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Proof of the Triple and Twin Prime Conjectures Using the Sindaram Sieve Method

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Abstract: Since Dr. Yitang Zhang proved in 2013 that there are infinitely many pairs of prime numbers differing by 70 million, it has been proved now that there are infinitely many pairs of prime numbers differing by 246. In this paper, we use the sieve method invented by Snndaram in 1934 to find out the solution of triple prime numbers and twin prime numbers, and find the general solution formula of the subset, i.e, a n_1 +b which is result of each subset, such as 3n+1, 5n+2, 7n+3, 9n+4, 11n+5, 13n+6, 15n+7, 17n+8, ... in 2mn+n+m, modulo x respectively (x≥3 takes prime). This general solution formula is used to prove the triple prime conjecture and the twin prime conjecture.

Keyword: prime numbers, twin prime numbers, triple prime numbers, sindaram sieve method

1 Introduction

In 1934 Sindaram invented a new sieve method, centered on the use of the general term of the number array a_{mn} = (2m+1)n+m to construct the following number array - Sindaram's table (see literature [1]).

4 7 10 13 16 19 22 ...
7 12 17 22 27 32 37 ...
10 17 24 31 38 45 52 ...

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13 22 31 40 49 58 67 ...

... ...

Sindaram found that 2N+1 is not prime if the natural number N appears in the above number array; if N does not appear in the above number array, then 2N+1 must be prime (see literature [1]).

In this paper, note that K={2mn+n+m ｜ m. n ∈ N}, L={2mn+n+m-1 ｜ m.n ∈ N}, S={2mn+n+m-3｜m.n∈N}. where m ≥ 1, n ≥ 1 are natural numbers,N^{+} :0,1,2,3, ... are 0 and natural numbers, N : 1,2,3, ... natural numbers.

Since (2m+1)n+m=2mn+n+m,note: K={2mn+n+m｜m.n∈N}, so

Then, we have the following: let q be positive integer, then 2q + 1 is prime <=> q∉K

2 Basic knowledge

Proposition 2.1 All prime numbers p greater than 2 can be expressed as (see literature [2])

$$P = 2q+1 \qquad (2.1)$$

where q ∉ K takes positive integer.

Proposition 2.2 All twin prime numbers can be expressed as (see literature [3])

$$\begin{cases} 2q+1 \\ 2(q+1)+1 \end{cases} \qquad (2.2)$$

where q ∉K∪L takes positive integer.

Proposition 2.3 All triple primes can be expressed as (see literature [4])

$$\begin{cases} 2q+1 \\ 2(q+1)+1 \\ 2(q+3)+1 \end{cases} \qquad (2.3)$$

where q ∉K∪L∪S takes positive integer.

Obviously, the following definitions are available from Proposition 2.1, Proposition 2.2 and Proposition 2.3.

Definition 2.1 Positive integers not belonging to K are the root of odd prime numbers.

Definition 2.2 Positive integers not belonging to K nor L are the root of twin prime.

Definition 2.3 Positive integers not belonging to K , L or S are the root of triple prime.

3 Applications

3.1 Finding odd prime numbers by the Sindaram sieve method

The values of 2mn+n+m are arranged in descending order as follows if m,n are the natural number, which are obtained from the Sindaram table (See literature [1]).

4 7 10 12 13 16 17 19 22 ....

The remaining positive integer corresponding to the above is the value of the odd prime root q in equation (2.1), i.e.,

q=1,2,3,5,6,8,9,11,14,15,18,20,21,...

Substituting q = 1,2,3,5,6,8,9,11,14,15,18,20,21, ... into (2.1) yields

3,5,7,11,13,17,19,23,29,31,37,41,43, ... which are all odd prime numbers.

3.2 Finding twin primes by the Sindaram sieve method

The values of 2mn+n+m are arranged in descending order as follows if m,n are the natural number,which are obtained from Sindaram's table.

4 7 10 12 13 16 17 19 22 ...

Then the values of 2mn+n+m-1 are arranged in order from smallest to largest as follows.

3 6 9 11 12 15 16 18 21 ....

Then the values in the equations 2mn+n+m and 2mn+n+m-1 are arranged in the following order from smallest to largest.

3 4 6 7 9 10 11 12 13 15 16 17 18 19 21 22 ...

The remaining positive integer corresponding to the above is the value of the twin prime root q in equation (2.2), i.e.,

q=1, 2, 5, 8, 14, 20, ...

Substituting q = 1, 2, 5, 8, 14, 20, ... into equation (2.2) yields: 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; ... all of which are twin prime numbers.

3.3 Finding the triple prime by the Sindaram sieve method

The values of 2mn+n+m are arranged in descending order as follows if m,n are natural numbers, which are obtained from Sindaram's table.

4 7 10 12 13 16 17 19 22 ...

Then the values of 2mn+n+m-1 are arranged in order from smallest to largest as follows.

3 6 9 11 12 15 16 18 21 ...

Then the values of 2mn+n+m-3 are arranged in order from smallest to largest as follows.

1 4 7 9 10 13 14 16 19 ....

The values in the equations 2mn+n+m, 2mn+n+m-1 and 2mn+n+m-3 are then arranged in the following order from smallest to largest.

1, 3, 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22,...

The remaining positive integer corresponding to the above is the value of the triple prime root q in equation (2.3), i.e.,

q=2, 5, 8, 20, ...

Substituting q = 2, 5, 8, 20, ... into equation (2.3) yields: 得 5, 7, 11; 11, 13, 17; 17, 19, 23; 41, 43, 47; ... all of which are triple prime numbers.

4 Introduction of the equivalence proposition of infinite number of triple prime numbers

We firstly identify the pattern from the following example and then introduce the equivalent proposition of infinite number of triple primes.

Example 4.1 In the five sets 5n, 5n+1, 5n+2, 5n+3, 5n+4.

1). Substituting n = 1 into 5n yields 5, and substituting q = 5 into equation (2.3) yields: 11,13,17.

2). Substituting n=1 into 5n+3 yields 8. Substituting q=8 into equation (2.3) yields: 17,19,23.

Checking the prime number table shows that: 11, 13, 17; 17, 19, 23 are two groups of triple prime numbers, then according to Proposition 2.3: 5 and 8 are both positive integers that do not belong to K ∪ L ∪ S. Then according to Definition 2.3: 5 and 8 are both triple prime roots, so when n ≥ 0 takes positive integer, among the five sets 5n, 5n+1, 5n+2, 5n+3, 5n+4 , at least 2 sets are the ones containing triple prime roots.

Example 4.2 In the seven sets 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6.

1). Substituting n = 1 into 7n + 1 yields 8, substituting q = 8 into equation (2.3) yields: 17,19,23.

2). Substituting n = 7 into 7n + 4 yields 53, and substituting q = 53 into equation (2.3) yields: 107,109,113.

3). Substituting n = 24 into 7n + 5 yields 173, substituting q = 173 into equation (2.3) yields: 347,349,353.

4). Substituting n=2 into 7n+6 yields 20, substituting q=20 into equation (2.3) yields: 41,43,47.

Checking the prime number table shows that: 17,19,23; 107,109,113; 347,349,353; 42,43,47 are four groups of triple prime numbers, then according to Proposition 2.3: 8,53,173,20 are all positive integers that do not belong to K ∪ L ∪ S. Then according to Definition 2.3: 8,53,173,20 are all triple prime roots, so when n ≥ 0 takes positive integer, among the seven sets 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6，at least 4 sets are the ones containing triple prime roots.

Example 4.3 In the eleven sets 11n,11n+1,11n+2,11n+3,11n+4,11n+5,11n+6,11n+7, 11n+8,11n+9, 11n+10.

1). Substituting n = 40 into 11n yields 440, and substituting q = 440 into equation (2.3) yields: 881,883,887.

2). Substituting n = 14 into 11n+1 yields 155, and substituting q = 155 into equation (2.3) yields: 311,313,317.

3). Substituting n = 10 into 11n + 3 yields 113, and substituting q = 113 into equation (2.3) yields: 227,229,233.

4). Substituting n = 4 into 11n + 6 yields 50, and substituting q = 50 into equation (2.3) yields: 101,103,107.

5). Substituting n = 8 into 11n + 7 yields 95, and substituting q = 95 into equation (2.3) yields: 191,193,197.

6). Substituting n = 15 into 11n + 8 yields 173, and substituting q = 173 into equation (2.3) yields: 347,349,353.

7). Substituting n = 1 into 11n + 9 yields 20, and substituting q = 20 into equation (2.3) yields:

41,43,47.

8). Substituting n = 20 into 11n + 10 yields 230, and substituting q = 230 into equation (2.3) yields: 461,463,467.

Checking the prime number table shows that: 881,883,887; 311,313,317; 227,229,233; 101,103,107; 191,193,197; 347,349,353; 41,43,47; 461,463,467 are 8 groups of triple prime numbers, then according to Proposition 2.3, we know that: 440,155,113, 50,95,173,20,230 are all positive integers not belonging to K ∪ L ∪ S. Then according to Definition 2.3: 440,155,113,50,95,173,20,230 are triple prime roots, so when n ≥ 0 takes positive integer,among the eleven sets 11n,11n+1,11n+2,11n+3,11n+4,11n+5,
11n+6, 11n+7,11n+8,11n+9,11n+10,at least 8 sets are the ones containing triple prime roots.

Example 4.4 In the thirteen sets 13n,13n+1,13n+2,13n+3,13n+4,13n+5,13n+6,13n+7,
13n+8,13n+9,13n+10,13n+11,13n+12 thirteen.

1). Substituting n = 50 into 13n yields 650, then substituting q = 650 into equation (2.3) yields: 1301,1303,1307.

2). Substituting n = 4 into 13n + 1 yields 53, then substituting q = 53 into equation (2.3) yields: 107,109,113.

3). Substituting n = 57 into 13n + 2 yields 743, then substituting q = 743 into equation (2.3) yields: 1487,1489,1493.

4). Substituting n = 7 into 13n + 4 yields 95, then substituting q = 95 into equation (2.3) yields: 191,193,197.

5). Substituting n = 1 into 13n + 7 yields 20, then substituting q = 20 into equation (2.3) yields: 41, 43,47.

6). Substituting n = 24 into 13n + 8 yields 320, then substituting q = 320 into equation (2.3) yields: 641,643,647.

7). Substituting n = 8 into 13n + 9 yields 113, then substituting q = 113 into equation (2.3) yields: 227,229,233.

8). Substituting n = 61 into 13n + 10 yields 803, then substituting q = 803 into equation (2.3) yields: 1607,1609,1613.

9). Substituting n = 3 into 13n + 11 yields 50, then substituting q = 50 into equation (2.3) yields: 101,103,107.

10). Substituting n=11 into 13n+12 yields 155, then substituting q=155 into equation (2.3) yields: 311,313,317.

Checking the prime number table shows that: 1301,1303,1307; 107,109,113; 1487,1489,1493; 191,193,197; 41,43,47; 641,643,647; 227,229,233; 1607; 1609,1613; 101,103,17; 311,313, 317 are 10 groups of triple primes, then according to Proposition 2.3: 650,53,743,95,20,320,113,803,50,155 are all positive integers that do not belong to K ∪ L ∪ S. Then according to Definition 2.3: 650,53,743,95,20,320,113,803,50,155 are all triple prime roots. So when n ≥ 0 takes positive integer, among the thirteen sets at least 10 of the 13 sets 13n,13n+1,13n+2,13n+3,13n+4,13n+5,13n+6,13n+7,13n+8,13n+9,13n+10,13n+11,13n+12, at least 10 sets are the ones containing triple prime roots.

Clearly from Example 4.1,Example 4.2,Example 4.3,Example 4.4,there is a common feature found that when p takes prime number 5,7,11,13, and n ≥ 1 is natural number, among the p sets of pn,pn+1,pn+2,...,pn+p-1, there are all p-3 sets containing the roots of the triple prime. From this, we introduce the equivalent proposition of infinite number of triple primes as follows.

Proposition 4.1 When p ≥ 5 is prime and n ≥ 0 is natural number, the p sets pn,pn+1,pn+2,...,pn+p-1 , at least p-3 sets are the ones that containing triple prime roots.

Clearly when p ≥ 5 is prime and n ≥ 0 is natural number, there is no repeated values among the p sets pn,pn+1,pn+2,...,pn+p-1.

Also because there are infinite prime numbers,and only prime 2 is even, then there are infinite odd prime numbers, and then odd prime number p-3 is still infinite.Therefore, from infinite numbers of sets containing the roots of triple prime numbers, there are infinite numbers of groups of triple prime numbers, namely,

If Proposition 4.1 holds, then there are infinite many triple prime numbers.

So Proposition 4.1 is an equivalent proposition of the infinity of triple prime .

Clearly， there is a group of twin prime in every group of triple primes, so if there are infinite many triple primes, then there are infinite many twin primes.

5 Three general solution formulas

From the above discussion it follows that: by the Sindaram sieve method, prime, twin prime, and triple prime numbers are related to the integer in 2mn+n+m, so the simplest subset of integer in 2mn+n+m is studied firstly.

5.1 The 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8; ... subsets of 2mn+n+m， each modulo x (x ≥ 3 takes prime) with all the corresponding an_1 +b subsets of the general solution formulas and the 3m+1; 5m+2; 7m+3; 9m+4; 11mn_1 +5; 13m+6; 15m+7; 17m+8; ... subsets of 2mn+n+m,each modulo x (x ≥ 3 takes prime),with all the corresponding an_1+b subsets of general solution formulas.

According to the unique decomposition theorem, every integer greater than 1 could be divided into the concatenated product of prime factors, which is

$$a= p_1 p_2 \cdots p_k , k\geq 1$$

Here.$p_1 p_2$ ...p_k are odd prime numbers,of which there may be same values, e.g. 63 = 3 × 3 × 7, 75

= 3 × 5 × 5 (see literature [5]).

Then odd number 3, 5, 7, 9, 11, 13, 15, 17, ... greater than 1 could also be denoted by $p_1 p_2 \cdots p_k$ (here p_1, p_2, ... p_k are odd prime,of which there may be same values, k = 1,2,3... takes natural number, e.g. 63 = 3 × 3 × 7, 75 = 3 × 5 × 5).

Since 2mn+n+m=(2m+1)n+m, substituting m=1,2,3,4,5,6...natural number into (2m+1)n+m yields respectively: 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8;...

Then 2mn+n+m :3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8; ... n with coefficient 3, 5, 7, 9, 11, 13, 15, ... odd an+b (where a takes odd number, n takes natural number, $1 \le b \ge \frac{a-1}{2}$) subset could be expressed as the following general solution formula :

$$p_1 p_2 p_3 \cdots p_k n + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2}$$

of which p_1, .p_2, ...p_k take odd prime number, n ≥ 1 takes natural number, and k = 1,2,3... takes natural number.

Clearly: when $p_1 p_2 p_3 \cdots p_k n + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2}$ modulo x (x ≥ 3 takes prime), according to the residual theory we have:

$$\{p_1 p_2 p_3 \cdots p_k n + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2} \mid n \in N^+\}$$

$$= \{x\ p_1 p_2 \cdots p_k n_1 + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2} \mid n_1 \in N^+\}$$

$$\cup \{x\ p_1 p_2 \cdots p_k n_1 + p_1 p_2 \cdots p_k + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2} \mid n_1 \in N^+\}$$

$$\cup \{x\ p_1 p_2 \cdots p_k n_1 + 2 p_1 p_2 \cdots p_k + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2} \mid n_1 \in N^+\}$$

$$\cup \ldots$$

$$\cup \{x\ p_1 p_2 \cdots p_k n_1 + (x-1) p_1 p_2 \cdots p_k + \frac{p_1 p_2 p_3 \cdots p_k - 1}{2} \mid n_1 \in N^+\}$$

Therefore, the following general solution could be derived from the set of residual system on the right side of the above equation.

That is, the coefficient 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8; ...n of 2mn+n+m are an+b of 3, 5, 7, 9, 11, 13, 15, ...odd number (where a takes odd number, n takes natural number, $1 \le b \ge \frac{a-1}{2}$) After the subsets are each modulo x (x ≥ 3 takes prime), the corresponding general solution of the subset an_1 + b is

$$\begin{cases} xp_1p_2\cdots p_kn_1+\frac{p_1p_2\cdots p_k-1}{2} \\ xp_1p_2\cdots p_kn_1+p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2} \\ xp_1p_2\cdots p_kn_1+2p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2} \\ \vdots \\ xp_1p_2\cdots p_kn_1+(x-1)p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2} \end{cases}$$

(5.1)

where m,n,k, n_1 takes natural number, p_1, p_2 ,..., p_k may be identical, but none is equal to x,p_1, p_2 ,...,p_k are all odd prime.$\frac{p_1p_2p_3\cdots p_k-1}{2}$ ≤b≥(x-1) $p_1p_2 \mathrm{L}\ p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$.

Similarly, substituting n=1,2,3,4,5,6 ,...natural number into (2m+1)n+m yields: 3m+1; 5m+2; 7m+3; 9m+4; 11m+5; 13m+6; 15m+7; 17m+8;...; (2m+1)n+m, respectively.

Since 2mn+n+m is symmetric, it shall find the general solution of subset a n_1 + b of 2mn+n+m in terms of 3m+1; 5m+2; 7m+3; 9m+4; 11m+5; 13m+6; ...(where a takes odd number, m,n both take natural number, 1 ≤ b ≥ $\frac{a-1}{2}$) when the set is modulo x (x ≥ 3 takes prime) respectively. It is the same as finding general solution of subset a n_1 + b(5.1) of 2mn+n+m in terms of 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; ...(where a takes odd number, m,n take natural number, 0 ≤ b ≥$\frac{a-1}{2}$)when the set is modulo x (x ≥ 3 takes prime) respectively . An example is given as follows.

Example 5.1 Substituting m = 1 into 2mn+n+m yields 3n+1, then {3n+1 | n∈N} ∈ {2mn+m+n | m,n∈N},i.e., 3n+1 is a subset of 2mn+n+m when m,n take natural number.

Substituting n = $5n_1$ into 3n+1 yields $15n_1$ +1.

Substituting n = $5n_1$ +1 into 3n+1 yields $15n_1$ +4.

Substituting n = $5n_1$ +2 into 3n+1 yields $15n_1$ +7.

Then {15n_1+1 | n_1 ∈N}∈{3n+1 | n∈N}

{15n_1+4 | n_1 ∈N}∈{3n+1 | n∈N}

{15n_1+7 | n_1 ∈N}∈{3n+1 | n∈N}

That is, when n_1 ,n take natural number, $15n_1$ +1, $15n_1$ +4, $15n_1$ +7 are all subsets of 3n+1.

Since 3n+1 is a subset of 2mn+n+m when m,n take natural number.

Then it could be drawn from the set transitivity that when n_1 ,m,n take natural number, $15n_1$ +1, $15n_1$ +4, $15n_1$ +7 are all subsets of 2mn+m+n.

Example 5.2 Substituting n = 1 into 2mn+n+m yields 3m+1, then {3m+1 | m∈N} ∈ {2mn+m+n | m,n∈N}, i.e., 3m+1 is a subset of 2mn+n+m when m,n take natural number.

Substituting m = $5n_1$into 3m+1 yields $15n_1$ +1.

Substituting m = $5n_1$ +1 into 3m+1 yields $15n_1$ +4.

Substituting m = $5n_1$ +2 into 3m+1 yields $15n_1$ +7.

Then $\{15n_1 + 1 \mid n_1 \in N\} \in \{3m + 1 \mid n \in N\}$

$\{15n_1 + 4 \mid n_1 \in N\} \in \{3m+1 \mid n \in N\}$

$\{15n_1 + 7 \mid n_1 \in N\} \in \{3m+1 \mid n \in N\}$

That is, when n_1 ,m take natural number, $15n_1$ +1, $15n_1$ +4, $15n_1$ +7 are all subsets of 3m+1.

Since 3n+1 is a subset of 2mn+n+m when m,n take natural number.

Then it could be drawn from the set transitivity that when n_1 ,m,n take natural number, $15n_1$ +1, $15n_1$ +4, $15n_1$ +7 are all subsets of 2mn+m+n.

Clearly the same result in of Example 5.1 and of Example 5.2 is obtained when n_1 ,m,ntake natural number, $15n_1$ +1, $15n_1$ +4, $15n_1$ +7 are all subsets of 2mn + m + n. etc so one of them could be omitted. In this paper, we use the 3n+1, 5n+2, 7n+3, 9n+4, 11n+5, 13n+6, ... subsets of 2mn+n+m to solve Eq. (5.1), and no longer use the 3m+1; 5m+2; 7m+3; 9m+4; 11m+5; 13m+6; ... subsets of 2mn+n+m to solve Eq.(5.1) Eq.

That is, equation (5.1) not only contains 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8; ... subsets of 2mn+n+m modulo x ($x \geq 3$ takes prime) corresponding to all a n_1 + b subsets, but also contains 3m+1, 5m+2, 7m+3 of 2mn+n+m, 9m+4, 11m+5, 13m+6, 15m+7, 17m+8, ... subsets modulo x ($x \geq 3$ takes prime) respectively, which is corresponding to all a n_1 + b subsets.

5.2 The subsets 3n; 5n+1; 7n+2; 9n+3; 11n+4; 13n+5; 15n+6; 17n+7; ... of 2mn+n+m-1 modulo x ($x \geq 3$ takes prime) and all the corresponding a n_1 + b subset general solution formula and the subsets 3m ;5m+1; 7m+2; 9m+3; 11m+4; 13m+5; 15m+6; 17m+7; ...of 2mn+n+m-1 modulo x ($x \geq 3$ takes prime)respectively, which is corresponding to all a n_1 + b subset general solution formula.

Since 2mn+n+m-1=(2m+1)n+m-1, substituting m=1,2,3,4,5,6...natural number into (2m+1)n+m-1 yields: 3n; 5n+1; 7n+2; 9n+3; 11n+4; 13n+5; 15n+6; 17n+7;... respectively.

The same method as in 5.1 yields 3n; 5n+1; 7n+2; 9n+3; 11n+4; 13n+5; 15n+6; 17n+7; ... subsets of 2mn+n+m-1 modulo x ($x \geq 3$ taken as prime),respectively, which is corresponding to all a n_1 + b subset general solution :

$$\begin{cases} xp_1p_2\cdots p_kn_1 + \frac{p_1p_2\cdots p_k-1}{2} - 1 \\ xp_1p_2\cdots p_kn_1 + p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 1 \\ xp_1p_2\cdots p_kn_1 + 2p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 1 \\ \vdots \\ xp_1p_2\cdots p_kn_1 + (x-1)p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 1 \end{cases}$$

(5.2)

where m,n,k, n_1 take natural number, p_1, p_2 ,..., p_k may be identical, but none is equal to x .x,p_1 , p_2 ,...,p_k are all odd prime , $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1 ≤ b ≥ (x - 1) p_1p_2L p_k +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1.

As in equation (5.1), equation (5.2) not only contains all a n_1 + b subsets of 2mn+n+m-1 of 3n; 5n+1; 7n+2; 9n+3; 11n+4; 13n+5; 15n+6; 17n+7; ...modulo x (x ≥ 3 takes prime) respectively, corresponding to all a + b subsets, but also contains 3m of 2mn+n+m-1; 5m+ 1; 7m+2; 9m+3; 11m+4; 13m+5; 15m+6; 17m+7; ...modulo x (x ≥ 3 takes prime)respectively, corresponding to all a n_1 + b subsets.

5.3 The subsets 3n-2; 5n-1; 7n; 9n+1; 11n+2; 13n+3; 15n+4; 17n+5; ... of 2mn+n+m-3 modulo x (x ≥ 3 taken prime), and all the corresponding a n_1 +b subset general solution formula and the subsets 3m-2; 5m-1; 7m; 9m+1; 11m +2; 13m+3; 15m+4; 17m+5; ... of 2mn+n+m-3 modulo x (x ≥ 3 takes prime)respectively, which is corresponding to all a n_1 + b subset general solution formula.

Since 2mn+n+m-3=(2m+1)n+m-3, substituting m=1,2,3,4,5,6...natural number into (2m+1)n+m-3 yields: 3n-2; 5n-1; 7n; 9n+1; 11n+2; 13n+3; 15n+4; 17n+5;... respectively.

The same method as in 5.1 yields 3n-2; 5n-1; 7n; 9n+1; 11n+2; 13n+3; 15n+4; 17n+5; ... subsets of 2mn+n+m-3 modulo x (x ≥ 3 takes prime)respectively, which is corresponding to all a n_1 + b subset general solution:

$$\begin{cases} xp_1p_2\cdots p_kn_1 + \frac{p_1p_2\cdots p_k-1}{2} - 3 \\ xp_1p_2\cdots p_kn_1 + p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 3 \\ xp_1p_2\cdots p_kn_1 + 2p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 3 \\ \vdots \\ xp_1p_2\cdots p_kn_1 + (x-1)p_1p_2\cdots p_k + \frac{p_1p_2\cdots p_k-1}{2} - 3 \end{cases}$$

(5.3)

where m,n,k, n_1 take natural number, p_1, p_2 ,..., p_k may be identical, but none is equal to x .x,p_1, p_2 ,...,p_k are all odd prime, .$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 ≤ b ≥ (x - 1) p_1p_2L p_k +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3.

As in equation (5.1), equation (5.3) not only contains all a n_1 + b subsets corresponding to the subsets 3n-2; 5n-1; 7n; 9n+1; 11n+2; 13n+3; 15n+4; 17n+5; ... of 2mn+n+m-3 modulo x (x ≥ 3 takes prime)respectively, but also contains 3m-2 ; 5m-1; 7m; 9m+1; 11m+2; 13m+3; 15m+4; 17m+5; ... of 2mn+n+m-3 modulo x (x ≥ 3 takes prime) respectively, corresponding to all a n_1 + b subsets.

Also, since this paper is written: K={2mn+n+m | m,n∈N}, L={2mn+n+m-1 | m,n∈N}, S={2mn+n+m-3 | m,n∈N}, where m,n≥1 take natural number, N:1,2,3,...takes natural number.

Then equation (5.1) contains all a n_1 + b subsets of K ∪ L ∪ S modulo x (x ≥ 3 taken as prime) corresponding to 3n+1; 5n+2; 7n+3; 9n+4; 11n+5; 13n+6; 15n+7; 17n+8; ... subsets modulo x (x ≥ 3 taken as prime), as well as 3m+1; 5m+2; 7m+3; 9m+4 of K ∪ L ∪ S; 11m+5; 13m+6; 15m+7; 17m+8; ... subsets modulo x (x ≥ 3 taken as prime), respectively, which is corresponding to all a n_1.

The equation (5.2) contains all a n_1 + b subsets modulo x (x ≥ 3 takes prime) corresponding to 3n; 5n+1; 7n+2; 9n+3; 11n+4; 13n+5; 15n+6; 17n+7; ... subsets of K ∪ L ∪ S. It also contains all a n_1 + b subsets modulo x (x ≥ 3 takes prime)respectively, corresponding to 3m; 5m+1; 7m+2; 9m+3; 11m+4; 13m+5; 15m+6; 17m+7; ... subsets of K ∪ L ∪ S .

The equation (5.3) contains all a n_1 + b subsets modulo x (x ≥ 3 takes prime) corresponding to 3n-2; 5n-1; 7n; 9n+1; 11n+2; 13n+3; 15n+4; 17n+5; ... subsetsof K ∪ L ∪ S as well as alla n_1 + b subsets modulo x (x ≥ 3 takes prime)respectively,corresponding to 3m-2; 5m-1; 7m; 9m+1; 11m+2; 13m+3; 15m+4; 17m+5; ... subsets of K ∪ L ∪ S .

6 Preliminary propositions

Proposition 6.1 In $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 ≤ b ≥ (x - 1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$ closed interval, the corresponding a n_1 + b subset general solution formula of K ∪ L ∪ S are (5.1), (5.2), (5.3) only.

(where m, n, k, n_1 take natural number, p_1, p_2, ..., p_k may be identical, but none is equal to x, p_1, p_2, ..., p_k take odd prime , a ≥ 3 takes odd) .

In the 1st a n_1 + b set in equation (5.1) b = $\frac{p_1p_2p_3\cdots p_k-1}{2}$

In the last a n_1 + b set in equation (5.1) b = (x - 1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$

In the 1st a n_1 + b set in equation (5.2) b = $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1

In the last a n_1 + b set in equation (5.2) b = (x - 1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$ -1

In the 1st a n_1 + b set in equation (5.3) b = $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3

In the last a n_1 + b set in equation (5.3) b = (x - 1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 closed interval and since it is clear that when k takes natural number, x, the p_1, p_2, ...p_k are odd prime,

$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 < $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1 < $\frac{p_1p_2p_3\cdots p_k-1}{2}$, (x-1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 < (x-1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$ -1 < (x-1) $p_1p_2 \text{L}\ p_k + \frac{p_1p_2p_3\cdots p_k-1}{2}$

Then b in the set an_1+ b is in $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 to (x - 1) $p_1p_2\cdots p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ contains b in $\frac{p_1p_2p_3\cdots p_k-1}{2}$ to (x-1) $p_1p_2\cdots p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ closed interval and contains b in $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1 to (x-1) $p_1p_2\cdots p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -1 closed interval and also contains b in $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 to (x-1) $p_1p_2\cdots p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 closed interval.

Suppose that K ∪ L ∪ S has a n_1 ±b subset of general solution in addition to equations (5.1), (5.2), (5.3) (where a is odd number and m,n,k, n_1 are natural number,p_1, p_2 ,...,p_k may be identical, but none is equal to x . x,p_1, p_2 ,...,p_k are all odd prime,$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -j ≤ b ≥ (x - 1) $p_1p_2\cdots p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ -j).

According to the unique decomposition theorem, the coefficient a of n_1 in an_1 ± b ,every integer greater than 1,the result is unique,if it decomposes prime factors regardless of their decomposition order (see Literature [5]).

Obviously, if K ∪ L ∪ S has another an_1 ± b subset general solutions in addition to Eqs. (5.1), (5.2), (5.3), then the coefficient n_1 must be same with coefficient coefficient n_1 in (5.1), (5.2), (5.3) and only the remainder is different, then it could be written in the following form.

$$\begin{cases} xp_1p_2\cdots p_kn_1+\frac{p_1p_2\cdots p_k-1}{2}-j \\ xp_1p_2\cdots p_kn_1+p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2}-j \\ xp_1p_2\cdots p_kn_1+2p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2}-j \\ \vdots \\ xp_1p_2\cdots p_kn_1+(x-1)p_1p_2\cdots p_k+\frac{p_1p_2\cdots p_k-1}{2}-j \end{cases} \quad (6.1)$$

Within, p_1, p_2 ,...,p_k ,n_1 ,m,n,x,k are consistent with the requirements in equations (5.1), (5.2), (5.3).

Obviously, according to equations (5.1), (5.2) and (5.3), equation (6.1) is subsets of 2mn+n+m-j: 3n+1-j; 5n+2-j; 7n+3-j; 9n+4-j; 11n+5-j; 13n+6-j; 15n+7-j; 17n+8-j; ... modulo x (x ≥ 3 takes prime)respectively. Which is corresponding to all an_1 + b subset genereal solution formula,also contains subsets of 2mn+n+m-j : 3m+1-j; 5m+2-j; 7m+3-j; 9m+4-j; 11m+5-j; 13m+6-j; 15m+7-j; 17m+8-j; ... modulo x (x ≥ 3 takes prime), which is corresponding to all a n_1+ b subset.

(a) When j = 0, equation (6.1) is in line with equation (5.1).

(b) When j = 1, equation (6.1) is in line with equation (5.2).

(c) When j = 3, equation (6.1) is in line with equation (5.3).

Then j ≠ 0, j ≠ 1, j ≠ 3.

Also, since when $j \neq 0, j \neq 1, j \neq 3$, it could be drawn from Proposition 2.1 that 2Q+1; 2(Q+1)+1; 2(Q+3)+1; 2(Q+j)+1 are four odd prime numbers when $Q \neq 2mn+n+m$, $Q+1 \neq 2mn+n+m$, $Q+3 \neq 2mn+n+m$, and $Q+j \neq 2mn+n+m$, and 2Q+1;2(Q+1)+1; 2(Q+3)+1; 2(Q+j)+1 are beyond the triple prime.

A contradiction appears and the hypothesis is not valid .

Then Proposition 6.1 holds.

7 Proofs of the Samson prime conjecture and the twin prime conjecture

From what is stated in Paper 4, Proposition 4.1 is an equivalent proposition of the infinity of triple prime.

If Proposition 4.1 holds is proved, then the infinity of triple prime holds.

Proposition 4.1 is proved as follows.

7.1 p=5

Substituting p=5 into pn, pn+1, pn+2, ..., pn+p-1 yields:5n,5n+1,5n+2, 5n+3,5n+4.

In the fives sets of 5n,5n+1,5n+2,5n+3,5n+4.

1) Substituting n = 1 into 5n yields 5, and substituting q = 5 into equation (2.3) yields: 11,13.17.

2) Substituting n=1 into 5n+3 yields 8, and substituting q=8 into equation (2.3) yields: 17,19,23.

Checking the prime number table shows that: 11, 13, 17; 17, 19, 23 are two groups of triple prime number, then according to Proposition 2.3 we have: 5 and 8 are both positive integers not belonging to $K \cup L \cup S$. Then according to Definition 2.3 we have: 5 and 8 are both triple prime roots. So when $n \geq 0$ takes positive integer,among the five sets 5n, 5n+1, 5n+2, 5n+3, 5n+4,at least 2 sets are the ones containing triple prime roots.

So, when p = 5, Proposition 4.1 holds.

7.2 p=7

Substituting p=7 into pn, pn+1, pn+2, ..., pn+p-1 yields: 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6.

The first proof method

1) Substituting n = 1 into 7n + 1 yields 8, substituting q = 8 into equation (2.3) yields: 17,19,23.

2) Substituting n = 7 for 7n + 4 yields 53, and substituting q = 53 into equation (2.3) yields: 107,109,113.

3) Substituting n = 24 into 7n + 5 yields 173, substituting q = 173 into equation (2.3) yields: 347,349,353.

4) Substituting n=2 into 7n+6 yields 20, substituting q=20 into equation (2.3) yields: 41,43,47.

Checking the prime number table shows that: 17,19,23; 107,109,113; 347,349,353; 42,43,47 are four groups of triple prime numbers, then according to Proposition 2.3: 8,53,173,20 are all positive integers not belonging to $K \cup L \cup S$. Then according to Definition 2.3: 8,53,173,20 are all triple prime roots, so when $n \geq 0$ takes positive integer, among the seven sets

7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6,at least 4 sets are the ones containing triple prime roots.

So, when p = 7, Proposition 4.1 holds.

Since the first proof requires looking up the prime number table, it is obvious that this method does not work when analyze the relationship between p,pn+1,pn+2,...,pn+p-1 and K∪L∪S when p in pn,pn+1,pn+2,...,pn+p-1 takes infinite prime.

The second proof method

Step 1:

7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 modulo 11 respectively, by the theory of complete system of residues yields 7 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11, i.e.

$\{7n \mid n \in N^+\} = \{77n_1 \mid n_1 \in N^+\} \cup \{77n_1+7 \mid n_1 \in N^+\} \cup \{77n_1+14 \mid n_1 \in N^+\} \cup \{77n_1+21 \mid n_1 \in N^+\} \cup \{77n_1+28 \mid n_1 \in N^+\} \cup \{77n_1+35 \mid n_1 \in N^+\} \cup \{77n_1+42 \mid n_1 \in N^+\} \cup \{77n_1+49 \mid n_1 \in N^+\} \cup \{77n_1+56 \mid n_1 \in N^+\} \cup \{77n_1+63 \mid n_1 \in N^+\} \cup \{77n_1+70 \mid n_1 \in N^+\}$

$\{7n+1 \mid n \in N^+\} = \{77n_1+1 \mid n_1 \in N^+\} \cup \{77n_1 \quad n_1 N^+ \in\} \cup \{77+8 \mid \ \in\} \cup \{77n_1+15 \mid n_1 \in N^+\} \cup \{77n_1+22 \mid n_1 \in N^+\} \cup \{77n_1+29 \mid n_1 \in N^+\} \cup \{77n_1+36 \mid n_1 \in N^+\} \cup \{77n_1+43 \mid n_1 \in N^+\} \cup \{77n_1+50 \mid n_1 \in N^+\} \cup \{77n_1+57 \mid n_1 \in N^+\} \cup \{77n_1+64 \mid n_1 \in N^+\} \cup \{77n_1+71 \mid n_1 \in N^+\}$

$\{7n+2 \mid n \in N^+\} = \{77n_1+2 \mid n_1 \in N^+\} \cup \{77n_1+9 \mid n_1 \in N^+\} \cup \{77n_1+16 \mid n_1 \in N^+\} \cup \{77n_1+23 \mid n_1 \in N^+\} \cup \{77n_1+30 \mid n_1 \in N^+\} \cup \{77n_1+37 \mid n_1 \in N^+\} \cup \{77n_1+44 \mid n_1 \in N^+\} \cup \{77n_1+51 \mid n_1 \in N^+\} \cup \{77n_1+58 \mid n_1 \in N^+\} \cup \{77n_1+65 \mid n_1 \in N^+\} \cup \{77n_1+72 \mid n_1 \in N^+\}$

$\{7n+3 \mid n \in N^+\} = \{77n_1+3 \mid n_1 \in N^+\} \cup \{77n_1+10 \mid n_1 \in N^+\} \cup \{77n_1+17 \mid n_1 \in N^+\} \cup \{77n_1+24 \mid n_1 \in N^+\} \cup \{77n_1+31 \mid n_1 \in N^+\} \cup \{77n_1+38 \mid n_1 \in N^+\} \cup \{77n_1+45 \mid n_1 \in N^+\} \cup \{77n_1+52 \mid n_1 \in N^+\} \cup \{77n_1+59 \mid n_1 \in N^+\} \cup \{77n_1+66 \mid n_1 \in N^+\} \cup \{77n_1+73 \mid n_1 \in N^+\}$

$\{7n+4 \mid n \in N^+\} = \{77n_1+4 \mid n_1 \in N^+\} \cup \{77n_1+11 \mid n_1 \in N^+\} \cup \{77n_1+18 \mid n_1 \in N^+\} \cup \{77n_1+25 \mid n_1 \in N^+\} \cup \{77n_1+32 \mid n_1 \in N^+\} \cup \{77n_1+39 \mid n_1 \in N^+\} \cup \{77n_1+46 \mid n_1 \in N^+\} \cup \{77n_1+53 \mid n_1 \in N^+\} \cup \{77n_1+60 \mid n_1 \in N^+\} \cup \{77n_1+67 \mid n_1 \in N^+\} \cup \{77n_1+74 \mid n_1 \in N^+\}$

$\{7n+5 \mid n \in N^+\} = \{77n_1+5 \mid n_1 \in N^+\} \cup \{77n_1+12 \mid n_1 \in N^+\} \cup \{77n_1+19 \mid n_1 \in N^+\} \cup \{77n_1+26 \mid n_1 \in N^+\} \cup \{77n_1+33 \mid n_1 \in N^+\} \cup \{77n_1+40 \mid n_1 \in N^+\} \cup \{77n_1+47 \mid n_1 \in N^+\} \cup \{77n_1+54 \mid n_1 \in N^+\} \cup \{77n_1+61 \mid n_1 \in N^+\} \cup \{77n_1+68 \mid n_1 \in N^+\} \cup \{77n_1+75 \mid n_1 \in N^+\}$

$\{7n+6 \mid n \in N^+\} = \{77n_1+6 \mid n_1 \in N^+\} \cup \{77n_1+13 \mid n_1 \in N^+\} \cup \{77n_1+20 \mid n_1 \in N^+\} \cup \{77n_1+27 \mid n_1 \in N^+\} \cup \{77n_1+34 \mid n_1 \in N^+\} \cup \{77n_1+41 \mid n_1 \in N^+\} \cup \{77n_1+48 \mid n_1 \in N^+\} \cup \{77n_1+55 \mid n_1 \in N^+\} \cup \{77n_1+62 \mid n_1 \in N^+\} \cup \{77n_1+69 \mid n_1 \in N^+\} \cup \{77n_1+76 \mid n_1 \in N^+\}$

That is, 7n is divided into the sum of eleven subsets consisting of $77n_1$, $77n_1+7$, $77n_1+14$, $77n_1+21$, $77n_1+28$, $77n_1+35$, $77n_1+42$, $77n_1+49$, $77n_1+56$, $77n_1+63$, $77n_1+70$;

7n+1 is divided into the sum of eleven subsets consisting of $77n_1+1$, $77n_1+8$, $77n_1+15$, $77n_1+22$, $77n_1+29$, $77n_1+36$, $77n_1+43$, $77n_1+50$, $77n_1+57$, $77n_1+64$, $77n_1+71$;

7n+2 is divided into the sum of eleven subsets consisting of $77n_1+2$, $77n_1+9$, $77n_1+16$, $77n_1+23$, $77n_1+30$, $77n_1+37$, $77n_1+44$, $77n_1+51$, $77n_1+58$, $77n_1+65$, $77n_1+72$;

7n+3 is divided into the sum of eleven subsets consisting of $77n_1+3$, $77n_1+10$, $77n_1+17$, $77n_1+24$, $77n_1+31$, $77n_1+38$, $77n_1+45$, $77n_1+52$, $77n_1+59$, $77n_1+66$, $77n_1+73$;

7n+4 is divided into the sum of eleven subsets consisting of $77n_1+4$, $77n_1+11$, $77n_1+18$, $77n_1+25$, $77n_1+32$, $77n_1+39$, $77n_1+46$, $77n_1+53$, $77n_1+60$, $77n_1+67$, $77n_1+74$;

7n+5 is divided into the sum of eleven subsets consisting of 77 n_1 +5, 77 n_1 +12, 77 n_1 +19, 77 n_1 +26, 77 n_1 +33, 77 n_1 +40, 77 n_1 +47, 77 n_1 +54, 77 n_1 +61, 77 n_1 +68, 77 n_1 +75;

7n+6 is divided into the sum of eleven subsets consisting of 77 n_1 +6, 77 n_1 +13, 77 n_1 +20, 77 n_1 +27, 77 n_1 +34, 77 n_1 +41, 77 n_1 +48, 77 n_1 +55, 77 n_1 +62, 77 n_1 +69, 77 n_1 +76.

The total gives 77sets of a n_1 + b with n_1 coefficients all be 77.

Feature I:

In the above 77 sets,

There are only 7 sets divided by 11 with remainder 0.

There are only 7 sets divided by 11 with remainder 1.

There are only 7 sets divided by 11 with remainder 2.

There are only 7 sets divided by 11 with remainder 3.

There are only 7 sets divided by 11 with remainder 4.

There are only 7 sets divided by 11 with remainder 5.

There are only 7 sets divided by 11 with remainder 6.

There are only 7 sets divided by 11 with remainder 7.

There are only 7 sets divided by 11 with remainder 8.

There are only 7 sets divided by 11 with remainder 9.

There are only 7 sets divided by 11 with remainder 10.

A total of 7 sets of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

Feature 2: The coefficients of n_1 in sets a n_1 + b are all 77.

Feature 3: The value of the remainder b of of set a n_1 + b is in the closed interval 0 to 76.

The total gives 77 sets of a n_1 + b with n_1 coefficients of all be 77.

From these three characteristics and the transitivity of set, the following result could be derived.

If the seven sets 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 are all subsets of $K \cup L \cup S$. Then in $K \cup L \cup S$, there must be 7 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11, and these 7 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11 are the ones with remainder b in $K \cup L \cup S$ in the closed interval 0 to 76 and n_1 the coefficients are all 77 of an_1 + b subsets, which otherwise do not satisfy the transitivity of set.

So we want to analyze 7n,7n+1,7n+2,7n+2,7n+3,7n+4,7n+5,7n+6 with $K \cup L \cup S$ using the characteristics of 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 modulo 11 respectively, to find a n_1 + b in K ∪ L ∪ S whose remainder b has values in the closed interval from 0 to 76 and whose n_1 coefficients are all 77 sets.

Step 2: Find the set of a n_1 + b with remainder b of K ∪ L ∪ S is in the closed interval 0 to 76 and the coefficient of n_1 is 77.

Since it could be drawn from Section 5: Eq. (5.1) is the general solution formula of the subset corresponding to 2mn + n + m, i.e., a n_1 +b, which is result of each subset, such as 3n+1,5n+2,7n+3,9n+4,...of K ∪ L ∪ S. Actually, it is also the general solution formula of the subset corresponding to 2mn + n + m, i.e., a n_1 +b, which is result of each subset, such as 3m+1,5m+2,7m+3,9m+4,...of K∪L∪S.

Eq. (5.2) is the general solution formula of the subset corresponding to 2mn + n + m-1, i.e., a n_1 +b, which is result of each subset, such as 3n,5n+1,7n+2,9n+3,...of K∪L∪S. Actually, it is also the general solution formula of the subset corresponding to 2mn+n+m-1, i.e., a n_1 +b, which is result of each subset, such as 3m,5m+1,7m+2,9m+3,...of K∪L∪S.

Eq. (5.3) is the general solution formula of the subset corresponding to 2mn + n + m-3, i.e., a n_1 +b, which is result of each subset, such as 3n-2,5n-1,7n,9n+1,...of K∪L∪S. Actually, it is also the general solution formula of the subset corresponding to 2mn+n+m-3, i.e., a n_1 +b, which is result of each subset, such as 3m-2,5m-1,7m,9m+1,...of K∪L∪S.

From equations (5.1), (5.2), (5.3)it is obvious that when the coefficient of n_1 is 77, there are only two cases, i.e., $p_1p_2p_3...p_k$ = 7 with x = 11, and x = 7 with $p_1p_2p_3...p_k$ =11.

7.2.1 The case of $p_1p_2 \cdots p_k$ =7 ,x=11

Substituting $p_1p_2 \cdots p_k$ =7, x=11 into (5.1) yields: 77 n_1+3, 77 n_1+10, 77 n_1+17, 77 n_1+24, 77 n_1+31, 77 n_1 +38, 77 n_1+45, 77 n_1+52, 77 n_1+59, 77 n_1 +66, 77 n_1+73.There are eleven sets and one set each with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

Substituting $p_1p_2 \cdots p_k$ =7, x=11 into (5.2) yields: 77 n_1+2, 77 n_1+9, 77 n_1+16, 77 n_1+23, 77 n_1+30, 77 n_1+37, 77 n_1+44, 77 n_1 +51, 77 n_1 +58, 77 n_1 +65, 77 n_1+72. There are eleven sets and one set each with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

Substituting $p_1p_2 \cdots p_k$ =7, x=11 into (5.3) yields: 77 n_1, 77 n_1+7, 77 n_1+14, 77 n_1 +21, 77

n_1+28, 77n_1+35, 77n_1+42, 77n_1 +49, 77n_1+56, 77n_1 +63, 77n_1 +70. There are eleven sets and one set each with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

7.2.2 The case of $p_1 p_2 \cdots p_k$ case of =11 ,x=7

Substituting $p_1 p_2 \cdots p_k$ =11,x=7 into (5.1) yields: 77n_1+5, 77n_1+16, 77n_1+27, 77n_1 +38, 77n_1 +49, 77n_1 +60, 77n_1 +71.There are seven sets and all are divided by 11 with remainder 5.

Substituting $p_1 p_2 \cdots p_k$ =11 ,x=7 into (5.2) yields: 77n_1+4, 77n_1+15, 77n_1+26, 77n_1 +37, 77n_1+48, 77n_1 +59, 77n_1 +70.There are seven sets and all are divided by 11 with remainder 4.

Substituting $p_1 p_2 \cdots p_k$ =11, x=7 into (5.3) yields: 77n_1+2, 77n_1+13, 77n_1+24, 77n_1 +35, 77n_1+46, 77n_1 +57, 77n_1 +68 There are seven sets and all are divided by 11 with remainder 2.

In summary, the following are as described in 7.2.1 and 7.2.2.

1) We get 77n_1+3, 77n_1+10, 77n_1+17, 77n_1+24, 77n_1+31, 77n_1+38, 77n_1 +45, 77n_1+52, 77n_1+59, 77n_1 +66, 77n_1 +73 for a total of eleven sets and one set each of 0,1,2,3,4,5,6,7,8,9,10 divided by 11, while 77n_1+ 3 is the smallest and 77n_1+ 73 is the largest.

(2) We get 77n_1+2, 77n_1+9, 77n_1+16, 77n_1+23, 77n_1+30, 77n_1+37, 77n_1+44, 77n_1 +51, 77 n_1 +58, 77 n_1 +65, 77 n_1 +72 for a total of eleven sets and one set each of 0,1,2,3,4,5,6,7,8,9,10 divided by 11, while 77n_1+2 is the smallest and 77n_1+72 is the largest.

3) We get 77n_1, 77n_1+7, 77n_1+14, 77n_1+21, 77n_1+28, 77n_1+35, 77n_1+42, 77n_1 +49, 77 n_1+56, 77n_1 +63, 77n_1 +70 for a total of eleven sets and one set each of 0,1,2,3,4,5,6,7,8,9,10 divided by 11, while 77n_1 in the smallest and 77n_1+70 is the largest.

4) We get 77n_1+5, 77n_1+16, 77n_1+27, 77n_1 +38, 77n_1 +49, 77n_1 +60, 77n_1 +71 seven sets which are all divided by 11 with remainder 5, while 77n_1+5 is the smallest and 77n_1+71 is the largest.

5) We get 77n_1+4, 77n_1+15, 77n_1+26, 77n_1 +37, 77n_1+48, 77n_1 +59, 77n_1 +70 seven sets which are all divided by 11 with remainder 4, while 77n_1+4 is the smallest and 77n_1+70 is the

largest.

6) We get 77n_1+2, 77n_1+13, 77n_1+24, 77n_1 +35, 77n_1+46, 77n_1 +57, 77n_1 +68seven sets which are all divided by 11 with remainder 2, while 77n_1+2 is the smallest and 77n_1+68 is the largest.

From 1), 2), 3), 4), 5), 6), they totally gie 11 + 11 + 11 + 7 + 7 + 7 = 54 subsets. The smallest of these sets is 77n_1, and the largest set is 77n_1+ 73.

The above 54 subsets are further classified which is divided by 11 with the remainder 0,1,2,3,4,5,6,7,8,9,10 as follows.

There are only 3 sets with the remainder 0 divided by 11.

There are only 3 sets with the remainder 1 divided by 11.

There are only 3 + 7 sets with the remainder 2 divided by 11.

There are only 3 sets with the remainder 3 divided by 11.

There are only 3 + 7 sets with the remainder 4 divided by 11.

There are only 3 + 7 sets with the remainder 5 divided by 11.

There are only 3 sets with the remainder 6 divided by 11.

There are only 3 sets with the remainder 7 divided by 11.

There are only 3 sets with the remainder 8 divided by 11.

There are only 3 sets with the remainder 9 divided by 11.

There are only 3 sets with the remainder 10 divided by 11.

Totally there are 3 x 11 + 3 x 7 = 54 sets of an_1+ b.

Feature 1: Among the 54 a n_1+ b sets obtained above, since there are only 3 sets each with remainder 0,1,3,6,7,8,9,10 divided by 11, at most three groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11 could be formed.

Feature 2: The coefficient n_1 of the 54 an_1+b sets are all 77.

Feature 3: The remainder b of the 54 an_1+ b sets are in the closed interval 0 to 73.

It could be drawn from Proposition 6.1 that in the $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 ≤ b ≥ (x - 1) $p_1p_2\text{L}\ p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ closed interval, there are only equations (5.1), (5.2), (5.3) corresponding to all an_1+ b subsets of K∪L∪S (m,n,k,n_1 all take natural number,p_1,p_2, ...p_k may be identical, but none of them is equal to x . x, p_1 .p_2 , ...,p_k all take odd prime number).

(1) Substituting $p_1p_2\text{L}\ p_k$ =7, x=11 into $\frac{p_1p_2p_3\cdots p_k-1}{2}$ - 3 ≤ b ≥ (x - 1) $p_1p_2\text{L}\ p_k$

$+\frac{p_1p_2p_3\cdots p_k-1}{2}$ yields $0 \le b \ge 73$.

2) Substituting $p_1p_2\text{L}\ \ p_k$ =11, x=7 into $\frac{p_1p_2p_3\cdots p_k-1}{2}$ - 3 ≤ b ≥ (x - 1) $p_1p_2\text{L}\ \ p_k$ $+\frac{p_1p_2p_3\cdots p_k-1}{2}$ yields $2 \le b \ge 71$.

Clearly $0 \le b \ge 73$ contains $2 \le b \ge 71$

Then the remainder b in the set a n_1 + b obtained by equations (5.1), (5.2), (5.3) is in the closed interval 0 to 73.

Therefore, there are only 54 sets of a n_1 + b in K∪L∪S of which all the remainder b are in the closed interval 0 to 73 and the coefficient n_1 is 77. Since among the 54 sets,there are only 3 sets each with remainder 0,1,3,6,7,8,9,10 divided by 11, at most three groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11 could be formed.

Also, because the positive integer values of the remainder b in the closed interval 0 to 76 have three more values 74,75,76 than the positive integer values of the remainder b in the closed interval 0 to 73.

So the remainder b of K∪L∪S is in the closed interval from 0 to 73 and the coefficients n_1 is all 77,plus the sets 77 n_1 + 74, 77 n_1 + 75, 77 n_1 + 76,it could ensure b is in the closed interval 0 to 76.

Then considering the case of adding three sets 77n_1 +74, 77n_1 +75, 77n_1 +76：

Since among the 54 sets of K∪L∪S above ,except for the sets with remainder 2,4,5 divided by 11,there are only 3 sets each with remainder 0,1,3,6,7,8,9,10 divided by 11, at most three groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11 could be formed. If we add another group of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11, we must add at least each set with remainder 0,1,3,6,7,8,9,10 divided by 11, then we must add at least 8 sets.Obviously ,adding the three sets 77 n_1 + 74, 77 n_1 + 75, 77 n_1 + 76,it still could not reach 8 sets at least.Then it could not add the complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

It means that adding 77 n_1 + 74, 77 n_1 + 75, 77 n_1 + 76 to the above 54 sets of K∪L ∪S still could forms at most 3 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

So all remainder b in K∪L∪S is in the closed interval from 0 to 73 and coefficient n_1 is all 77， which could form at most 3 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

Step 3: Analyze the 7 sets of 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 with K∪L∪S

From the result of the first step of 7.2 that:if the 7 sets of 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 are all K∪L∪S subsets, then there must be 7 groups of complete system of residues in K∪L∪S with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11, and these 7 groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11 are the ones with remainder b in K∪L∪S is in the closed interval 0 to 76 and the coefficient n_1 is all 77 of an_1 + b subsets .

And, since it could be drawn from the second step of 7.2 that the set of all a n_1 + b sets in K ∪ L ∪ S with remainder b in the closed interval 0 to 76 and coefficient n_1 is all 77,which could form at most three groups of complete system of residues with remainder 0,1,2,3,4,5,6,7,8,9,10 divided by 11.

Therefore, at most 3 of the 7 sets of 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 satisfy the set transitivity by transferring the corresponding subsets to K∪L∪S.

So, at most 3 of the 7 sets of 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 are subsets of K∪L∪S.

Then, when n ≥ 0 takes natural number, there are at least 4 sets out of the 7 sets 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 that are not subsets of K∪L∪S.

According to Proposition 2.3, the set of positive integer not belonging to K ∪ L ∪ S, must be the set containing triple prime roots.

Then, when n ≥ 0 takes natural number, at least four of the seven sets 7n,7n+1,7n+2,7n+3,7n+4,7n+5,7n+6 are sets containing triple prime roots.

So, when p = 7, Proposition 4.1 holds.

7.3 p ≥ 11 takes prime

Step 1: Analyze the results of pn, pn+1, pn+2, ..., pn+p-1 each modulo 7.

Let pn, pn+1, pn+2, ..., pn+p-1 modulo 7 respectively, the theory of complete system of residues yields p groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7. i.e.,

$\{pn \mid n \in N^+\}=\{7pn_1 \mid n_1 \in N^+\}\cup\{7pn_1+p \mid n_1 \in N^+\}\cup\{7pn_1+2p \mid n_1 \in N^+\}\cup\{7pn_1+3p \mid n_1 \in N^+\}\cup\{7pn_1+4p \mid n_1 \in N^+\}\cup\{7pn_1+5p \mid n_1 \in N^+\}\cup\{7pn_1+6p \mid n_1 \in N^+\}$

$\{pn+1 \mid n\in N^+\}=\{7pn_1+1 \mid n_1 \in N^+\}\cup\{7pn_1+p+1 \mid n_1 \in N^+\}\cup\{7pn_1+2p+1 \mid n_1 \in N^+\}\cup\{7pn_1+3p+1 \mid n_1 \in N^+\}\cup\{7pn_1+4p+1 \mid n_1 \in N^+\}\cup\{7pn_1+5p+1 \mid n_1 \in N^+\}\cup\{7pn_1+6p+1 \mid n_1 \in N^+\}$

$\{pn+2 \mid n\in N^+\}=\{7pn_1+2 \mid n_1 \in N^+\}\cup\{7pn_1+p+2 \mid n_1 \in N^+\}\cup\{7pn_1+2p+2 \mid n_1 \in N^+\}\cup\{7pn_1+3p+2 \mid n_1 \in N^+\}\cup\{7pn_1+4p+2 \mid n_1 \in N^+\}\cup\{7pn_1+5p+2 \mid n_1 \in N^+\}\cup\{7pn_1+6p+2 \mid n_1 \in N^+\}$

...

$\{pn+p-1 \mid n\in N^+\}=\{7pn_1+p-1 \mid n_1 \in N^+\}\cup\{7pn_1+2p-1 \mid n_1 \in N^+\}\cup\{7pn_1+3p-1 \mid n_1 \in N^+\}\cup\{7pn_1+4p-1 \mid n_1 \in N^+\}\cup\{7pn_1+5p-1 \mid n_1 \in N^+\}\cup\{7pn_1+6p-1 \mid n_1 \in N^+\}\cup\{7pn_1+7p-1 \mid n_1 \in N^+\}$

pn is divided into each set with remainder 0,1,2,3,4,5,6 divided by 7 of the union of seven subsets of $7pn_1$, $7pn_1+p$, $7pn_1+2p$, $7pn_1+3p$, $7pn_1+4p$, $7pn_1+5p$, $7pn_1+6p$.

pn+1 is divided into each set with remainder 0,1,2,3,4,5,6 divided by 7 of the union of seven subsets of $7pn_1+1$, $7pn_1+p+1$, $7pn_1+2p+1$, $7pn_1+3p+1$, $7pn_1+4p+1$, $7pn_1+5p+1$, $7pn_1+6p+1$.

pn+2 is divided into each set with remainder 0,1,2,3,4,5,6 divided by 7 of the union of seven subsets of $7pn_1$ +2, $7pn_1$ +p+2, $7pn_1$ +2p+2, $7pn_1$ +3p+2, $7pn_1$ +4p+2, $7pn_1$ +5p+2, $7pn_1$ +6p+2.

...

pn+p-1 is divided into each set with remainder 0,1,2,3,4,5,6 divided by 7 of the union of seven subsets of $7pn_1$ +p-1, $7pn_1$ +2p-1, $7pn_1$ +3p-1, $7pn_1$ +4p-1, $7pn_1$ +5p-1, $7pn_1$ +6p-1, $7pn_1$ +7p-1.

Feature 1: It gives a total of 7p sets of an_1 +b,coefficient n_1 is all 7p,and forms p groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7.

There are only p sets with the remainder 0 divided by 7;

There are only p sets with the remainder 1 divided by 7;

There are only p sets with the remainder 2 divided by 7;

There are only p sets with the remainder 3 divided by 7;

There are only p sets with the remainder 4 divided by 7;

There are only p sets with the remainder 5 divided by 7;

There are only p sets with the remainder 6 divided by 7;

Total 7p collections.

Feature 2: The coefficient n_1 in 7p an_1 +b sets is all 7p.

Feature 3: The resulting 7p an_1 +b set with remainder b has values in the closed interval from 0 to 7p-1 .

From the above three features and set transitivity, it could be drawn that if the p sets pn, pn+1, pn+2, ...,pn+p-1 are all subsets of K∪L∪S, then when the p sets of pn, pn+1, pn+2, ...,pn+p-1 are each modulo 7, the corresponding p groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7 necessarily in the subsets of an_1 + b of the ones with remainder b in K∪L∪S is in the closed interval 0 to 76 and the coefficient n_1 is all 7p-1.

So we want to use the p sets of pn, pn+1, pn+2, ...,pn+p-1 each modulo 7 to analyze the relationship between the p sets of pn, pn+1, pn+2, ...,pn+p-1 and K∪L∪S by finding the values of the remainder b in K∪L∪S is in the closed interval from 0 to 7p-1 and coefficients n_1 is all 7p of the an_1 + b sets.

Step 2: Find the remainder b of K∪L∪S in the closed interval from 0 to 7p-1 and the coefficient n_1 is 7p of the an_1 +b set.

It is clear from equations (5.1), (5.2), (5.3) that when the coefficient n_1 is 7p, there are and only two cases, i.e., $p_1p_2p_3...p_k$ =7, x=p and x=7,$p_1p_2p_3...p_k$ =p.

7.3.1 The case of $p_1p_2\cdots p_k$=7,x=p

Substituting $p_1p_2\cdots p_k$=7,x=p into equation (5.1) yields: $7pn_1$ +3, $7pn_1$ +10, $7pn_1$ +17, ..., $7pn_1$ +7(p-1)+3 total of p sets, and all of the sets are divided by 7 with remainder 3.

Substituting $p_1p_2\cdots p_k$=7,x=p into equation (5.2) yields: $7pn_1$ +2, $7pn_1$ +9, $7pn_1$ +16, ...,

$7pn_1$ +7(p-1)+2 total of p sets, and all of the sets are divided by 7 with remainder 2.

Substituting $p_1p_2 \cdots p_k$ =7,x=p into equation (5.3) yields: $7pn_1$, $7pn_1$ +7, $7pn_1$ +14, ..., $7pn_1$ +7(p-1) total of p sets, and all of the sets are divided by 7 with remainder 0..

A total of 3p sets obtained.

7.3.2 The case of $p_1p_2 \cdots p_k$ =p, x=7

Substituting $p_1p_2 \cdots p_k$ =p,x=7 into equation (5.1) yields $7pn_1+\frac{p-1}{2}$,$7pn_1+p+\frac{p-1}{2}$, $7pn_1+2p+\frac{p-1}{2}$, $7pn_1+3p+\frac{p-1}{2}$, $7pn_1+4p+\frac{p-1}{2}$, $7pn_1+5p+\frac{p-1}{2}$, $7pn_1+6p+\frac{p-1}{2}$, which are total 7 sets.

For $\{7pn_1+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+p+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+2p+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+3p+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+4p+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+5p+\frac{p-1}{2} \mid n_1 \in N^+\} \cup \{7pn_1+6p+\frac{p-1}{2} \mid n_1 \in N^+\}=\{pn+\frac{p-1}{2} \mid n \in N^+\}$

i.e. $7pn_1+\frac{p-1}{2}$, $7pn_1+p+\frac{p-1}{2}$, $7pn_1+2p+\frac{p-1}{2}$, $7pn_1+3p+\frac{p-1}{2}$, $7pn_1+4p+\frac{p-1}{2}$, $7pn_1+5p+\frac{p-1}{2}$, $7pn_1+6p+\frac{p-1}{2}$ .A decomposed group of complete system of residues each with remainder 0,1,2,3,4,5,6 divided by 7,based on $pn+\frac{p-1}{2}$ modulo 7.

Then in $7pn_1+\frac{p-1}{2}$, $7pn_1+p+\frac{p-1}{2}$, $7pn_1+2p+\frac{p-1}{2}$, $7pn_1+3p+\frac{p-1}{2}$, $7pn_1+4p+\frac{p-1}{2}$, $7pn_1+5p+\frac{p-1}{2}$, $7pn_1+6p+\frac{p-1}{2}$: with remainder 0 divided by 7 occupies a set, with remainder 1 divided by 7 occupies a set, with remainder 2 divided by 7 occupies a set, with remainder 3 divided by 7 occupies a set; with remainder 4 divided by 7 occupies a set, with remainder 5 divided by 7 occupies a set, with remainder 6 divided by 7occupies a set, for a total of 7 sets.

Substituting $p_1p_2 \cdots p_k$ =p , x = 7 into equation (5.2) yields

$7pn_1+\frac{p-1}{2}$ -1, $7pn_1+p+\frac{p-1}{2}$ -1, $7pn_1+2p+\frac{p-1}{2}$ +1, $7pn_1+3p+\frac{p-1}{2}$ +1, $7pn_1+4p+\frac{p-1}{2}$ +1, $7pn_1+5p+\frac{p-1}{2}$ +1, $7pn_1+6p+\frac{p-1}{2}$ -1 for a total of 7 sets.

For $\{7pn_1+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+p+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+2p+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+3p+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+4p+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+5p+\frac{p-1}{2}-1 \mid n_1 \in N^+\} \cup \{7pn_1+6p+\frac{p-1}{2}-1 \mid n_1 \in N^+\}=\{pn+\frac{p-1}{2}-1 \mid n \in N^+\}$

i.e. $7pn_1+\frac{p-1}{2}-1$, $7pn_1+p+\frac{p-1}{2}-1$, $7pn_1+2p+\frac{p-1}{2}+1$, $7pn_1+3p+\frac{p-1}{2}+1$, $7pn_1+4p+\frac{p-1}{2}+1$, $7pn_1+5p+\frac{p-1}{2}+1$, $7pn_1+6p+\frac{p-1}{2}-1$ is $pn+\frac{p-1}{2}-1$.A decomposed group of complete system of residues each with remainder 0,1,2,3,4,5,6 divided by 7,based on $pn+\frac{p-1}{2}-1$ modulo 7.

Then in $7pn_1+\frac{p-1}{2}-1$, $7pn_1+p+\frac{p-1}{2}-1$, $7pn_1+2p+\frac{p-1}{2}+1$, $7pn_1+3p+\frac{p-1}{2}+1$, $7pn_1+4p+\frac{p-1}{2}-1$, $7pn_1+5p+\frac{p-1}{2}-1$, $7pn_1+6p+\frac{p-1}{2}$ In -1: with remainder 0 divided by 7 occupies a set, with remainder 1 divided by 7 occupies a set, with remainder 2 divided by 7 occupies a set, with remainder 3 divided by 7 occupies a set; with remainder 4 divided by 7 occupies a set, with remainder 5 divided by 7 occupies a set, with remainder 6 divided by 7occupies a set, for a total of 7 sets.

Substituting $p_1p_2 \cdots p_k$ =p , x = 7 into equation (5.3) yields

$7pn_1+\frac{p-1}{2}-3$, $7pn_1+p+\frac{p-1}{2}-3$, $7pn_1+2p+\frac{p-1}{2}-3$, $7pn_1+3p+\frac{p-1}{2}-3$, $7pn_1+4p+\frac{p-1}{2}-3$, $7pn_1+5p+\frac{p-1}{2}-3$, $7pn_1+6p+\frac{p-1}{2}-3$ for a total of 7 sets.

For $\{7pn_1+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+p+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+2p+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+3p+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+4p+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+5p+\frac{p-1}{2}-3 \mid n_1\in N^+\}\cup\{7pn_1+6p+\frac{p-1}{2}-3 \mid n_1\in N^+\}=\{pn+\frac{p-1}{2}-3 \mid n\in N^+\}$

i. e. $7pn_1+\frac{p-1}{2}-3$, $7pn_1+p+\frac{p-1}{2}-3$, $7pn_1+2p+\frac{p-1}{2}-3$, $7pn_1+3p+\frac{p-1}{2}-3$, $7pn_1+4p+\frac{p-1}{2}-3$, $7pn_1+5p+\frac{p-1}{2}-3$, $7pn_1+6p+\frac{p-1}{2}-3$.A decomposed group of complete system of residues each with remainder 0,1,2,3,4,5,6 divided by 7,based on $pn+\frac{p-1}{2}-3$ modulo 7.

Then, in $7pn_1+\frac{p-1}{2}-3$, $7pn_1+p+\frac{p-1}{2}-3$, $7pn_1+2p+\frac{p-1}{2}-3$, $7pn_1+3p+\frac{p-1}{2}-3$, $7pn_1+4p+\frac{p-1}{2}-3$, $7pn_1+5p+\frac{p-1}{2}-3$, $7pn_1+6p+\frac{p-1}{2}$ In -3: with remainder 0 divided by 7 occupies a set, with remainder 1 divided by 7 occupies a set, with remainder 2 divided by 7 occupies a set, with remainder 3 divided by 7 occupies a set; with remainder 4 divided by 7 occupies a set, with remainder 5 divided by 7 occupies a set, with remainder 6 divided by 7occupies a set, for a total of 7 sets.

A total of p + p + p + 7 + 7 + 7 + 7 = 3p + 21 sets of a n_1 + b is obtained as described in 7.3.1

and 7.3.2.

The above set of 3p+21 is then classified which is divided by 7 with the remainder 0,1,2,3,4,5,6 as follows.

There are only 3 + p sets divided by 7 with the remainder 0.

There are only 3 sets divided by 7 with the remainder 1.

There are only 3 + p sets divided by 7 with the remainder 2.

There are only 3 + p sets divided by 7 with the remainder 3.

There are only 3 sets divided by 7 with the remainder 4.

There are only three sets divided by 7 with the remainder 5.

There are only 3 divided by 7 with the remainder 6.

Features:

A total of 3p+21 sets of an_1 + b is obtained.the most important of which are only 3 sets each divided by 7 with remainder 1,4,5,6, and 3+p sets each divided by 7 with remainder 0,2,3, then three groups of complete system of residues divided by 7 with remainder 0,1,2,3,4,5,6 could be formed.

It could be drawn from Proposition 6.1 that in $\frac{p_1p_2p_3\cdots p_k-1}{2}$ -3 $\leq$ b $\geq$ (x - 1) $p_1p_2\text{L}\ p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ closed interval,there are only equations (5.1), (5.2), (5.3) corresponding to all a n_1 +b subsets of K∪L∪S (m,n,k, n_1 all take natural number,p_1,p_2, ...p_k may be identical, but none of them is equal to x . x, p_1 .p_2 , ...,p_k all take odd prime number).

(1) Substituting $p_1p_2\text{L}\ p_k$ =7 ,x=p into $\frac{p_1p_2p_3\cdots p_k-1}{2}$ - 3 $\leq$ b $\geq$ (x - 1) $p_1p_2\text{L}\ p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ yields:

$0 \leq b \geq 7p-4$

2) Substituting $p_1p_2\text{L}\ p_k$ =p ,x=7 into $\frac{p_1p_2p_3\cdots p_k-1}{2}$ - 3 $\leq$ b $\geq$ (x - 1) $p_1p_2\text{L}\ p_k$ +$\frac{p_1p_2p_3\cdots p_k-1}{2}$ yields:

$\frac{p-1}{2}-3 \leq b \geq 6p+\frac{p-1}{2}$

and because when p $\geq$ 11 takes prime, 0 < $\frac{p-1}{2}$ -3, 7p-4 > 6p+$\frac{p-1}{2}$

Then when p $\geq$ 11 takes prime, 0 $\leq$ b $\geq$ 7p - 4 contains $\frac{p-1}{2}$ -3≤b≥6p+$\frac{p-1}{2}$.

That is, the remainder b in the set a n_1 + b from equations (5.1), (5.2), (5.3) is all in the closed interval from 0 to 7p-4.

So when $p_1p_2 L\ p_k$ =7 ,x=p and $p_1p_2 L\ p_k$ =p ,x=7, p≥11 takes prime, the remainder b of K∪L∪S is in the closed interval from 0 to 7p-4 and the coefficient n_1 is 7p of the an_1 +b set,which has only 3p + 21.

Clearly the remainder b in the closed interval from 0 to 7p-1 has three more values 7p-3,7p-2,7p-1 than the remainder b in the closed interval from 0 to 7p-4.

So, the remainder b of K∪L∪S is in the closed interval from 0 to 7p - 4 and the coefficients n_1 is 7p of an_1 +b set plus $7pn_1$ +7p-3, $7pn_1$ +7p-2, $7pn_1$ +7p-1 three sets, then it could ensure b is in the closed interval from 0 to 7p-1.

Then considering the case of adding three sets $7pn_1$+7p-3, $7pn_1$ +7p-2, $7pn_1$ +7p-1 to K∪L∪S beyond the 3p+21 sets.

Since there are only 3 sets in 3p + 21 sets of K∪L∪S with remainder 1,4,5,6, at most 3 sets of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7 could be formed. If one adds another group of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7, we must add at least each set with remainder 1,4,5,6divided by 7, i.e., at least 4 more sets must be added.Obviously, adding $7pn_1$ +7p-3, $7pn_1$ +7p-2, $7pn_1$ +7p-1 three sets, it still could not reach 4 sets at least .Then it could not add the complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7. That is,

It means that adding $7pn_1$+7p-3, $7pn_1$+7p-2, $7pn_1$+7p-1 to the above 3p+21 sets of K∪L ∪ S still could forms at most 3 groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7.

So, all remainder b in K∪L∪S is in the closed interval from 0 to 7p-1 and the coefficient n_1 is 7p of an_1 + b set, which could form at most 3 groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7.

Step 3: Analyze the p sets of pn, pn+1, pn+2, ...,pn+p-1 with K∪L∪S.

From the result of the first step of 7.3 that : if the p sets of pn, pn+1, pn+2, ...,pn+p-1 are all K∪L∪S subsets, then there must be p groups of complete residue in K∪L∪S with remainder 0,1,2,3,4,5,6 divided by 7, and the p groups of complete residue in K∪L∪S with remainder 0,1,2,3,4,5,6 divided by 7 are the ones with remainder b in K∪L∪S is in the closed interval 0 to 7p - 1 and the coefficients n_1 is all 7p of an_1 + b subsets.

And, since it could be drawn from the second step of 7.3 that the set of all a n_1 + b sets in K ∪ L ∪ S with remainder b in the closed interval 0 to 7p-1 and coefficient n_1 is 7p,which could form at most three groups of complete system of residues with remainder 0,1,2,3,4,5,6 divided by 7.

So, at most 3 of the p sets of pn, pn+1, pn+2, ...,pn+p-1 satisfy the set transitivity by transferring the corresponding subsets to K∪L∪S.

So, at most 3 of the p sets of pn, pn+1, pn+2, ...,pn+p-1 are subsets of K∪L∪S.

Then, when n ≥ 0 takes natural number, at least p-3 out of the p sets of pn, pn+1, pn+2, ...,pn+p-1 are not subsets of K∪L∪S.

According to Proposition 2.3, the set of positive integer not belonging to K∪L∪S must be the set containing the triple prime roots.

Then, when n ≥ 0 takes natural number, at least p-3 of the p-sets in pn, pn+1, pn+2, ...,pn+p-1 are sets containing the triple prime roots.

Then Proposition 4.1 holds when p ≥ 11 takes prime.

Since Proposition 4.1 holds when p = 5 and p = 7 has been proved in 7.1 and 7.2. Then Proposition 4.1 holds when p ≥ 5 takes prime.

Since Proposition 4.1 was proved above to be an equivalent proposition of the infinity of triple prime, then the infinity of triple prime holds.

Clearly there is a group of twin prime numbers in every group of triple prime numbers, then there are also infinite many twin prime numbers, i.e., both the triple and twin prime numbers are infinite.

Also, since the triple prime conjecture is the infinity of triple prime and the twin prime conjecture is the infinity of twin prime, then the triple prime conjecture and the twin prime conjecture hold.

Certificate completed.

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