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The Trigonometric Function Sin(n/2)π that Satisfies the Dirichlet Feature

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Abstract: Find the trigonometric function sin(n/2)π that satisfies the Dirichlet feature, then analyze the real parts of non trivial zeros of the L-function, some are 1/2, some are not 1/2, and identify the reasons for this situation.We conclude that the Riemann hypothesis is correct, then, we use the Euler Beta function remainder formula to verify whether it is correct when the real part of the non trivial zero point s is 1/2.
Keywords: Exceptional zero, Dirichlet feature, Euler Beta function remainder formula

We know that the Dirichlet feature is a function $\chi(n)$ defined on an integer with the range of real numbers C, the expression of the Dirichlet function is: $L(s,\chi)=\sum_{n=1}^{+\infty}\frac{\chi(n)}{n^s}$

Now we have found a trigonometric function $\sin\frac{(n)}{2}\cdot\pi$.(n is an integer),then let：

$$\sin\frac{(n)}{2}\cdot\pi=\chi(n)$$

And then:

1） $\chi(n+N)=\sin\frac{(n+N)}{2}\cdot\pi=\sin\frac{(n)}{2}\cdot\pi=\chi(n)$

when N is an even period constant,such as $N=4$,then:

$$\sin\frac{(n+N)}{2}\cdot\pi=\sin\frac{(n)}{2}\cdot\pi$$

Article history: Received August 25, 2023 Accepted September 10, 2023 Available September 12, 2023
To cite this paper: Li Xiao Hui (2023). The Trigonometric Function Sin(n/2)π that Satisfies the Dirichlet Feature. SQPreprints, Vol1, Issue1, Pages 37-41.
Doi: https://doi.org/10.55375/SQPreprints.2023.3

2) $\chi(1)=\sin\frac{(1)}{2}\cdot\pi=1$

3) $\chi(nm)=\sin\frac{(nm)}{2}\cdot\pi=\left(\sin\frac{(n)}{2}\cdot\pi\right)\left(\sin\frac{(m)}{2}\cdot\pi\right)=\chi(n)\chi(m)$ (n and m are are integers)

Proof:if n or m are even numbers, then：

$$\chi(nm)=\sin\frac{(nm)}{2}\cdot\pi=\left(\sin\frac{(n)}{2}\cdot\pi\right)\left(\sin\frac{(m)}{2}\cdot\pi\right)=\chi(n)\chi(m)=0$$

So when n and m are both odd numbers, because the minimum period of x in $\sin x\cdot\pi$ is 2, the minimum period of n in $\sin\frac{(n)}{2}\cdot\pi$ is 4. We can set：

$$n=4p+f \qquad m=4q+g \qquad \left(f,g\in N^{+}\right)(p,q \text{ is an integer})$$

so: $f=1$ or $f=3$, and $g=1$ or $g=3$

We have： $n\cdot m=(4p+f)(4q+g)=16pq+4pg+4qf+fg$

So:

$$\chi(nm)=\sin\frac{(nm)}{2}\cdot\pi=\sin\frac{(f\cdot g)}{2}\cdot\pi=\left(\sin\frac{(n)}{2}\cdot\pi\right)\left(\sin\frac{(m)}{2}\cdot\pi\right)=\left(\sin\frac{(f)}{2}\cdot\pi\right)\left(\sin\frac{(g)}{2}\cdot\pi\right)=\chi(n)\chi(m)$$

4) $\chi(x)=0 \quad (\gcd(x,N)\neq 1)$

Let[2] : $x=d+e\cdot i=2s$

$$\Rightarrow \chi(x)=\chi(d+e\cdot i)=\sin\left(\frac{d}{2}+\frac{e}{2}i\right)\cdot\pi=0 \qquad (d+e\cdot i\in C)$$

So: $d/2=1/2+k$

$$\Rightarrow \chi(x)=\sin\left((1/2+k)+\frac{e}{2}i\right)\cdot\pi=0 \qquad (k \text{ is an integer})$$

We can obtain the relationship expression between x and s as follows:

$$x=1+2k+e\cdot i=2s \qquad (1)$$

From the above, it can be found that $\sin\frac{(n)}{2}\cdot\pi$ satisfies all four conditions, and it is a Dirichlet feature.

Next, let's analyze the Dirichlet function.

the Dirichlet L-function is：

$$L(s,\chi)=\sum_{n=1}^{+\infty}\frac{\chi(n)}{n^s} \qquad (2)$$

When n is an odd number, then $\chi(n)=\sin\frac{(n)}{2}\cdot\pi=1$

the L-function is equivalent to the Euler Zeta function: $\sum_{n=1}^{+\infty}\frac{1}{n^s}$

When n is an even number, then $\chi(n)=\sin\frac{(n)}{2}\cdot\pi=0$

the L-function is: $L(s,\chi)=0$

so the trivial zero of the L-function is consistent with the trivial zero of the Riemannian zeta function[2].

According to equation (1), it can be concluded that the expression for the non trivial zero point s of the L-function is:

$$s=\left(\frac{1}{2}+k\right)+\frac{e}{2}\cdot i \qquad (3)$$

In equation (1),when $2k=0$, then $x=1+e\cdot i=2s$ and $L(s,\chi)=0$, these points on the complex plane are both non trivial zeros of the L-function and Riemannian Zeta function.

But when $2k\neq 0$, $x=1+2k+e\cdot i$,It is a non trivial zero of an L-function.but it is not a non trivial zero of a Riemannian Zeta function.

In summary:

when $2k=0$,the real parts of the non trivial zeros of the L-function and the Riemannian Zeta function are both all $1/2$. When $2k\neq 0$, the real part of the non trivial zero of the L-function is not on 1/2.

Therefore, the Riemann hypothesis holds, and the Landau Siegel Zeros Conjecture is not entirely valid.

We concluded that the Riemann hypothesis is correct, then, we use the Euler Beta function remainder formula to verify whether it is correct when the real part of the non trivial zero point s is 1/2.

The Euler Beta function remainder formula:

$$\Gamma(x)\Gamma(1-x)=\frac{\pi}{\sin x\cdot\pi} \qquad x\in(0,1)$$

Let's set the function :

$$f(x)=\Gamma(x)\Gamma(1-x)\cdot\sin x\cdot\pi=\pi \qquad (4)$$

So:

$$f(1-x)=\Gamma(1-x)\Gamma(1-(1-x))\cdot\sin(1-x)\cdot\pi=\Gamma(1-x)\Gamma(x)\sin(1-x)\cdot\pi=\Gamma(1-x)\Gamma(x)\cdot\sin x\pi$$

We can immediately obtain:

$$f(x)=f(1-x)=\pi \qquad (5)$$

From equation (3), when $k=0$, then $s=\frac{1}{2}+\frac{e}{2}\cdot i$ and $L(s,\chi)=0$, these points on the complex plane are both non trivial zeros of the L-function and Riemannian Zeta function.

so now : $s=\frac{1}{2}+\frac{e}{2}\cdot i$

then from equation (4),so:

$$f(s)=\Gamma\left(\frac{1}{2}+\frac{e}{2}\cdot i\right)\Gamma\left(1-\frac{1}{2}-\frac{e}{2}\cdot i\right)\cdot\sin\left(\frac{1}{2}\pi+\frac{e}{2}\cdot i\cdot\pi\right)$$

$$f(1-s)=\Gamma\left(1-\frac{1}{2}-\frac{e}{2}\cdot i\right)\Gamma\left(\frac{1}{2}+\frac{e}{2}\cdot i\right)\cdot\sin\left(\frac{1}{2}\pi-\frac{e}{2}\cdot i\cdot\pi\right)$$

And because:

$$\sin\left(\frac{1}{2}\pi+\frac{e}{2}\cdot i\cdot\pi\right)=\cos\frac{e}{2}\cdot i\cdot\pi$$

$$\sin\left(\frac{1}{2}\pi-\frac{e}{2}\cdot i\cdot\pi\right)=\cos\frac{e}{2}\cdot i\cdot\pi$$

So,we obtained a result that coincides with the real part of non trivial zero s being 1/2:

$$f(s)=f\left(\frac{1}{2}+\frac{e}{2}\cdot i\right)=f\left(\frac{1}{2}-\frac{e}{2}\cdot i\right)=f(1-s) \qquad (6)$$

Now we can express the Euler Beta function remainder formula in the form of complex numbers:

$$\Gamma\left(\frac{1}{2}+bi\right)\Gamma\left(\frac{1}{2}-bi\right)=\frac{\pi}{\cos(bi\cdot\pi)} \qquad \left(0<1/2+bi<1\right)$$

$eg: b=\frac{1}{4}$, $i^2=-1$, then:

$(bi)\cdot(-bi)=-i^2(b)^2=(b)^2$

$(bi)\cdot(-bi)=(bi)\cdot(i^2bi)=i^4(b)^2=(bi^2)^2=\left(\left(\sqrt{b}\cdot i\right)^2\right)^2$

$\Rightarrow\left(\left(\sqrt{b}\right)^2\right)^2=\left(\left(\sqrt{b}\cdot i\right)^2\right)^2\Rightarrow\sqrt{b}=\sqrt{b}\cdot i\Rightarrow\sqrt{\frac{1}{4}}=\sqrt{\frac{1}{4}}\cdot i\Rightarrow\frac{1}{2}=\frac{i}{2}$

Immediately obtain : $\frac{1}{4}i=\frac{i}{2}\times\frac{1}{2}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

So:

$$\Gamma\left(\frac{1}{2}+bi\right)\Gamma\left(\frac{1}{2}-bi\right)=\Gamma\left(\frac{1}{2}+\frac{1}{4}i\right)\Gamma\left(\frac{1}{2}-\frac{1}{4}i\right)=\frac{\pi}{\cos\left(\frac{1}{4}\cdot\pi\right)}=\Gamma\left(1-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)=\frac{\pi}{\sin\left(\frac{1}{4}\cdot\pi\right)}=\sqrt{2}\cdot\pi$$

Conclusion: The real parts of all non trivial zeros of Riemann zeta functions are 1/2.and the real parts of non trivial zeros of the L-function are not all on 1/2.

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