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Mathematical Model in Middle School: Concept, Characteristics and Educational Value

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Abstract: The mathematical model builds a bridge between mathematics and the outside world. The definition of the mathematical model in middle school is given in this article based on existing literature and teaching practice, which is a reflection of the mathematical relations or structure (prototypes) in middle school mathematics through mathematical abstraction. This article also reveals the characteristics of middle school mathematical models: structuredness, issue-related nature, unity, and practicality. Last but not least, the educational value of the mathematical model in middle school is analyzed to promote mathematical understanding, induce problem generation and reduce cognitive load.

Keywords: mathematical model in middle school, concept, characteristic, educational value

1. Introduction

Mathematical models have a long history. For instance, a natural number is invented to simulate the number of prey. With the use of numbers, people have built various mathematical models gradually. In ancient China, The Nine Chapters on the Mathematical Art has begun to systematically use mathematical models: 246 topics are classified into nine categories, that is, nine different mathematical models, hence the name "Nine Chapters". The questions set in each chapter are typical realistic prototypes selected from a large number of practical problems, which were transformed into a mathematical model through "shu" (that is, algorithm). Some chapters are dedicated to discussing the application of certain mathematical models, such as "Pythagorean" and "Equations".[1] In modern times, research on mathematical models has reached an unprecedented state, and researchers have constructed several mathematical models, realizing the wide application of mathematical models in many fields of science, society and engineering technology. Based on those application fields, biological mathematical models, medical mathematical models, geological mathematical models, meteorological mathematical models, economic mathematical models, sociological, mathematical models, physical mathematical models, chemical mathematical models,

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etc., are built. It can be seen that mathematical models build a bridge between mathematics and the real world.[2] At the same time, "*General High School Mathematics Curriculum Standards (2017 Edition)*"[2] (hereinafter referred to as "*Curriculum Standards 2017*") proposes six core competencies: mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operations, and data analysis. Among them, mathematical modeling is the ability to mathematically abstract real problems, use mathematical language to express problems, and use mathematical methods to construct models to solve problems. While mathematical models are the foothold of mathematical modeling, they are the most direct product of examining real problems from a mathematical perspective, considering them using mathematical thinking and expressing them with mathematical language. Hence mathematical models have attracted the attention of many mathematical researchers. For instance, Zengsheng Wu[3] expounded the expression form of mathematical models and the psychological mechanism of understanding and constructing mathematical models; Dayong Pu[4] proposed that model thinking is the basic way for students to experience and understand the relationship between mathematics and the outside world; Huijuan Yang et al.,[5] proposed that mathematics teaching should let students discover mathematical models, solve mathematical models and promote mathematical models; Liulin Fu, Li Chen et al.,[6]-[8] explained the meaning of methods of mathematical model; Lili Zhang, Zhong Jin, Cheng Xiang et al.,[8]-[10] introduced the application of some common models of solving problems in high school mathematics, etc. It is not difficult to find that the research results mostly focus on the psychological mechanism, process, and application of mathematical model construction. But what is a mathematical model? What are the characteristics of the mathematical model? What is the educational value of mathematical models? These problems are hardly involved. In order to make the research on this series of problems more targeted, the middle school mathematics model is used for analysis in this paper.

2. The Concept of Mathematical Model in Middle School

Although researchers have constructed a series of mathematical models, the concept of a mathematical model has not been uniformly defined. Qiyuan Jiang and others believe that a mathematical model can be described as a mathematical structure obtained by using appropriate mathematical tools to make some necessary simplified assumptions on a certain object in the real world for a specific purpose according to it's unique internal law.[11] Peiling Qian points out that a mathematical model is a structure in which the researcher illustrates the main characteristics and main relations of the process and phenomenon of the object studied in a formalized mathematical language generally or approximately according to the research purpose.[12] Zengsheng Wu proposes that a mathematical model refers to an intuitive object with quantitative relation and spatial structure after mathematical abstraction. It is observable, descriptive, and manipulable (being able to split, combine, and transform). In essence, a mathematical model is a collection. It is the outcome of abstracting, separating, recombining, and logically reasoning toward objective objects' structure, characteristics and relations.[3] Fanzhe Kong defines the mathematical model as an expression of mathematical structures that intends to describe and study the law of motion and change of objective phenomenon according to the problem's actual situation and the research object's characteristics. Mathematical models are formed by using methods such as mathematical abstraction and generalization. Mathematical models include mathematical formulas, logical principles, specific algorithms, and mathematical concepts.[13] It is not difficult to see that many researchers have different definitions of mathematical models. Some emphasize the purpose of modeling, and some put emphasis on the characteristics of practical problems. But all definitions involve the process of mathematical abstraction and the core of the model -- mathematical structure. Based on the process of mathematical abstraction and mathematical structure, we can define the mathematical model in middle school as a mathematical relation or structure reflecting the specific problems or systems (prototypes) in middle school mathematics in the process of mathematical abstraction. According to this, in the context of middle school mathematics, the concepts, formulas, theorems, laws, etc., are all mathematics models in middle school. For example, Pythagorean Theorem, Sine Theorem, Cosine Theorem, Summation Formula of Arithmetic Progression, Summation Formula of Geometric Progression, Exponential Operation Rule, Logarithm Operation Rule, Absolute value inequality, Weiseneek Weisenböck's inequality, etc., are all mathematical models in middle school.

3. Characteristics of Mathematical Model in Middle School

The mathematical model in middle school results from mathematical abstraction and is more of a general reflection or simulation of the prototype of reality.[12] Therefore, the mathematical model in middle school has specific and profound connotations.

3.1. Structuredness

Structuredness is the most basic feature of the mathematical model in middle school. French mathematician Boutbaki points out, "Mathematics is not about quantity, but about structure." The construction of a mathematical model requires mathematical abstraction, non-essential mathematical information removal, retains of mathematical relation or structure that highlights the essence, and reflection of the core of knowledge in the most concise form. Therefore, as an important part of mathematics, the mathematical model in middle school naturally inherits the structure of mathematics. As a matter of fact, it's the result of mathematical abstraction in that the formation is always accompanied by mathematization, and the final result is presented as a mathematical relation or structure. This is because the process of constructing mathematical models is essentially a process of topology and topological transformation. This conclusion is drawn from the network topology architecture of the human brain and the formation mechanism of mathematical models.

3.2. Issue-Related Nature

Issue-related nature is a prominent feature of mathematical models in middle school. The logical starting point of mathematical modeling is to solve problems, and its ultimate goal is to problems being solved. While mathematical models in middle school are a high-level generalization of a class of middle school mathematical problems, bearing the basic approach and way of thinking to solve this kind of mathematical problem. Building a model generally goes through the course of problem discovery, problem posing, problem analysis, problem-solving, and problem reflection. It can be seen that the entire modeling process starts with the problem and ends with it. It is the mathematical problems that help the modeling develop, deepen and improve. To some extent, the formation process of the mathematical model is the process of a class of mathematical problems being solved continuously and perfectly.

3.3. Unity

Mathematical models in middle school are highly overarching. Mathematical models have mathematical significance. They can be analyzed theoretically, calculated, and reasoned, and deduce some definite results according to them.[14] That is to say, mathematical models in middle school are the high-level generalization and characterization of certain kinds of mathematical problems. Additionally, the formation of this new type of mathematical problem can be regarded as the deduction of the model in hand. The construction of the mathematical model is based on the mathematical knowledge system and the problem-solving process under the reasoning method, which is rich in ideas and methods needed in relevant activities. It has reference and guiding significance for the development of mathematics activities. As an important prototype, the mathematical model in middle school provides tools and methods for solving mathematical problems.

3.4. Practicality

Mathematical models in middle schools are universally applicable. *Curriculum Standards 2017* lists mathematical modeling activities and mathematical inquiry activities as one of the four main lines of high school mathematics teaching. The intention is to cultivate students' mathematical application ability and mathematical innovation consciousness. At the same time, the mathematical structure or relations carried by the mathematical model serves as an important prototype, which provides ideas and tools for solving mathematical problems. This is also the original intention of constructing a mathematical model - "use the model". Therefore, the mathematical model in middle school is an essential form of mathematics. [2]

4. The Educational Value of Mathematical Model in Middle School

Curriculum Standards 2017 points out that core literacy in mathematics includes: mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical operations, and data analysis. Among them, mathematical modeling is the ability to mathematically abstract real problems, express problems in mathematical language, and use mathematical methods to build models to solve problems. [2] It is not difficult to see that the development of mathematical modeling literacy is finally implemented with mathematical models. At the same time, the construction of mathematical models is often accompanied by the development of mathematical abstraction, logical reasoning, mathematical operations, and other literacy. Therefore, mathematical models are widely used in mathematics teaching activities in middle school. It has unique educational value, which is to promote mathematical understanding, induce problem generation, and reduce cognitive load.

4.1. Promote Mathematical Understanding

Benjamin Bloom (1956) believes that understanding is the ability to properly organize facts and skills through the effective use, analysis, synthesis, and evaluation.[15] John Dewey (1993) points out that understanding is the result of learners inquiring into the meaning of things.[15] From a cognitive perspective, "Understanding is of greater value than knowing," because education is based on understanding, without which education is impossible.[16] Therefore, teaching for understanding has become a major goal of teaching activities. Focusing on the subject of mathematics, mathematical understanding not only determines whether the learners' cognitive process being smooth or not, but also profoundly affects learners' mastery of mathematical knowledge and application.[17] Herbert and Carpenter believe that "If a mathematical concept, method or fact is understood, then it will become a part of the personal internal network. The number and strength of the connection determines the degree of understanding". Professor Shiqi Li believes that in the process of learning a mathematical concept, principle, or theorem, if you can organize an appropriate and effective cognitive structure psychologically and make it a part of your internal knowledge network, then it means understanding.[18] The process of constructing a mathematical model is the process of mathematization (both horizontal and vertical). Accompanied by multi-sensory participation, the process often goes through a series of thinking activities such as abstraction, association, analysis, comparison, evaluation, and reflection. Those thinking activities are natural catalysts to promote mathematical understanding. At the same time, mathematical models, as carriers of mathematical relations and structures, are closely connected and deeply integrated with mathematical knowledge, strategies, and thinking. Mathematical models positively promote the construction of knowledge network, understanding of the essence of mathematics, and improvement of cognitive structure.

Example 1: A reverse teaching expert of “Summation Formula of Geometric Progression”

Teaching Objectives: Use the model below (hereinafter referred to as Model 1) to deepen students' understanding of Dislocation Subtraction.

$$S_n = \begin{cases} na_1, q = 1 \\ \dfrac{a_1(1-q^n)}{1-q}, q \neq 1 \end{cases}$$

(Model 1)

Teaching Process:

Teacher: From Model 1, we can obtain: $S_n - qS_n = a_1 - a_1q^n$(1). How to get $q \times S_n$?

Student: Use $q \times S_n$ to get $qS_n = a_1q + a_1q^2 + \cdots + a_1q^{n-1} + a_1q^n$(2)

Teacher: How can we change the structure to the left side of (1) ?

Student: By subtracting.

Teacher: How to subtract?

Student: Subtract the identical items in (1) and (2) by doing so:

$$S_n = a_1 + a_1q + a_1q^2 + \cdots + a_1q^{n-1}$$

$$qS_n = a_1q + a_1q^2 + a_1q^3 + \cdots + a_1q^n$$

Teacher: What's the character of the position of those identical items?

Student: They are dislocated.

Teacher: So we call it "Dislocation Subtraction".

Teacher: Why should we use $q \times S_n$?

Student: In order to generate the identical items.

Teacher: Can $\frac{1}{q} \times S_n$, $q^2 \times S_n$, $q^3 \times S_n$, achieve similar effect?

Student: Yes.

Teacher: What's the difference between $q \times S_n$ and $\frac{1}{q} \times S_n$, $q^2 \times S_n$, $q^3 \times S_n$?

Student: More identical items can be generated by subtracting with $q \times S_n$, $\frac{1}{q} \times S_n$ and S_n.

Teacher: Excellent. Normally we use $q \times S_n$ to complete Dislocation Subtraction. And what's the essence of Dislocation Subtraction?

Student: Subtraction.

Teacher: Great. You hit the essence of the problem directly: the subtraction is exactly what Dislocation Subtraction and the reverse additive method have in common.

By using Model 1 reversely, we can deepen the students' understanding of $q\times S_n$. ①Students get a preliminary understanding from the perspective of model structure. ② Students' Cognition is deepened by understanding the difference and connections among $q\times S_n$, $\frac{1}{q}\times S_n$, $q^2\times S_n$ and $q^3\times S_n$ based on a comparative perspective. ③Based on the commonness of dislocation subtraction and reverse additive method, the thought of transformation and equation is infiltrated in this process. By setting up inquiry activities, the number and intensity of connections in mathematical knowledge are increased, the construction of students' internal knowledge network is promoted, and the understanding of dislocation subtraction is deepened.

4.2. Induce Problem Generalization

Curriculum Standards 2017 points out that students' ability to discover, raise, analyze and solve problems can be further cultivated by studying high school courses.[2] Among them, the ability to raise questions is particularly important. This is because questions are the starting point and driving force of thinking, and asking questions is often more creative than solving them. Hilbert points out, "As long as a branch of science can raise a large amount of questions, it is full of vitality, while the lack of questions indicates the termination or decline of independent development". Specifically, in terms of mathematics, questions can be seen as the lifeline of it; yet it is hard to ask a meaningful mathematical question. As Einstein said, "It is more important to ask a question than to solve it, because asking a question is often creative." As a high-level generalization of a class of problems, the mathematical model itself leads and represents a class of mathematical problems. Therefore, the mathematical model has the function of generating problems, being regarded as the deduction of the model in hand. As a matter of fact, many models in mathematics have induced a large number of new problems, and they are known as "the chicken that lays golden eggs."

Example 2:

Let $a_1, a_2, b_1, b_2 > 0$, use $\frac{a_1^2}{b_1}+\frac{a_2^2}{b_2}\geq\frac{(a_1+a_2)^2}{b_1+b_2}$ to induce problems.

The model in Example 2 is evolved from the Cauchy-Buniakowsky-Schwarz inequality, which has the characteristics of concise form, beautiful symmetry, and beauty of unity. A large number of mathematical problems can be obtained by direct assignment of a_1, a_2, b_1, b_2 or promoting the use of the model.

Problem 1: If $x+y=1$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}$.

Problem 2: If $2x+y=1$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}$.

Problem 3: If $x+3y=1$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}$.

Problem 4:If $2x+3y=1$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}$.

Problem 5: If $ax+by=1(a,b>0)$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}$.

Problem 6:If $x+y+z=1$,then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}+\frac{z^2}{4}$.

Problem 7:If $ax+by+cz=1(a,b,c>0)$, then we can find the minimum value of $\frac{x^2}{2}+\frac{y^2}{3}+\frac{z^2}{4}$.

The model in Example 2 is the "root" of the above series of new problems. Among them, problems 1-5 are the direct assignment of parameters in the model, and problems 6-7 are deduced from the promoted form of the model (If $a_1,a_2,a_3,b_1,b_2,b_3>0$, then $\frac{{a_1}^2}{b_1}+\frac{{a_2}^2}{b_2}+\frac{{a_3}^2}{b_3}\geq\frac{(a_1+a_2+a_3)^2}{b_1+b_2+b_3}$). It is not difficult to see that the model is the cradle of problem generation, and the problem generated by the model has the characteristics of directness, high efficiency, rules to rely on, and traces to follow. Applying the philosopher Popper's point of view: the growth of science and knowledge always starts with the models and ends with them -- models that are deeper and deeper, models that are more and more able to induce new problems.[19] Despite the exaggeration, the model does burst with the power of problem generation!

4.3. Reduce Cognitive Load

Psychologist John Sweller first proposed the Cognitive Load Theory in 1988. The so-called cognitive load refers to the total amount of cognitive structure (mainly the intellectual activity on working memory) added to the learner by the current task provider when the individual is performing cognitive activities in a certain task environment.[20] Cognitive Load Theory mainly examines students' learning results and problem solutions from the perspective of resource allocation based on Resource Limitation Theory and Schema Theory. Resource Limitation Theory holds that: all kinds of cognitive activities in the process of problem-solving or learning need to consume cognitive resources. If the total amount of resources required by cognitive activities exceeds that required by the individual, it is considered that the cognitive load is too heavy. And Schema Theory emphasizes that a schema is a cognitive structure that enables people to classify information according to the way it will be used.[20] The practice has shown that three basic elements affect cognitive load: the individual's previous experience, the intrinsic nature of learning materials and the organization and presentation of materials.[20] Among them, prior experience mainly refers to the quantity and quality of the images in the individual's long-term memory. On the condition that previous experience has been identified, cognitive load is primarily determined by the essential characteristics, organization, and presentation of the learning material. Mathematical models in middle school represent the mathematical relations or structure of a particular problem or system (prototype) in middle school mathematics. It presents fragmented and disordered information in a highly harmonious, unified and holistic manner, integrating a large amount of information into a small number of intuitive information units, reducing the number of information blocks entering the working memory, and ensuring the effective development of learners' information processing. The middle school mathematical model is the result of a high degree of abstraction, which reduces the irrelevant cognitive load (and reduces the total cognitive load accordingly), and allows more resources to be invested in learning-related activities. For example, the formula of the distance between two points “ $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ “ is an intuitive presentation after a high degree of abstraction. The abstraction discards irrelevant cognitive loads such as the location of the point, the size and properties of positive and negative of the horizontal and vertical coordinates of the point, and whether the horizontal and vertical coordinates of the point is a number or a formula. The middle school mathematical model is a kind of schema, a kind of high-quality schema. Under the guidance of these schemata, students can quickly and correctly classify the current problems into a certain category, which is usually matched with the "prototype" stored in the brain and related to some problem-solving steps or strategies.[20] It promotes information processing activities to be based on the nearest cognitive area, avoids them from developing in the original and primary state, and reduces the brain's "warm-up", "startup", "acceleration" and "energy consumption". Here’s and example: “Given the two sides and included angle of the triangle, find the third side.” There is no need for learners to return to the primary state of the trilateral relationship and triangular relationship, that is, "warm-up"; and no need for the second stage of using Pythagorean Theorem, that is, “startup"; and no need for the law of sines, the state of "acceleration". The problem's solution can be obtained by directly using the model - the law of cosines. To sum up, the mathematical model in middle school represents problems and methods or a kind of example. Under the circumstances of example learning, learners do not need to go through the process of "means -- end", and do not need to carry out wrong problem-solving strategies and learn by mistake. Learners only need to pay attention to the questions related to the acquisition of schema in the examples and effectively reduce cognitive load, especially by eliminating irrelevant ones.[20]

Conclusions and Discussion

This paper answers the three questions raised at the beginning of the article and mainly works in three aspects: it defines the concept of the mathematical model in middle school, reveals its characteristics of it, and deeply analyzes its educational value of it. As for the general steps of middle school mathematical model construction, the psychological process of middle school mathematical model construction, the application of middle school mathematical model, etc., will be discussed in another article.

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