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Exploration of Continuous Compound Interest in Complex Numbers

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Abstract: In the past, the interest rate functions, including continuous compound interest rate functions, were solved in real numbers. Scholars believe that interest rate functions in the other numeral system are meaningless for most financial markets. In the development of the times, financial events such as negative interest rates continue to break out, and more financial black swan events can happen beyond the current financial investment theory; The exploration of the formula of continuous compound interest rate from the perspective of complex numbers will help us find more features of the law of the financial market.

Keywords: Complex Numbers, Continuous Compound Interest Rate, Finance Market, Negative Asset

1. Introduction

In the current financial textbooks, it is generally believed that negative interest is meaningless to most financial problems. However, in actual financial investment cases, failure and loss happened frequently. Financial events, including negative oil prices have taken place, too; the negative asset's price is the real case now, as well as the valuation, pricing, and economic models of virtual assets urge us to upgrade investment theory. Especially, the interest rate expression also needs to be explored and upgraded. Complex function plays a key role in the development of natural science. In 2022, the team of Professor Pan Jian Wei of the University of science and technology of China proved the necessity of complex numbers description for the quantum microphysical world through quantum computing experiments (Chen, 2022). This paper also attempts to use complex numbers and complex functions to deduce and analyze the formula expression of continuous compound interest rates in finance.

2.Interest Rate in Complex Numbers

The cumulative function $a(t)$, which was defined in the book ***Introduction to Financial Mathematics*** published by Peking University Press (Wu, Huang & He, 2013). is the time variable $t \geq 0 \,\&\, t \in N$. that book does not define the scope of $a(t)$. In addition, it is specially stated, "if the function shows a downward trend, it will produce negative interest, which is not a problem in mathematics, but it is

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meaningless in most financial problems. Negative interest will occur only when the investment principal cannot be recovered, and the cumulative function is a fixed value, indicating no interest, which sometimes occurs" (Wu, Huang & He, 2013).

This paper extends the cumulative function a (t) to the complex numbers to explore its expression and significance. We can deduce the expression of complex numbers cumulative function from the definition of the basis of cumulative function: The cumulative function $b(t) \in C$ is defined as the expression of complex variable function, $t \geq 0, t \in N$.

Now let's Define interest rate expression:

$$\lambda_{t1,t2} = \left[\frac{b(t2) - b(t1)}{b(t1)}\right] \tag{1}$$

From above, an interest rate is also a complex number.

The functions of compound interest rate in complex expression can be deduced as $b(t), t \in N$,
$\lambda \in C$.

2.1 CONTINUOUS COMPOUND INTEREST RATE IN COMPLEX NUMBERS

Like the definition for the compound interest rate in Real numbers, we define the express for the compound interest rate in Complex numbers as:

$$b(t) = (1 + \lambda)^t \tag{2}$$

The definition of a continuous compound interest rate could be

$$b(t) = \lim_{m \to \infty} \left(1 + \frac{\lambda}{m}\right)^{mt} \tag{3}$$

here $m \to \infty, m, t \in N; \lambda \in C$

It can be deduced according to the limit's formula in the complex numbers:

$$\mathrm{b(t) = e^{\lambda t}} \tag{4}$$

In addition, from the perspective of the definition of interest force:

The definition of interest force function is: here $t \geqslant 0, \psi \in C$

$$\psi(t) = \frac{b^{'}(t)}{b(t)} = \frac{dln[b(t)]}{dt} \tag{5}$$

According to the definition of accumulation function (Wu, 2013) and elementary complex numbers theory (Lavrentieff & Shabat,2006)):

$$b(t) = exp\left(\int_0^t \psi(t)dt\right) \tag{6}$$

here $t \geqslant 0; \psi, b(t) \in C$

Because according to the definition of continuous compound interest rate, the continuous compound interest rate in the interval $(0, t)$ is a fixed value, here is a complex fixed value:

$$b(t) = e^{\psi t} \tag{7}$$

Here $\psi = \alpha + i\beta$ α, β is the complex number fixed value, and the letter i is the imaginary number symbol: According to the Euler Formula:

$$b(t) = e^{\alpha t} \times [\cos(\beta t) + i sin(\beta t)] \tag{8}$$

here $t \in N, \alpha, \beta \in R$

Equivalent to:

$$b(t) = e^{\alpha t} \times [\cos(\beta t + 2k\pi t) + i sin(\beta t + 2k\pi t)] \tag{9}$$

here $t \in N, \alpha, \beta, k \in R$

When the interest force function is a complex fixed value:

According to expression (2) and expression (7):

$$e^{\psi t} = (1 + \lambda)^t \tag{10}$$

So, we could find,

$$\psi = \ln(1 + \lambda) \tag{11}$$

It can be seen that the expression of real compound interest rate is the real part of the complex compound interest rate expression, when:

$\cos(\beta t + 2k\pi t) = 1$ *and*

$i\sin(\beta t + 2k\pi t) = 0$

$$b(t) = e^{\alpha t} \tag{12}$$

So, the continuous compound interest rate of complex numbers contains the real continuous compound interest rate.

2.2 Simple Interest in Complex Numbers

Similarly, the functions for continuous compound interest rates could be extended to complex numbers, and the functions of the simple interest rate could also be extended from real numbers to complex numbers.

Let's also define λ, which is the complex number symbol of a simple interest rate. According to the definition of simple interest rate, the cumulative function expression is:

$$b(t) = 1 + \lambda \times t \tag{13}$$

here $\lambda \in C, t \in N$.

Here λ is a complex fixed value,

$$\lambda = \alpha + i\beta \tag{14}$$

here $\alpha, \beta \in R$

so, the expression of the simple interest rate in complex numbers for a cumulative function is:

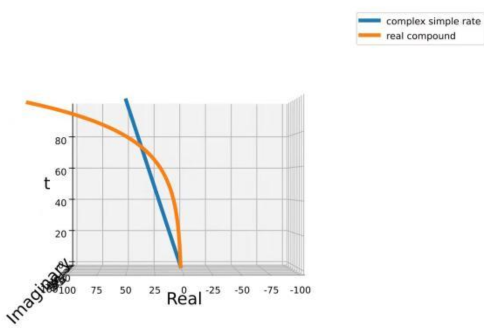
$$b(t) = (1 + \alpha t) + i\beta t \tag{15}$$

The $(1 + \alpha t)$ is that the real number part of the complex number for the simple interest rate is consistent in the real numbers, and there is the imaginary number part $i\beta t$ in the expression of the complex number for the simple interest rate. From the perspective of investment yield, it can be regarded as a part of invisible yield.

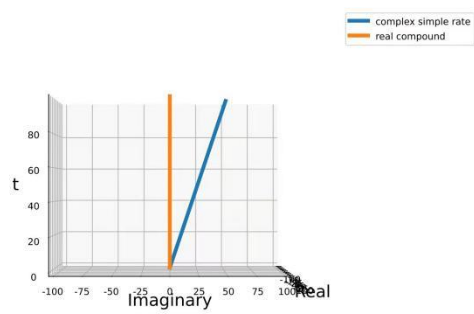
3. CHART COMPARISON BETWEEN REAL NUMBERS INTEREST RATE AND COMPLEX NUMBERS INTEREST RATE

We have finished the deduction for the expression of the simple interest rate, the compound interest rate, and the continuous compound interest rate from real numbers to complex numbers. What difference between them? We could plot the chart for two types of expression.

3.1 CHART OF CONTINUOUS COMPOUND INTEREST RATE IN REAL NUMBERS AND SIMPLE INTEREST RATE IN COMPLEX NUMBERS



(a) Figure 1



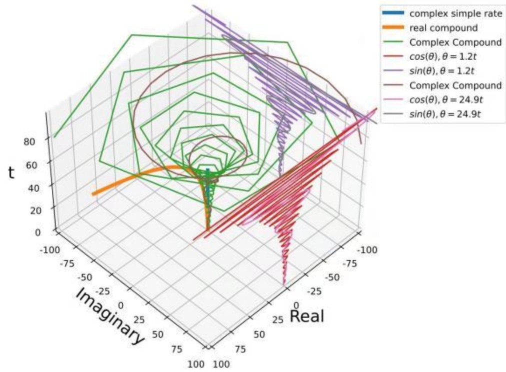
(b) Figure 2

As shown in Figure 1, the projection of real numbers' continuous compound interest rate trend graph and complex numbers' simple interest rate trend graph on the real plane is consistent with each other trend in real numbers.

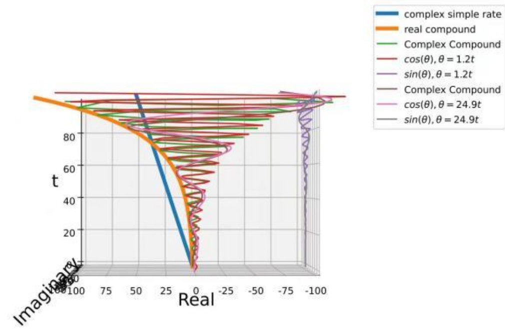
As it is shown in Figure 2, the complex numbers of simple interest rates have an inclination angle in the Y plane due to the imaginary numbers.

The actual numbers of continuous compound interest rates have no imaginary number part, so the projection on the imaginary numbers plane is a straight line.

3.2 CHART OF CONTINUOUS COMPOUND INTEREST RATE IN REAL NUMBERS AND CONTINUOUS COMPOUND INTEREST RATE IN COMPLEX NUMBERS



(c) Figure 3



(d) Figure 4

Figure 3 shows the trend of complex number continuous compound interest rates in three-dimensional space.

The imaginary part parameters of the two spiral rising trend lines are 1.2 t and 24.9 t, respectively. It can be seen that the larger coefficient makes greater angular velocity and makes more broken lines of the displacement trend in space, and when the trend is projected onto the plane of the real numbers, the cumulative function will jump violently between positive and negative values. The smaller coefficient makes the smaller angular velocity, makes the more minor rotation frequency, creates a smoother displacement trend in space, and makes the trend projected to the plane of real numbers relatively flat. The angle of an imaginary part could be understood as the choice of investment direction. The investment that is changed frequently will be made to the sharp fluctuation of accumulation function.
In Figure 4, it can be seen that the outer contour of the spiral trend of the complex number continuous compound interest rate is the trend line of the real number continuous compound interest rate. It is consistent with the result of our previous expression derivation.

4. CONCLUSION

The expression of continuous compound interest rates in complex numbers reveals more abundant trend characteristics. The imaginary number of continuous compound interest rates in complex numbers provides the rationality of the existence of negative interest rates. The complex number expression also suggests multiple corresponding solutions to the same yield curve, which is determined by the multi-valued property of the complex numbers. The complex number expression also reveals the characteristics of sharp volatility of yield.

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