



## Influence of Structural Break on the Power of ADF Unit Root Test

Xinchen Chen

School of Economics, Huazhong University of Science and Technology, Hubei, China

[chenxinchen19@hust.edu.cn](mailto:chenxinchen19@hust.edu.cn)

**Abstract:** With the changes in economic structure and social development, the paths of most economic variables show the characteristics of structural changes. Traditional unit root tests, such as ADF and PP tests, often lead to wrong conclusions about data stationarity characteristics when considering such structural variation factors. This paper considers the test power of ADF statistic with one structural break using Monte Carlo simulation when the data generating process has double breaks in both intercept and slope. The results show that when there is a structural break in the data generating process if the conventional unit root test is performing without considering this break, the ADF test is still efficient when the degree of the structural break is relatively low. Furthermore, this paper discusses the influence of the size of the sample, break position and break degree on the power of the ADF statistic test under the premise of a low break degree.

**Keywords:** Structural Break, ADF Test, Test Power

### 1. INTRODUCTION

#### 1.1 Background

Granger and Newbold (1974) put forward the concept of spurious regression, arguing that if the variables used for regression analysis are non-stationary, the corresponding regression model may also be spurious regression. Therefore, when studying time series, we should first check whether it is stationary because stationary and non-stationary variables have entirely different economic meanings and statistical properties. The traditional view holds that if a time series has a stable trend, then the impact of random shocks on its overall trend is only temporary, and a deterministic time trend function dominates the long-term drive of corresponding economic variables. If the time series has its unit root, any random shock will have a long-term impact, resulting in a permanent effect. If the difference of the time series with the unit root can be stationary, it is called the difference stationary or stochastic trend process. To test whether there is a unit root, scholars have proposed various unit root test methods to determine the stationarity of time series.

The DF (ADF) unit root test proposed by Dickey and Fuller (1979, 1981) is the most widely used unit root test method in empirical analysis. However, scholars have continuously improved it because its test power is not high in the limited sample. Phillips and Perron (1987) use the zero-frequency spectral density estimation test (PP test) to verify that current shocks will permanently affect their long-term levels for most macroeconomic variables. Perron (1989) first introduces the concept of structural break based on the ADF test. They found that if the real DGP (Data Generating Process) is a (trend) stationary process with a structural break, the DF (ADF) test statistic is difficult to

reject the null hypothesis of the unit root, which means the test power tends to zero. The DF (ADF) test statistic misjudges a (trend) stationary process with a structural break as a unit root process, later known as the "Perron phenomenon."

## **1.2 Literature Review**

Perron and Vogelsang (1992) abandon the exogenous assumption of structural break and test all possible structural breakpoints. Then they argue that the real exchange rate between the United States and the United Kingdom, the United States, and Finland are stationary variables with horizontal structural changes, confirming purchasing power parity. At the same time, Zivot and Andrews (1992), Banerjee, Lumsdaine, and Stock (1992) also question the exogenous assumption of Perron's (1989) structural breakpoint. They believe that the position of the test is conditionally dependent, and the test may fail if the structural changes of the original data are not significant. They advocate using the method of endogenous testing to determine the actual structural breakpoint while testing the unit root through the data mining method. The endogenous assumptions in the above studies are all based on the unknown timing of the structural break. Sen (2003) believes that we should also regard the form of the structural break as unknown in this case. He believes that a severe loss of test power will happen if the setting of the break form in the test model does not match the actual data generating process. Therefore, he suggests following the principle from general to particular to find errors in practical application and determine the form and position of the break.

In summary, the different assumptions of endogenous and exogenous structural breaks, the differences in the setting of test models, and the use of critical values often lead to different conclusions. Due to these debates, domestic scholars have paid more attention to the research on the unit root test with a structural break. Wang Shaoping (2003) proposes the unit root test that structural break occurs in a specific interval and carries out simulation experiments. Wang Shaoping and Li Zinai (2003) analyze the structural break of China's exchange rate by using the endogenous and exogenous structural break test and CusumSQ diagnosis. They believe that the data generating process of the RMB exchange rate obeys the unit root process with a structural break, rather than the trend stationary process of a structural break.

Zhang Jianhua and Tu Taotao (2007) use the Monte Carlo method to discuss the validity of the unit root test of economic variables with a structural change point. When there is a structural break in DGP, the regular unit of this change is not considered. The root test does not fail only under certain conditions.

Nie Qiaoping and Ye Guang (2008) believe that when the real DGP is a (trend) stationary process with a single linear structural break, the asymptotic properties of the DF unit root test are not the same as the finite sample properties. The test power is affected not only by the break degree but also by the break position. The appearance and severity of the "Perron phenomenon" are sensitive to the break location. In addition, the autocorrelation structure of the random disturbance term will also affect the power of the DF unit root test. That is to say, when the degree of autocorrelation is different, the degree of the "Perron phenomenon" is also different. Nie Qiaoping and Feng Lei (2008) also propose a set of unit root test procedures considering structural break based on the Perron test under the "innovative outlier model" test analysis framework and analyze the program's limited performance through Monte Carlo simulation in a finite sample.

## **1.3 Research purpose and methodology**

The purpose of this paper is to analyze the validity of the unit root test of economic variables with a structural breakpoint from multiple perspectives. The research method in this paper is to use Monte Carlo simulation to generate the data to be tested. Under the model of double breaks of intercept and slope, we try to examine the efficacy of conventional unit root tests when factors such as sample size, degree of the structural break, and location of structural break are changed.

## **2. THE INFLUENCE OF STRUCTURAL BREAK ON THE POWER OF ADF TEST**

For convenience, this paper adopts the basic linear model:

$$Y_t = a + b_t + \mu$$

The random disturbance term  $\mu$  is assumed to be white noise in the data generating process, where  $t=1, 2, 3,$

4...1000—the sample size  $N=1000$ . Moreover, we assume that the data meets a structural break when the sample period is 501.

$Y_t$  generated according to the linear model above should be a trend stationary process. However, in the case of a structural break, it is easy to invalidate the ADF test and obtain wrong conclusions. Therefore, it is necessary to perform regression analysis and unit root test in sections. Thus, we will use Monte Carlo simulation analysis to explore the power of the ADF test for  $Y_t$  based on both ignoring structural breaks and considering structural breaks in the case of large and small break degrees. To simplify, we consider the case when  $Y_t$  is a continuous variable in this paper.

### 2.1 A significant degree of structural break

When the structural break is not considered, the specific steps are as follows:

1. When  $t \in [1, 500]$ , let  $a=8$ ,  $b=1.5$ , and when  $t \in [501, 1000]$ , let  $a=-742$ ,  $b=3$ . Generate a set of data  $Y_t$ .
2. ADF test is performed directly on  $Y_t$  during the 1000 sample period. Since  $Y_t$  has a non-zero mean and a time trend, the constant term and time trend term should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.
3. Repeat steps 1 and 2 above 100 times.

The test results are shown in the following figure:

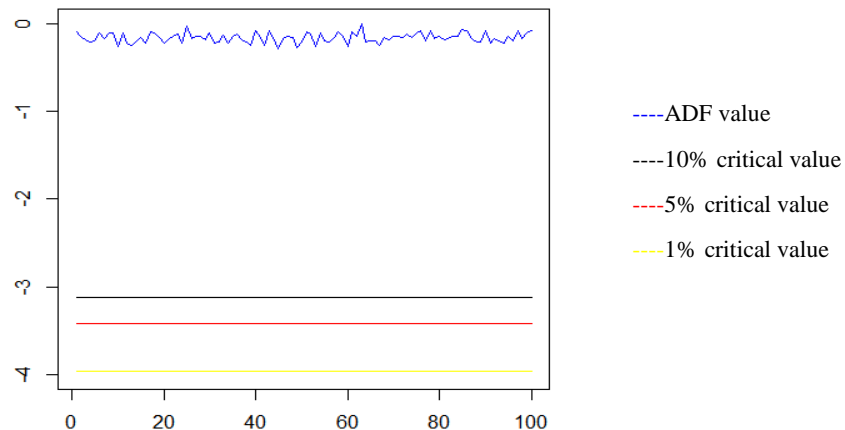


Figure 1. ADF test result under the condition of ignoring structural break

It can be seen from Figure 1 that if the ADF test is carried out directly without considering the structural break, the ADF values obtained from 100 tests are all greater than the 10% critical value, which means that all tests cannot reject the null hypothesis. Thus,  $Y_t$  is rejected as a trend stationary process, and the ADF test power is 0.

When considering structural breaks, the specific steps are as follows:

1. When  $t \in [1, 500]$ , let  $a=8$ ,  $b=1.5$ , and when  $t \in [501, 1000]$ , let  $a=-742$ ,  $b=3$ . Generate a set of data  $Y_t$ .
2. Perform an ADF test on the first 500 samples. Since  $Y_t$  has a non-zero mean and has a time trend, constant terms and time trend terms should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.
3. Carry out the ADF test on the first 500 samples. Since  $Y_t$  has a non-zero mean and has a time trend, constant terms and time trend terms should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.
4. Repeat steps 1 and 2 above 100 times.

The test results are as follows:

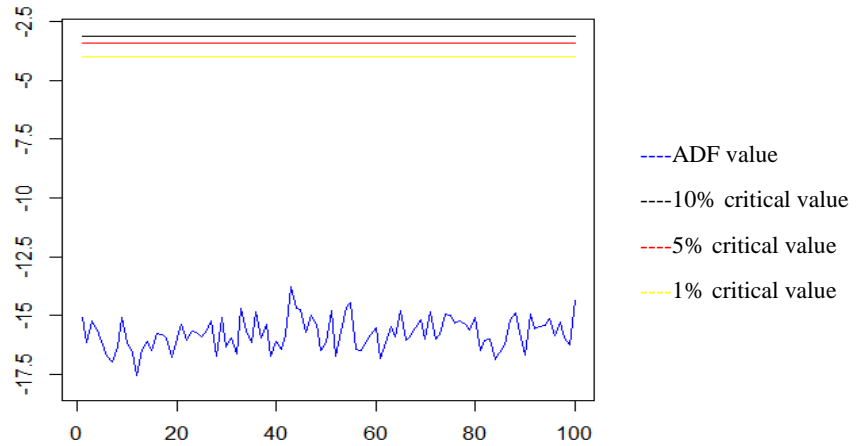


Figure 2. ADF test result of the first 500 sample periods when a structural break is considered

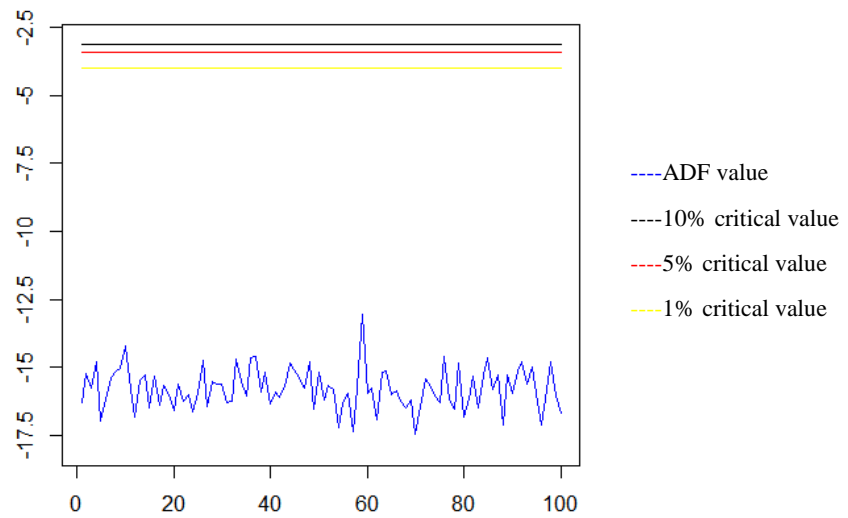


Figure 3. ADF test result of the post-500 sample period when a structural break is considered

We can learn from Figures 2 and 3 when the structural break is considered. The unit root test is performed on the samples before and after the break. The ADF values obtained from 100 experiments are less than the critical value of 1%. It means that all tests reject the null hypothesis.  $Y_t$  is accepted as a trend stationary process while the ADF test power is 100 at this time.

## 2.2 A minor degree of structural break

When structural breaks are not considered, the specific steps are as follows:

1. When  $t \in [1, 500]$ , let  $a=8$ ,  $b=1.5$ ,  $t \in [501, 1000]$ , let  $a=3$ ,  $b=1.51$ . Generate a set of data  $Y_t$ .
2. ADF test is performed directly on  $Y_t$  during the 1000 sample period. Since  $Y_t$  has a non-zero mean and a time trend, the constant term and time trend term should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.
3. Repeat steps 1 and 2 above 100 times.

The test results are shown in the figure below

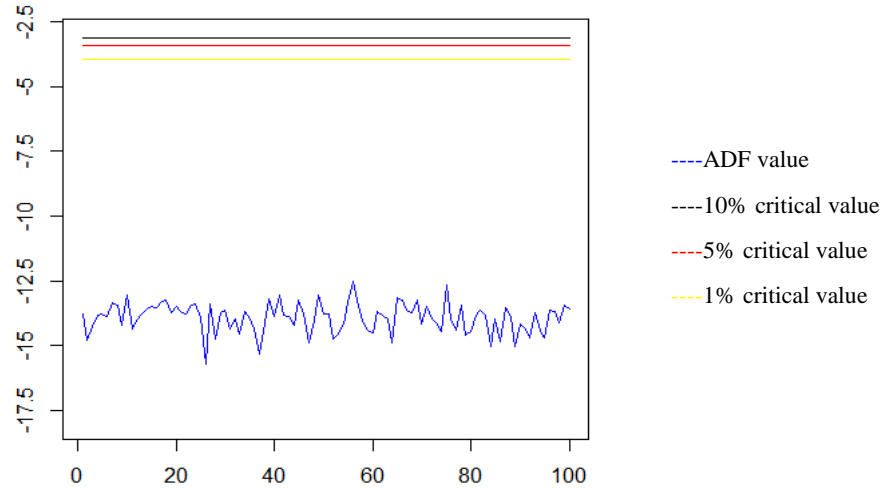


Figure 4. ADF test result ignoring structural break

As shown in Figure 4, if the ADF test is carried out directly without considering the structural break, the ADF values obtained from 100 tests are all less than the 1% critical value. That is to say, all tests reject the null hypothesis, and it is concluded that  $Y_t$  is in the 1000 sample period. Thus,  $Y_t$  is accepted as a trend stationary process, and the ADF test power is 100 at this time.

When considering structural breaks, the specific steps are as follows:

$t \in [1, 500]$ , let  $a=8$ ,  $b=1.5$ , when  $t \in [501, 1000]$ , let  $a=3$ ,  $b=1.51$ . Generate a set of data  $Y_t$ .

2. Perform an ADF test on the first 500 samples. Since  $Y_t$  has a non-zero mean and has a time trend, constant terms and time trend terms should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.

3. The ADF test is performed on the last 500 samples. Since  $Y_t$  has a non-zero mean and a time trend, the constant term and time trend term should be included in the unit root test. The R language determines the lag order and automatically selects according to the AIC guidelines.

4. Repeat steps 1 and 2 above 100 times.

The test results are as follows:

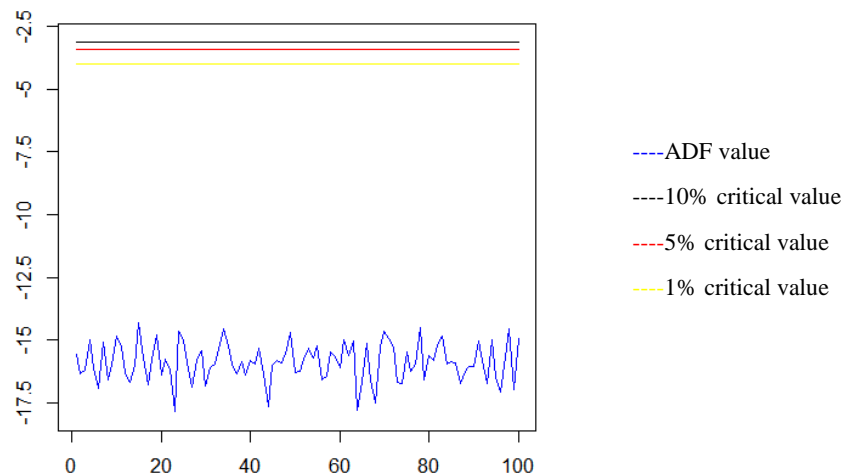


Figure 5. ADF test results of the first 500 sample periods when structural breaks are considered

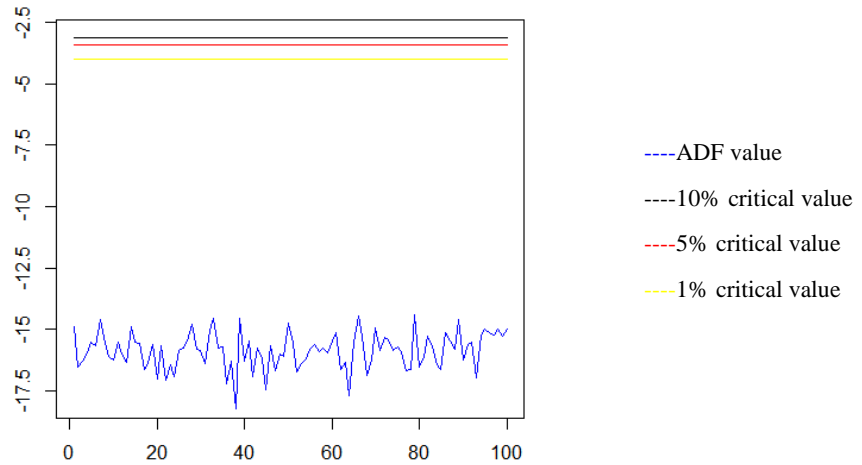


Figure 6. ADF test results of the post-500 sample period when a structural break is considered

We can learn from Figures 5 and 6 when the structural break is considered. The unit root test is performed on the samples before and after the break. The ADF values obtained in 100 trials are less than the critical value of 1%. It means that all tests reject the null hypothesis.  $Y_t$  is a trend stationary process in both sample periods, while the test power of ADF is 100 at this time.

It can be seen from the above analysis that when a structural break occurs during the generation of time series data, if the degree of structural break is relatively large, the conventional ADF test will fail. While if the degree of structural break is relatively low, the traditional ADF test is practical, and the test power can even be as high as 100%. Therefore, the following article will take the occurrence of a small degree of a structural break as the premise, and study the impact of sample size, break position and break degree on the efficacy of the ADF test. That is, to find out the conditions under which the ADF test will not "fail."

### 3. IMPACT OF SAMPLE SIZE CHANGES ON THE POWER OF THE UNIT ROOT TEST

This part mainly studies the influence of sample size on the test power of ADF unit root. The specific test steps are as follows:

1. Keep the structural break position and the degree of the slope of the other two factors unchanged, and the total number of samples is  $N$ . The breaks occur at  $N/2$ , and  $Y_t$  is continuous at the breakpoint. When  $t \in [1, N/2]$ ,  $a=8$ ,  $b=1.5$ , when  $t \in [N/2+1, N]$ ,  $b=1.51$ ,  $a=8-0.005N$
2. Regardless of structural changes, we perform ADF unit root tests on  $Y_t$  with different sample sizes and compare the effects of sample size changes on unit root tests.
3. The number of repetitions of the test is 100 times.

Plot the test results as a graph as shown in Figure 7:

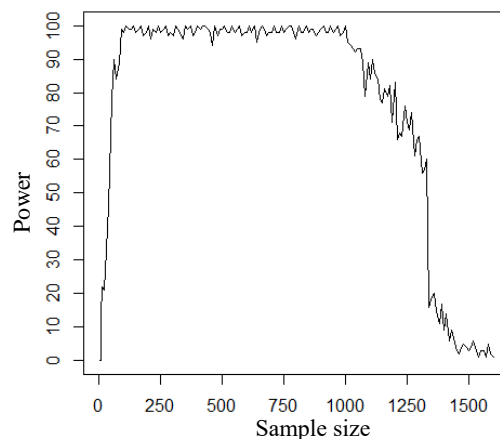


Figure 7 Line chart of ADF test power under different sample sizes

As shown in Figure 7, the test power of ADF shows a trend of increasing--flat--decreasing with the increase of sample size, which shows an inverted "U" shape. When the sample size is between 100 and 1000, the test power

reaches 100, and the ADF test is entirely valid. When the sample size is less than 100 or greater than 1000, the probability of failure of the ADF test will increase. When the sample size is larger than 1600, the power of the ADF test drops to 0 and fails. Therefore, when there are structural changes, the unit root test will be practical only when the sample size is relatively small, between 100 and 1000. If the sample size is too small or too large, the possibility of ADF test failure will increase.

#### 4. THE EFFECT OF POSITION CHANGE ON THE POWER OF UNIT ROOT TEST

This part mainly studies the influence of the positional changes of the structural breakpoints in the time series on the power of the ADF test. The specific test steps are as follows:

1. Keep the total number of samples  $N$  and the degree of the slope unchanged, that is, when  $t \in [1, \lambda N]$ ,  $a=8$ ,  $b=1.5$ , when  $t \in [\lambda N+1, N]$ ,  $b=1.51$ ,  $a=8-0.01 * \lambda N$ , the break occurs at  $\lambda N$ , and  $Y_t$  is continuous at the breakpoint,  $\lambda$  is the break coefficient,  $N=1000$ .
2. Regardless of structural changes, perform unit root test on sequences  $Y_t$  with different breakpoint positions and compare the effect of different breakpoint positions on the power of unit root test.
3. The number of repetitions of the test is 100 times.

Plot the test results as a graph as shown in Figure 8:

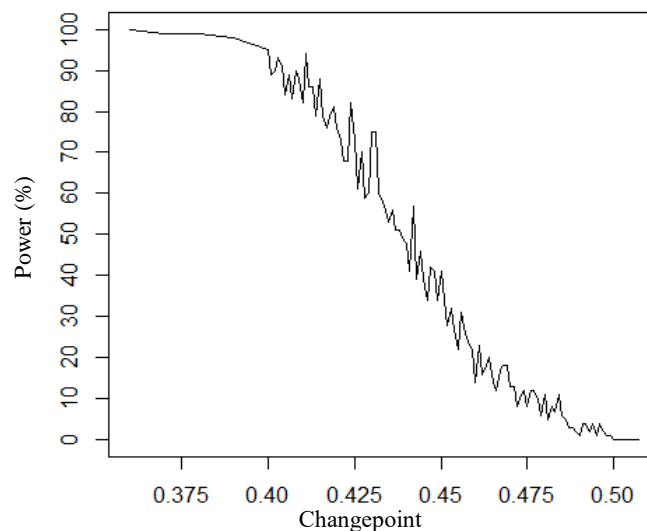


Figure 8 Line chart of the power of ADF test under different break positions

It can be seen from Figure 8 that the higher the break position, the higher the power of the ADF test. When the break position is around 0.5, the test power drops to 0. That is to say, when the position of the structural breakpoint gradually moves backward in the time series, the ADF unit root test will slowly fail until the samples after the structural change increase to half of the total number, and the ADF test will fail.

#### 5. THE EFFECT OF BREAK DEGREE CHANGE ON THE POWER OF UNIT ROOT TEST

This part mainly studies the impact of break degree changes on the efficacy of the ADF test. The specific test steps are as follows:

1. Keep the sample size  $N$  and break position  $N/2$  unchanged, and  $Y_t$  is continuous at the breakpoint, when  $t \in [1, N/2]$ ,  $a=8$ ,  $b=1.5$ , when  $t \in [N/2+1, N]$ ,  $b=1.5+w$ ,  $a=8-N/2*w$ ,  $N=1000$ , and  $w$  is the degree of the slope.
2. Ignoring the structural change, we perform an ADF unit root test on  $Y_t$  in 1000 samples to compare the effect of break degree on the power of the unit root test.
3. The experiment was repeated 100 times.

The test results are drawn into the following three figures, in which Figure 9 is the power of the ADF test when the slope is gradually increased, Figure 10 is the power of the ADF test when the slope is slowly reduced. Figure 11 is the synthesis of the first two figures, considering the effect of slope change on the power of the ADF test comprehensively.

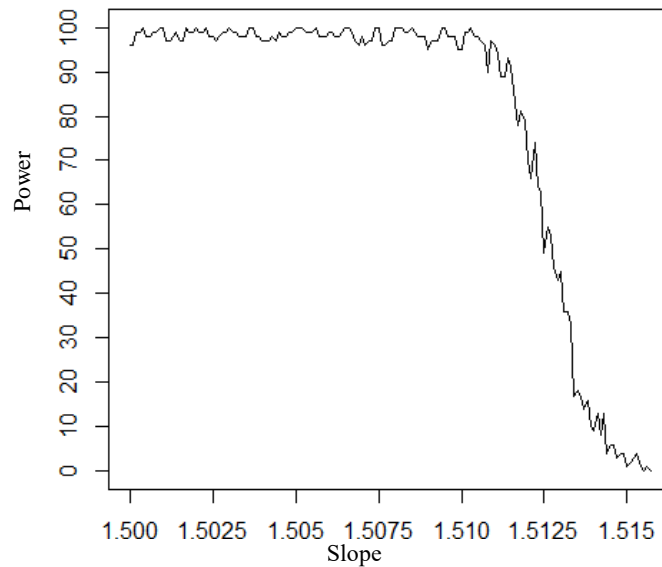


Figure 9 Line graph of ADF test power when the slope increases

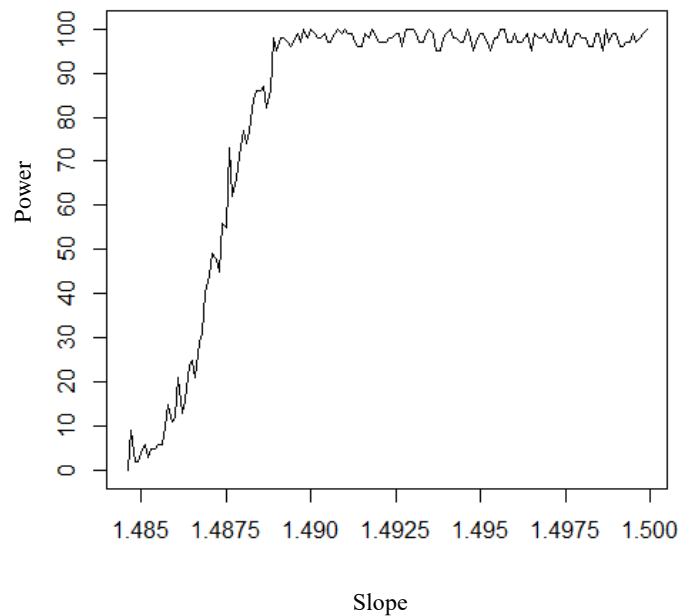


Figure 10 Line graph of ADF test power with decreasing slope



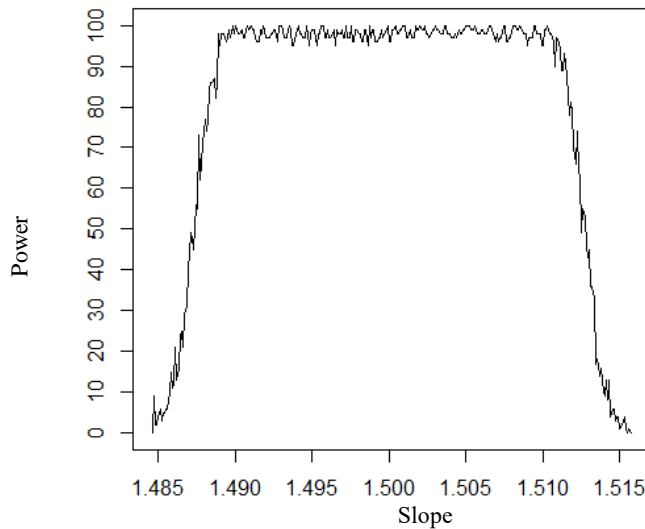


Figure 11 A comprehensive line chart of the power of the ADF test when the slope changes

Based on the conclusion of the third part, if we do not consider the structural change, the ADF test will not fail when the structural break degree is small. From the above three figures, we know that the change of the break degree is susceptible to the result of the unit root test, and it exhibits a symmetrical shape. When the absolute change of the slope is less than 0.01, the power of the ADF test is 100, but when the complete change of the slope exceeds 0.01, the power of the test decreases with the increase of the slope. When the absolute change exceeds 0.015, the power of the test down to 0, in all, without considering the structural break, if the ADF unit root test does not fail, the break degree must be within a small range. The ADF unit root test result will gradually fail beyond this range.

## 6. CONCLUSION AND POLICY SUGGESTIONS

The comprehensive analysis of the power of the ADF statistic test in the case of two breaks of intercept and slope in the data generating process shows that: ignoring the large degree structural break in the data generating process to carry out the ADF test will lead to severe distortion of the test results. When the degree is relatively small, the extreme sample size, the later position of structural break, and the significant slope changes will all reduce the power of the ADF statistic test. Therefore, when using ADF for unit root test, we should be cautious when the unit root process generates the data, and it is necessary to analyze, discriminate or test whether there is a structural break in the data, as well as the sample size, break position, and the slope. If it contains a structural break, it is necessary to select an appropriate testing method according to the characteristics of the structural break.

In practical application, the evolution of social processes and the influence of the external political and economic environment may bring about structural changes in time series. Therefore, it has important theoretical and practical significance to investigate the unit root characteristics of time series combined with structural change analysis. Results show that stochastic trends dominate only stock prices, RMB nominal exchange rate, and real exchange rate significantly. It is more appropriate to treat them as a unit root process of structural breaks. While other variables—including real GDP, imports, and exports — are dominated by deterministic trends, more accurate predictions and perceptions can be obtained by modeling them as a smooth transition trend stationary process. Furthermore, considering the results of this paper, the stationarity test, including a structural break, is suggested to be carried out on other variables in China macroeconomic system in further studies.

## REFERENCES

- [1] Banerjee, A., RL Lumsdaine & JH Stock, 1992, Recursive and sequential tests of the unit-root and trend-break hypothesis: theory and international evidence [J]. *Journal of Business and Economic Statistics*, 10, 271-287.
- [2] Dickey D A., Fuller W A., 1979, Distribution of the estimators for autoregressive time series with a unit root [J]. *Journal of the American Statistical Association* (74), 427-431.
- [3] Dickey D A., Fuller W A., 1981, Likelihood ratio statistics for autoregressive time series with a unit root [J]. *Econometrica*, (49), 1057-1072.
- [4] Granger C, Newbold P., 1974, Spurious regressions in econometrics [J]. *Journal of Econometrics*, (2), 111-120.
- [5] Perron, P. & P.C.B, Phillips, 1987, Does GNP have a unit root? [J]. *Economics Letters*, 23, 135-145.
- [6] Perron, P., 1989, The great crash, the oil price shock and the unit-root hypothesis [J]. *Econometrica*, 57, 1519-1554.
- [7] Perron, P.& TJ Vogelsang, 1992, Non-stationary and level shifts with an application to purchasing power parity [J]. *Journal of Business and Economic Statistics*, 10, 301-320.
- [8] Sen, A., 2003, On unit-root tests when the alternative is a trend-break stationary process [J]. *Journal of Business and Economic Statistics*, 21, 174-184.
- [9] Zivot, E. & D. Andrews, 1992, Further evidence on the great crash, the oil price shock and the unit-root hypothesis [J]. *Journal of Business and Economic Statistics*, 10, 251-270.
- [10] Fang Lin, Zou Weixing. A comparative study of various unit root test methods[J]. *Quantitative Economics and Technical Economics Research*, 2007(01): 151-160.
- [11] He Yunqiang. The influencing factors of the unit root test and the discussion on the test efficacy [D]. *Lanzhou University of Finance and Economics*, 2015.
- [12] Luan Huide. Unit root test with structural catastrophe—review of the literature[J]. *Quantitative and Technical Economics Research*, 2007(03): 152-161.
- [13] Nie Qiaoping, Ye Guang. Analysis of the influence of single-shot linear structural break on DF unit root test—further research on "Perron Phenomenon"[J]. *Statistical Research*, 2008(09): 71-79.
- [14] Wang Shaoping: "Several frontier theories and applications of macro-metrics" [M], *Nankai University Press*, 2003.
- [15] Wang Shaoping, Li Zinai. Empirical analysis of structural sudden change and RMB exchange Rate[J]. *World Economy*, 2003(08): 22-27.
- [16] Zhang Jianhua, Tu Taotao. "Pseudo-Test" for the unit root of structural catastrophe time series[J]. *Quantitative Economics and Technical Economics Research*, 2007(03): 142-151.