hardware and software components. For the hardware configuration, students are required to equip themselves with laptops, while the software configuration necessitates the installation of Wireshark, a premier and widely used network protocol analyzer. Widely recognized for its utility in packet analysis, Wireshark offers functionalities such as live capture, offline analysis, versatile display filters, and compatibility across multiple platforms.

The measurement platform is described in Figure 2. Based on this platform, students will acquire proficiency in capturing packets for diverse network traffic types under the instructor's guidance. The available actual network traffic types include but are not limited to, online video watching, online gaming, online meetings, and other real network traffic with their laptops serving as terminals.

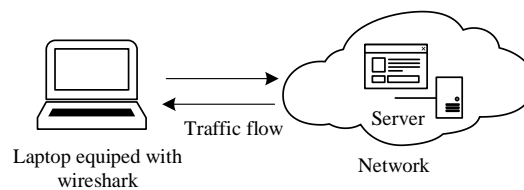


FIG.2 MEASUREMENT PLATFORM

Based on the collected traffic data, the analysis of traffic characteristics, which includes the statistical distribution of packet lengths and inter-arrival time, is required. With the help of network calculus theory and especially the concept of arrival curve, a robust traffic model is encouraged to be constructed, capturing the essence of the traffic flow. This work is also organized in groups with 2 to 3 members. Each group will present their work and discuss traffic characteristics with all other students and teachers.

This practical part of the course consolidates the students' theoretical knowledge and equips them with well-prepared skills to handle complex network analysis and optimization.

5 COURSE EVALUATION

Upon completing the course, a course survey is encouraged to be finished by students, as outlined in Table 2. The survey comprises ten questions covering aspects such as the overall assessment of the course content, difficulty, uniqueness, interest, and more.

TABLE 2 GRADUATE CORSE SURVEY QUESTIONNAIRE

| Question | Choices | | | |
|---|--|-----------------------------------|--|---|
| 1. What motivated you to enroll in this course? | <input type="radio"/> Advisor's guidance | <input type="radio"/> In exchange | <input type="radio"/> For NC knowledge | <input type="radio"/> Same research field |

| | | for credit | | |
|---|-------------------|--------------|------|-----------|
| Grade levels | Needs Improvement | Satisfactory | Good | Excellent |
| 2. How content are you with the overall caliber of the course? | ⊙ | ⊙ | ⊙ | ⊙ |
| 3. What did you like most and least about the course content, structure, and delivery? | ⊙ | ⊙ | ⊙ | ⊙ |
| 4. How effective were the instructor's teaching methods and communication skills? | ⊙ | ⊙ | ⊙ | ⊙ |
| 5. How would you evaluate the course in terms of its positive impact on your learning? | ⊙ | ⊙ | ⊙ | ⊙ |
| 6. How would you rate the difficulty level of the course? | ⊙ | ⊙ | ⊙ | ⊙ |
| 7. Would the course enhance your knowledge, skills, and abilities in the subject area? | ⊙ | ⊙ | ⊙ | ⊙ |
| 8. Do you think students in the coming year should enroll in this course? | ⊙ | ⊙ | ⊙ | ⊙ |
| 9. Level of appeal | ⊙ | ⊙ | ⊙ | ⊙ |
| 10. Level of academic workload | ⊙ | ⊙ | ⊙ | ⊙ |
| <i>Table Note: "Needs Improvement," "Satisfactory," "Good," and "Excellent" represent "1", "2", "3", and "4" points respectively.</i> | | | | |

The primary objective of the first question is to investigate why students choose this course. This helps the author better understand students' motivations for enrollment and enables targeted adjustments to the teaching methods for the second-year course. These questions provide four options, allowing students to choose the one that best suits their circumstances.

In the subsequent nine questions of the survey, the same set of four response options, i.e., "Needs Improvement", "Satisfactory", "Good" and "Excellent", has been employed. Upon completing the survey, the teacher calculates scores for each question based on assigned values for respective options. The course survey provides instructors with comprehensive feedback and information, aiding them in gaining a nuanced understanding of student needs and course efficacy. This, in turn, facilitates continuous improvement and optimization of teaching methodologies and content. A detailed description of the survey results is provided in the following part.

As depicted in Figure 3, the first question reveals that over 75% of students cite the opportunity to learn about network calculus as their primary reason for enrolling in the course. This result indicates that theoretical knowledge of network calculus is becoming increasingly significant within the students' academic framework. For the overall course evaluation by students, as illustrated in Figure 4, the overall evaluation scores for each year exceed 3.5, indicating a favorable response to the course. This suggests that the course has received positive feedback in terms of its alignment with students' academic requirements, the content delivered, and the teaching methods employed by the instructor.

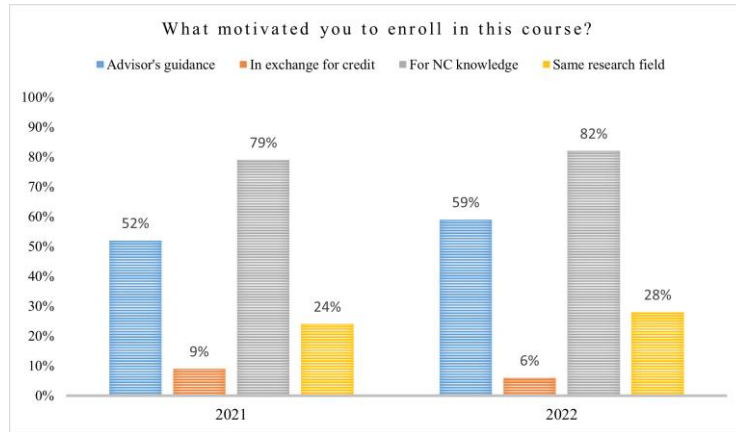


FIG.3 THE SURVEY RRESULTS OF QUESTION 1 IN TABLE 2

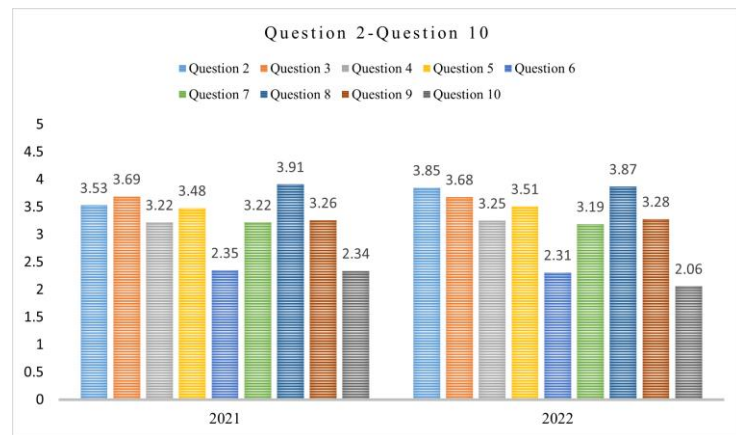


FIG.4 THE RESULTS OF QUESTION 2-10

The scores for questions three to five also consistently fall above 3.0 on average, indicating that the instructor's course structure, organization, and teaching methods are effective. Furthermore, it is noteworthy that over 75% of students believe that completing the course has been very beneficial for them. Approximately half of the students find the course difficulty moderate and the academic workload appropriate. The overwhelming majority of students unanimously believe that the course has significantly enhanced their competence in both the research field and personal capabilities. Additionally, over 90% of students are willing to recommend this course to future cohorts, reflecting the course's popularity among students. The detailed analysis of the course survey questionnaire results is now complete.

6 SUMMARY

Network calculus is an important method of analysis for quality of service guarantees, and it can be applied in various scenarios. To the best of our knowledge, this course is the first course focusing on network calculus theory and its application in China. In this course, both fundamental theoretical knowledge and practical applications will be included. After joining this course, students will master basic contents of network calculus theory and learn how to use the theory to solve practical problems.

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