



素数分布规律的发现(II)以及黎曼猜想的直接证明

——素数分布规律的最新定理(2)

The Discovery of the Law of the Distribution of Prime Numbers (II) and the Direct Proof of the Riemann Conjecture

---The Latest Theorem of the Distribution Law of Prime Numbers (2)

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摘要: 黎曼猜想有二层含义: 一是指黎曼对素数在自然数的分布规律的有关假设和推测。用语言文字描述, 即素数规律所包含的信息都在实部 $x=1/2$ 的这条直线上。不限具体的数学表示形式。二是指黎曼猜想的具体的数学表达形式, 给出了黎曼 ζ 函数。也就是说, 存在与素数分布有关的复数 $s = \sigma + it$, 使得所有非平凡零点都在实部 $\sigma = 1/2$ 的直线上。为此, 本文作者基于素数通项公式的首先发现, 并利用该公式直接、巧妙地证明了黎曼假设的素数所有信息都在实部等于 $1/2$ 的直线上。因此, 推断黎曼猜想成立。

Abstract: Riemann Conjecture has two meanings: First, it refers to Riemann Hypothesis and speculation about the distribution law of prime numbers in natural numbers. To put it into words, the law of prime numbers contains all the information on this line with the real part $x = 1/2$. Not limited to specific mathematical representations. The second refers to the concrete mathematical

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expression of Riemann Conjecture, and gives the Riemann zeta function. That is, there are complex numbers $s = \sigma + it$, related to the distribution of prime numbers such that all nontrivial zeros are on a line with the real part $\sigma = 1/2$. Therefore, based on the first discovery of the general term formula of prime numbers, the author of this paper directly and skillfully proves that all the information of Riemann's assumption of prime numbers is on the line with the real part equal to $1/2$. Therefore, we conclude that the Riemann Conjecture is true.

关键词: 素数分布规律, 素数通项公式, 黎曼猜想, 直接初等证明

Key words: Distribution of prime numbers, Prime number general term formula, Riemann Conjecture, Direct elementary proof.

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第二部分 素数分布规律的最新定理(2)

Part II Latest theorems on the Distribution of prime Numbers (2)

§ 4 素数分布出现规律的若干新定理

§4 Some new theorems of the occurrence rule of prime number distribution

由于有序素数集合是有序正整数集合的子集。有序合数集合是有序正整数的补集。因此，在有序素数集合里，根据以上在有序素数序列中两个相邻素数之间的合数个数按奇数分布规律原理的发现。我们可以推得以下素数分布出现规律的若干新定理。

Since the set of ordered primes is a subset of the set of ordered positive integers. The set of ordered composite numbers is the complement of ordered positive integers. Therefore, in the set of ordered prime numbers, according to the above discovery of the principle of odd distribution of the number of composite numbers between two adjacent prime numbers in the sequence of ordered prime numbers. We can derive some new theorems for the occurrence of the distribution of prime numbers.

定理 1. 素数通项公式

Theorem 1. Prime number general term formula

任意一个素数等于它在素数序列中的序数加上此素数之前的所有合数个数和。

Any prime number is equal to its ordinal number in the prime sequence plus the sum of all composite numbers preceding this prime number.

P_m 表示位于第 m 位的素数； m 为素数序列当中的序数，即 m 也是素数个数； $S_m(P_m)$ 表示在素数数列中从第一个素数 1 开始，到第 m 个素数的所有合数个数之和，也就是前 m 个素数的间隔合数个数级数之和。最初三个素数 1、2、3，由于没有合数间隔，这里定义它们的合数间隔个数为 0，因此，素数序列或者素数数列的通项公式表示为：

P_m represents the prime number in the m position; m is an ordinal number in the sequence of prime numbers, that is, m is also a prime number; $S_m(P_m)$ represents the sum of all composite numbers in the prime sequence from the first prime number 1 to the m th prime number, that is, the sum of the interval composite number series of the first m primes. The first three prime numbers 1, 2, 3, since there is no composite interval, the number of their composite interval is defined here as 0, so the general formula of the prime number sequence or prime number series is expressed as:

$$P_m = m + S_m(P_m) \quad (4.1)$$

若令 $AAS_m = S_m(P_m)$ ，上式(4.1)有时简化表示为：

If let $S_m = S_m(P_m)$, the above formula (4.1) is sometimes simplified as:

$$P_m = m + S_m$$

其中: Among them:

$$S_m = S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

式中: n 为无穷级数项数, K_m 为两个相邻素数间隔合数的个数, m 为素数序数; $n > m$; $n, m \in \mathbb{N}^*$ 。 $S_m(P_m)$ 作者称为素数有序数列的特征无穷级数函数。它是决定每个素数在非零自然数序列分布出现规律一个重要函数变量。若将 $S_m(P_m)$ 展开, 即,

In the formula, n is the number of infinite series terms, K_m is the number of composite numbers between two adjacent prime numbers, m is the ordinal number of prime numbers; $n > m$; $n, m \in \mathbb{N}^*$. $S_m(P_m)$ authors called the prime ordered series of characteristic infinite series function. It is an important function variable that determines the distribution of every prime number in the non-zero sequence of natural numbers. If $S_m(P_m)$ is expanded, that is,

$$\begin{aligned} S_1(P_1) &= 0, \text{ be written as } \sum 0 \\ S_2(P_2) &= 0 + 0, \text{ be written as } \sum 0 \\ S_3(P_3) &= 0 + 0 + 0, \text{ be written as } \sum 0 \\ S_4(P_4) &= 0 + 0 + 0 + 1, \text{ be written as } \sum 1 \\ S_5(P_5) &= 0 + 0 + 0 + 1 + 1, \text{ be written as } \sum 2 \\ S_6(P_6) &= 0 + 0 + 0 + 1 + 1 + 3, \text{ be written as } \sum 5 \\ S_7(P_7) &= 0 + 0 + 0 + 1 + 1 + 3 + 1, \text{ be written as } \sum 6 \\ S_8(P_8) &= 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3, \text{ be written as } \sum 9 \\ S_9(P_9) &= 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + 1, \text{ be written as } \sum 10 \\ S_{10}(P_{10}) &= 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + 1 + 3, \text{ be written as } \sum 13 \\ S_{11}(P_{11}) &= 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + 1 + 3 + 5, \text{ be written as } \sum 18 \\ &\dots\dots \\ S_m(P_m) &= b_1 + b_2 + b_3 + \cdots + b_m, \text{ be written as } \sum \infty \end{aligned}$$

现就素数的通项表达式推导和证明如下。

The general term expression formula of prime numbers is derived and proved as follows.

根据以上有序素数集合的定义, 将素数集合中的所有元素, 包括单位素数 1 在内, 由小到大, 依次排序, 所形成的一系列数。本文笔者则称为素数序列, 或者素数数列。如下所列:

According to the above definition of the ordered prime number set, all the elements in the prime number set, including the unit prime number 1, from small to large, in order to form a list of numbers. In this paper, it is called prime number sequence, or prime number series. Listed below:

$$1, 2, 3, 5, 7, 11, \cdots, P_m, \cdots, \infty$$

其中, 在以上素数数列中, 位于首位的一项, 称为素数数列的第 1 项 $P_1 = 1$; 位于第二位的一项, 称为第 2 项 $P_2 = 2$; 位于第三位的一项, 称为第 3 项 $P_3 = 3$; 位于第四位的一项, 称为第 4 项 $P_4 = 5$; $\dots\dots$, 位于第 m 位的一项, 称为第 m 项 P_m .

In the above prime number series, the first term is called the first term of the prime number series $P_1 = 1$; The second term is called the second term $P_2 = 2$; The third term is called item 3 $P_3 = 3$. The item in fourth place is called item 4 $P_4 = 5$; ... The term located in the m position is called the m term P_m .

假设 m 为正整数, 即 $m \in N^*$; m 为所有素数从 1 开始由小到大依次排列的序数; 前 m 项级数 $S_m(P_m)$ 为素数 p_m 之前的所有合数的个数之和。最初三个素数没有合数间隔, 是空集 \emptyset , 合数个数定义为 0。并规定第一个素数 1 即 $P_1: 1=1$, 此前的合数个数为零, 记作 $\Sigma 0$; 第二个素数 2 即 $P_2: 2=2$, 此前的合数个数之和为零, 记作 $\Sigma 0$; 第三个素数 3 即 $P_3: 3=3$, 此前的合数个数之和为零, 记作 $\Sigma 0$; 从第四个素数 5 即 $P_4: 5=4+1$ 开始, 此前的合数个数之和为 1, 记作 $\Sigma 1$; 第五个素数 7 即 $P_5: 7=5+2$, 此前的合数个数之和为 2, 记作 $\Sigma 2$; 第六个素数 11 即 $P_6: 11=6+5$, 此前的合数个数之和为 5, 记作 $\Sigma 5$, …… , 依此类推。根据素数分布原理规律, 则有:

Let m be a positive integer, that is, $m \in N^*$; m is the ordinal number that all prime numbers are arranged from 1 in descending order; The first m -term series $S_m(P_m)$ is the sum of all composite numbers before the prime p_m . The first three prime numbers have no composite interval and are the empty set \emptyset , with the composite number defined as 0. The first prime number 1 is $P_1: 1 = 1+0$, and the previous composite number is zero, recorded as $\Sigma 0$; The second prime number 2 is $P_2: 2 = 2+0$, where the sum of the previous composite numbers is zero, is written as $\Sigma 0$; The third prime 3 is $P_3: 3 = 3+0$, the sum of the previous composite numbers is zero, is written as $\Sigma 0$; Starting from the fourth prime number 5 is $P_4: 5=4+1$, the sum of the previous composite numbers is 1, which is written as $\Sigma 1$; The fifth prime number 7 is $P_5: 7=5+2$, the sum of the previous composite numbers of 2, is written as $\Sigma 2$; The sixth prime number 11 is $P_6: 11=6+5$, the sum of the previous composite numbers of 5, is written as $\Sigma 5$,……. And so on. According to the principle of prime number distribution, there are:

- 第 1 个素数 $P_1: 1=1+\Sigma 0$;
- 第 2 个素数 $P_2: 2=2+\Sigma 0$;
- 第 3 个素数 $P_3: 3=3+\Sigma 0$;
- 第 4 个素数 $P_4: 5=4+\Sigma 1$;
- 第 5 个素数 $P_5: 7=5+\Sigma 2$;
- 第 6 个素数 $P_6: 11=6+\Sigma 5$;
- 第 7 个素数 $P_7: 13=7+\Sigma 6$;
- 第 8 个素数 $P_8: 17=8+\Sigma 9$;
- 第 9 个素数 $P_9: 19=9+\Sigma 10$;
- 第 10 个素数 $P_{10}: 23=10+\Sigma 13$;
- 第 11 个素数 $P_{11}: 29=11+\Sigma 18$;
- 第 12 个素数 $P_{12}: 31=12+\Sigma 19$;
- 第 13 个素数 $P_{13}: 37=13+\Sigma 24$;
- 第 14 个素数 $P_{14}: 41=14+\Sigma 27$;
- 第 15 个素数 $P_{15}: 43=15+\Sigma 28$;
- 第 16 个素数 $P_{16}: 47=16+\Sigma 31$;
- 第 17 个素数 $P_{17}: 53=17+\Sigma 36$;
- 第 18 个素数 $P_{18}: 59=18+\Sigma 41$;
- 第 19 个素数 $P_{19}: 61=19+\Sigma 42$;
- 第 20 个素数 $P_{20}: 67=20+\Sigma 47$;
- 第 21 个素数 $P_{21}: 71=21+\Sigma 50$;
- 第 22 个素数 $P_{22}: 73=22+\Sigma 51$;
- 第 23 个素数 $P_{23}: 79=23+\Sigma 56$;
- 第 24 个素数 $P_{24}: 83=24+\Sigma 59$;

第 25 个素数 P_{25} : $89=25+\Sigma 64$;

第 26 个素数 P_{26} : $97=26+\Sigma 71$;

第 27 个素数 P_{27} : $101=27+\Sigma 74$;

.....

第 $m-1$ 个素数 $P_{m-1} = m-1 + S_{m-1}(P_{m-1})$;

第 m 个素数 $P_m = m + S_m(P_m)$;

.....

根据以上素数分布的原理规律, 可归纳推得:

第 m 个素数通项公式(4.1):

The first prime P_1 : $1 = 1 + \Sigma 0$;

The second prime P_2 : $2 = 2 + \Sigma 0$;

The third prime P_3 : $3 = 3 + \Sigma 0$;

The fourth prime P_4 : $5 = 4 + \Sigma 1$;

The fifth prime P_5 : $7 = 5 + \Sigma 2$;

The sixth prime P_6 : $11 = 6 + \Sigma 5$;

The 7th prime P_7 : $13 = 7 + \Sigma 6$;

The 8th prime P_8 : $17 = 8 + \Sigma 9$;

The 9th prime P_9 : $19 = 9 + \Sigma 10$;

The 10th prime P_{10} : $23 = 10 + \Sigma 13$;

The 11th prime P_{11} : $29 = 11 + \Sigma 18$;

The 12th prime P_{12} : $31 = 12 + \Sigma 19$;

The 13th prime P_{13} : $37 = 13 + \Sigma 24$;

The 14th prime P_{14} : $41 = 14 + \Sigma 27$;

The 15th prime P_{15} : $43 = 15 + \Sigma 28$;

The 16th prime P_{16} : $47 = 16 + \Sigma 31$;

The 17th prime P_{17} : $53 = 17 + \Sigma 36$;

The 18th prime P_{18} : $59 = 18 + \Sigma 41$;

The 19th prime P_{19} : $61 = 19 + \Sigma 42$;

The 20th prime P_{20} : $67 = 20 + \Sigma 47$;

The 21st prime P_{21} : $71 = 21 + \Sigma 50$;

The 22nd prime P_{22} : $73 = 22 + \Sigma 51$;

The 23rd prime P_{23} : $79 = 23 + \Sigma 56$;

The 24th prime P_{24} : $83 = 24 + \Sigma 59$;

The 25th prime P_{25} : $89 = 25 + \Sigma 64$;

The 26th prime P_{26} : $97 = 26 + \Sigma 71$;

The 27th prime P_{27} : $101 = 27 + \Sigma 74$;

...

The $m-1$ th prime $P_{m-1} = m-1 + \Sigma S_{m-1}(P_{m-1})$;

The m -th prime $P_m = m + \Sigma S_m(P_m)$;

...

According to the principle of the distribution of prime numbers above, it can be concluded that:

Formula for the general term of the m prime number (4.1) :

$$P_m = m + S_m(P_m)$$

其中, 素数的特征无穷级数:

Where, the characteristic infinite series of prime numbers:

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \dots + \infty$$

式中: $\{K_m\} = 0, 0, 0, 1, 1, 3, 1, 3, 1, 3, 5, 1, 5, 3, \dots, \infty$ 作者称为素数的特征无穷有序数列。

Where: $\{K_m\} = 0, 0, 0, 1, 1, 3, 1, 3, 1, 3, 5, 1, 5, 3, \dots, \infty$ A series of characteristic infinite ordered numbers that authors call prime numbers.

对于第 $m+1$ 个素数, 则有:

For the $m+1$ prime number, there is:

$$P_{m+1} = (m+1) + S_{m+1}(P_{m+1}) \quad (4.2)$$

将(4.2)减(4.1), 可得

Subtract (4.2) from (4.1) to get:

$$P_{m+1} - P_m = [S_{m+1}(P_{m+1}) - S_m(P_m)] + 1 \quad (4.3)$$

显然, 上式(4.1)和(4.3)对于素数序列中的前 1、2 和 3 三个素数是成立的。

Obviously, the above equations (4.1) and (4.3) are true for the first 1, 2, and 3 primes in the prime number sequence.

从第 3+1 个素数开始, 若令

Start with the 3rd +1 prime number, Joing

$$S_{m+1}(P_{m+1}) - S_m(P_m) = K_{m+1} \quad (4.4)$$

并将(4.4)代入(4.3), 可得到

And by substituting (4.4) into (4.3), we get

$$P_{m+1} - P_m = K_{m+1} + 1 \quad (4.5)$$

其中, Among them, $m+1 \geq 4$. $m \in \mathbb{N}^*$.

$K_{m+1} = 1$, 或 3, 或 5, 或 7, 或 9, \dots , 或 $2n-1$, \dots , ($n, m \in \mathbb{N}^*$), 均为奇数。如图 9 所示。

$K_{m+1} = 1, \text{ or } 3, \text{ or } 5, \text{ or } 7, \text{ or } 9, \dots, \text{ or } 2n-1, \dots, (n, m \in \mathbb{N}^*),$ both are odd. As shown in Figure 9.

存在 $2n-1$ 个合数
There are 2n minus 1 composite numbers
 $\dots P_m \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \dots P_{m+1} \dots$

图 9. 任意两个相邻素数之间的间隔合数个数分布示意图

Figure 9. Schematic diagram of the number distribution of interval composite numbers between any two adjacent prime numbers

说明: 当相邻的两个素数插入的合数个数为偶数个数 $2n$ 或者是 $2(n+1)$ 时, 两个相邻素数的数位总是多一个或者是少一个, 违反自然数秩序公理。所以, 连接任意两个相邻素数的合数个数必然是奇数个数。n 为正整数, 即 $n \in \mathbb{N}^*$ 。

Description: When the combined number of two adjacent prime numbers inserted is an even number $2n$ or $2(n+1)$, Two adjacent prime numbers always have one more or one less digit, which

violates the axioms of the natural number order. So, connect The composite number of any two adjacent prime numbers must be an odd number. N is a positive integer, i.e. $n \in \mathbb{N}^*$.

代数推导证明方法 1. 除最初三个素数 1、2、3 外, 假设在 2 个相邻素数之间插入的合数个数为偶数个数时, 由于 p_m 和 p_{m+1} 是所有素数(包括单位素数 1)由小到大, 按 1, 2, 3, 5, 7, 11, \dots , p_{m-1} , p_m , p_{m+1} , \dots , 依次排序当中的 2 个相邻素数之间空位个数均为奇数, 因此, 插入合数个数为偶数个数时, 两素数之间必然多出一个或者是缺少一个空位, 违反自然数次序公理, 两者相互矛盾。所以, 除最初三个素数 1、2、3 外, 在任意 2 个相邻素数的合数个数必然是奇数。即从素数 5 开始, 任意两个相邻的素数在自然数序列中总是按奇数间隔规律出现的。如图 9 所示。这就是本文笔者首次发现的素数分布原理规律。定理 1 成立。
□

Algebraic derivation proof 1. In addition to the first three prime numbers 1, 2, 3, assuming that the number of composite numbers inserted between two adjacent prime numbers is an even number, since p_m and p_{m+1} is all prime numbers (including the unit prime number 1) from small to large, according to 1, 2, 3, 5, 7, 11, \dots , p_{m-1} , p_m , p_{m+1} , \dots . The number of vacancies between the two adjacent prime numbers in order is odd, so when the number of inserted composite numbers is even, there must be one more or a lack of vacancies between the two prime numbers, which violates the axioms of the order of natural numbers, and the two contradict each other. Therefore, except for the first three prime numbers 1, 2, and 3, the composite number of any two adjacent prime numbers must be odd. That is, starting with the prime number 5, any two adjacent prime numbers always appear in the sequence of natural numbers at odd intervals. As shown in Figure 9. This is the principle of prime number distribution discovered by the author for the first time. Theorem 1 is true. □

根据笔者发现的素数在自然数序列中的分布原理和规律可知, 除素数 1、2 和 3 外, 其他任意相邻的素数之间间隔的合数个数均为奇数。所以, 素数数列通项公式成立。

According to the distribution principle and law of prime numbers found by the author in the sequence of natural numbers, we can see that except for prime numbers 1, 2 and 3, the number of composite numbers separated by any other adjacent prime numbers is odd. Therefore, the general term formula of prime number series is valid.

以上定理 1, 本文笔者称之为第一素数分布出现定理, 或者素数通项公式定理。特别需要注意的是: 本定理与经典素数定理, 两者的概念和含义是不同的, 不能混淆。

The above theorem 1 is called the first prime number distribution theorem, or the prime number general term formula theorem. In particular, it should be noted that the concept and meaning of this theorem and the classical prime number theorem are different and should not be confused.

根据以上素数的推算方法, 我们可以验证素数通项公式(4.1)是成立的。

Similarly, according to the above prime number calculation method, we can verify whether the prime number general term formula (4.1) is valid.

例如 1, 查表 4, 由于第 1, 2, 3 个素数没有合数间隔, 因此合数个数均为 0; 根据定理 1 素数通项公式(4.1):

For example, 1, look at Table 4, because the 1, 2 and 3 prime numbers have no composite interval, so the composite number is 0; According to theorem 1 prime number general term formula (4.1):

$$P_m = m + S_m(P_m)$$

则有: There are: $P_1 = 1 + 0 = 1$; $P_2 = 2 + 0 = 2$; $P_3 = 3 + 0 = 3$.

例如 2, 查表 4, 第 11 个素数的素数间隔合数个数之和为 18; 根据定理 1 素数通项公式(4.1), 则有:

For example, 2, look at Table 4, the sum of the number of prime interval composite numbers of the 11th prime number is 18; According to theorem 1 prime number general term formula (4.1), then:

$$P_{11} = 11 + 18 = 29$$

例如 3, 查表 4, 第 61 个素数的素数间隔合数个数之和为 220; 根据定理 1 素数通项公式(4.1), 则有:

For example, 3, look at Table 4, the sum of the number of prime interval composite numbers of the 61st prime number is 220; According to theorem 1 prime number general term formula (4.1), then:

$$P_{61} = 61 + 220 = 281$$

例如 4, 查表 4, 第 296 个素数的素数间隔合数个数之和为 1637; 根据定理 1 素数通项公式(4.1), 则有:

For example, 4, look at Table 4, the sum of the combined number of prime intervals of the 296th prime number is 1637; According to theorem 1 prime number general term formula (4.1), then:

$$P_{296} = 296 + 1637 = 1933$$

例如 5, 查表 4, 第 1000 个素数的素数间隔合数个数之和为 6907; 根据定理 1 素数通项公式(4.1), 则有:

For example, 5, look at Table 4, the sum of the combination number of prime intervals of the 1000rd prime number is 6907; According to theorem 1 prime number general term formula (4.1), then:

$$P_{1000} = 1000 + 6907 = 7907$$

例如 6, 查表 4, 第 1230 个素数的素数间隔合数个数之和为 8743; 根据定理 1 素数通项公式(4.1), 则有:

For example, 6, look at Table 4, the sum of the prime-interval composite numbers of the 1230th prime number is 8743; According to theorem 1 prime number general term formula (4.1), then:

$$P_{1230} = 1230 + 8743 = 9973$$

据文献资料验证: 素数 370261 之后紧接着就是 111 个合数[2], 为奇数; 素数 1686994940955803 之后有 923 个合数; 素数 418032645936712127 之后有 1369 个合数等等, 这些合数个数都是奇数。目前已找到的素数 18361375334787046697, 最大间隔是有 1549 个合数[2], 也是奇数。没有任何一个反例。这些都是支持定理 1 成立的证据之一。

According to literature verification: The prime number 370261 is followed by 111 composite numbers [2], which are odd; The prime number 1686994940955803 is followed by 923 composite numbers; The prime number 418032645936712127 is followed by 1369 composite numbers and

so on, which are all odd numbers. The prime number 18361375334787046697 has a maximum interval of 1549 composite numbers [2], which are also odd. There isn't a single counterexample. This is all part of the evidence for theorem 1.

通过以上举例验证, 可见定理 1 给出的素数通项公式(4.1)是正确的。

Through the above examples, it can be seen that the formula (4.1) of prime numbers given by theorem 1 is correct.

根据以上等式(4.5), 很明显, 我们可以得到以下推论:

From the above equation (4.5), it is obvious that we can draw the following inferences:

推论 1. 相邻素数对, 或者广义孪生素数。即不小于 5 的任意相邻的 2 个素数间隔的合数个数均为奇数。

Corollary 1. Adjacent Prime Pairs, or Generalized Twin Primes. That is, the number of composite numbers between any two adjacent prime numbers not less than 5 is odd.

证明. 根据定理 1, 可得第 m 个素数通项公式(4.1):

Proof. According to theorem 1, the general term formula for the m th prime number (4.1) can be obtained:

$$P_m = m + S_m(P_m)$$

对于第 $m-1$ 个相邻素数, 则有:

For the adjacent prime number $m-1$, there is:

$$P_{m-1} = m - 1 + S_{m-1}(P_{m-1}) \quad (4.6)$$

将(4.1)减(4.6), 可得,

Subtract (4.1) from (4.6), it is obtained

$$P_m - P_{m-1} = m - (m - 1) + [S_m(P_m) - S_{m-1}(P_{m-1})]$$

即, That is,

$$P_m - P_{m-1} = (m - m) + 1 + [S_m(P_m) - S_{m-1}(P_{m-1})]$$

上式由于 $m - m + 1 \equiv 1$ 。可见有无穷多对相邻素数对, 不论 m 为任何非零自然数, 总有:

Because $m - m + 1 \equiv 1$. It can be seen that there are infinitely many pairs of adjacent prime numbers, and whether m is any non-zero natural number, there are always:

$$P_m - P_{m-1} = [S_m(P_m) - S_{m-1}(P_{m-1})] + 1 \quad (4.7)$$

当 $m \geq 4$ 时, 则得,

When $m \geq 4$, it is obtained

$$P_m - P_{m-1} = K_m + 1 \quad (4.8)$$

其中, $K_m = 1$, 或 3, 或 5, 或 7, \dots , 或 $2n - 1$, \dots ; $n, m \in N^*$

Among them, $K_m = 1, \text{or} 3, \text{or} 5, \text{or} 7, \dots, \text{or} 2n - 1, \dots$; $n, m \in N^*$

式中: $3 \leq p_{m-1} < p_m$; $m \geq 4$; $n, m \in N^*$; K_m 是 2 个相邻素数间隔的合数的个数。根据不小于素数 5 之间的合数的个数是奇数的分布原理规律, 即 2 个相邻素数间隔的合数个数为 1, 或 3, 或 5, 或 7, \dots , 或 $2n - 1$, 以至无穷大。推论 1 成立。□

Where: $3 \leq p_{m-1} < p_m$; $m \geq 4$; $n, m \in N^*$; K_m is the number of composite numbers separated by two adjacent prime numbers. According to the distribution principle that the number of composite numbers between not less than the prime number 5 is odd, that is, the number of composite numbers between two adjacent prime numbers is 1, or 3, or 5, or 7, ... Or $2n - 1$, or even infinity.

Corollary 1 is true. \square

推论 2. 孪生素数，或者狭义孪生素数。

所谓孪生素数是指两个相邻素数之间间隔的合数个数是 1 的两个素数。

Corollary 2. Twin Primes, or Narrow Twin Primes.

Twin Primes are two primes in which the number of composite numbers separated by two adjacent primes is 1.

证明. 根据定理 1 的推论 1，则有式(4.8)：

Proof. According to the corollary 1 of theorem 1, the formula (4.8) is:

$$P_m - P_{m-1} = K_m + 1$$

其中， $K_m = 1$ ，或 3，或 5，或 7， \dots ，或 $2n - 1$ ， \dots ； $m \geq 4$ ； $n, m \in N^*$

Among them, $K_m = 1, \text{or} 3, \text{or} 5, \text{or} 7, \dots, \text{or} 2n - 1, \dots$ ； $m \geq 4$ ； $n, m \in N^*$

因为，2 个相邻素数间隔合数个数为 1，即 $K_m = 1$ ($m \geq 4$ ； $m \in N^*$)。

将 $K_m = 1$ 代入

Because, 2 adjacent prime interval number is 1, that is, $K_m = 1$ ($m \geq 4$ ； $m \in N^*$)。

Substitute $K_m = 1$ into

$$P_m - P_{m-1} = K_m + 1$$

即，That is,

$$P_m - P_{m-1} = (1)_m + 1 \quad (4.9)$$

式中： $P \neq 1, 2, 3$ ； $m \geq 4$ ； $m \in N^*$ 。

Where: $P \neq 1, 2, 3$ ； $m \geq 4$ ； $m \in N^*$ 。

也就是说，任意 2 个孪生素数之差总是等于 2。推论 2 成立。 \square

That is, the difference between any two twin prime numbers is always equal to 2.

Corollary 2 is true. \square

推论 3. 任意一个素数可表示为它的相邻素数($P \neq 2$)加上一个偶数(包括个位数 0)。

Corollary 3. Any prime number can be represented as its adjacent prime number ($P \neq 2$) plus an even number (including the single digit 0).

证明. 若定义零为偶数，对于最初 3 个素数 1, 2, 3，显然推论 3 是成立的。

Proof. If zero is defined as even, for the first three prime numbers, 1, 2, 3, it is clear that the corollary 3 holds.

当 $m+1 \geq 4$ ， $P \neq 2$ 时，根据以上定理 1，则有：

When $m+1 \geq 4$, $P \neq 2$, according to the above theorem 1, there is:

$$P_{m+1} - P_m = K_{m+1} + 1$$

因为 $K_{m+1} = 1$ ，或 3，或 5，或 7，或 9， \dots ，或 $2n - 1$ ， \dots ，($m+1 \geq 4$ ； $n, m \in N^*$)，为奇数。由于 $K_{m+1} + 1$ 必是偶数，并令

Because $K_{m+1} = 1, \text{or} 3, \text{or} 5, \text{or} 7, \text{or} 9, \dots, \text{or} 2n - 1, \dots$, ($m+1 \geq 4$ ； $n, m \in N^*$), is odd.

Because $K_{m+1} + 1$ must be even, and make

$$K_{m+1} + 1 = (2n - 1)_m + 1 \quad (n, m \in N^*)$$

可得，it is available

$$P_{m+1} - P_m = (2n - 1)_m + 1$$

即，i.e.

$$P_{m+1} = P_m + (2n)_m \quad (4.10)$$

式中: $P \neq 2$; $m \geq 4$; $m \in N^*$.

Where: $P \neq 2$; $m \geq 4$; $m \in N^*$.

也就是说, 任意一个素数可表示为它的相邻素数($P \neq 2$)加上一个偶数。所以推论 3 成立。□

That is, any prime number can be represented as its adjacent prime number ($P \neq 2$) plus an even number. So corollary 3 is true. □

集合推导证明方法 2.

Set derivation proof method 2.

根据有序正整数集合的定义, 这里令全体有序正整数集合:

According to the definition of the set of ordered positive integers, here let the set of all ordered positive integers:

$$U = \{1, 2, 3, 4, 5, 6, 7, \dots, n\}, \text{ 即, which is } U = N^*$$

其中, 在全体有序正整数集合中, 所有有序素数集合:

Where, in the set of all ordered positive integers, the set of all ordered primes: AABBB

$$P = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \dots\}$$

更一般地, More generally,

$$P = \{P_m\} = \{P_1, P_2, P_3, \dots, P_m\}$$

又假设, 有序正整数集合 B 为有序素数集合的补集, 也就是所有正整数当中除素数之外所构成的一个有序合数集合。并定义如果两个素数之间没有合数间隔, 为空集 \emptyset , 定义合数元素个数为 0。则有:

It is also assumed that the set of ordered positive integers B is the complement of the set of ordered prime numbers, that is, an ordered composite number set composed of all positive integers except the prime numbers. It is also defined that if there is no composite interval between two prime numbers, it is the empty set \emptyset , and the number of composite elements is defined as 0. There is:

$$B = \{\emptyset, \emptyset, \emptyset, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, \dots\}$$

若合数 $b_{r,m}$ 为素数 P_m 间隔合数, r 为合数元素个数, m 为有序素数的序数。因此, 所有素数的有序合数集合, 我们可以更一般地, 表示为:

If the composite number $b_{r,m}$ is the prime P_m interval composite number, r is the number of composite elements, and m is the ordinal number of ordered primes. Therefore, the set of ordered composite numbers of all prime numbers can be expressed, more generally, as:

$$B = \{b_{r,m}\} = \{b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}\},$$

其中, 有序合数 $b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}$,

Where, the ordered composite number $b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}$,

分别依次为有序素数 $P_1, P_2, P_3, \dots, P_{m-1}, P_m$ 的 r 个连续前继数, 也是每个对应素数所间隔合数。同时, 有序素数 $P_1, P_2, P_3, \dots, P_{m-1}, P_m$ 分别依次又是有序合数 $b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}$ 的后继数。

The r consecutive precedents of the ordered prime $P_1, P_2, P_3, \dots, P_{m-1}, P_m$, respectively, are also the composite numbers separated by each corresponding prime. At the same time, the ordered prime number $P_1, P_2, P_3, \dots, P_{m-1}, P_m$ is in turn the ordered composite

number $b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}$, the successor of $P_1, P_2, P_3, \dots, P_{m-1}, P_m$.

根据有序非零自然数集, 则有:

According to the set of ordered non-zero natural numbers, there is:

$$P \cup C_U P = U \quad (U = N^*)$$

即, That is,

$$P \cup B = N^*$$

也就是, That is,

$$\{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \dots, P_{m-1}, P_m\} \cup \{\emptyset, \emptyset, \emptyset, \{4\}, \{6\}, \{8, 9, 10\}, \{12\}, \{14, 15, 16\}, \{18\}, \{b_{r,m-1}\}, \{b_{r,m}\}, \dots\} = N^*$$

$$U = \{\emptyset, 1\}, \{\emptyset, 2\}, \{\emptyset, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9, 10, 11\}, \{12, 13\}, \{14, 15, 16, 17\}, \{18, 19\}, \dots, \{b_{r,m-1}, P_{m-1}\}, \{b_{r,m}, P_m\}, \dots$$

因为, $P = \{P_m\} = \{P_1, P_2, P_3, \dots, P_m\}$, 是所有有序正整数集合 U 的子集合。所有素数元素从小到大依次排列所形成的一个有序素数数列, 其中 m 为素数数列的排次序数。

Because $\{P_m\} = \{P_1, P_2, P_3, \dots, P_m\}$, is a subset of the set of all ordered positive integers U . An ordered prime sequence of all prime elements arranged from smallest to largest, where m is the ordering ordinal of the prime sequence.

$B = \{b_{r,m}\} = \{b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}\}$, 是数列 $\{P_m\}$ 中, 两个连续素数元素之间间隔补集 $C_U P$ 元素的个数, 前 m 项级数 $S_m = \sum \text{Card}(C_U A)$, 是前 m 个补集元素个数之和, 即补集前 m 个元素的个数级数。也就是:

$B = \{b_{r,m}\} = \{b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}\}$, is the number of interval complement $C_U P$ elements between two consecutive prime elements in the sequence $\{P_m\}$, and the first m term series $S_m = \sum \text{Card}(C_U A)$ is the sum of the first m complement elements, that is, the number series of m elements before the complement. That is,

$$\{b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m-1}, b_{r,m}\} \cup \{P_1, P_2, P_3, \dots, P_m\} = N^*$$

更一般地, More generally,

$$\{b_{r,1}, P_1\} \cup \{b_{r,2}, P_2\} \cup \{b_{r,3}, P_3\}, \dots, \{b_{r,m-1}, P_{m-1}\} \cup \{b_{r,m}, P_m\} = N^*$$

根据自然数的定义, 每个自然数都是由 n 个元素构成的一个特殊有序集合。即,

$$0 = \emptyset, 1 = \{0\}, 2 = \{0, 1\}, 3 = \{0, 1, 2\}, \dots, n+1 = \{0, 1, 2, 3, \dots, n\} = n \cup \{n\}$$

According to the definition of natural numbers, every natural number is a special ordered set of n elements. That is,

$$0 = \emptyset, 1 = \{0\}, 2 = \{0, 1\}, 3 = \{0, 2\}, \dots, n+1 = \{0, 1, 2, 3, \dots, n\} = n \cup \{n\}$$

也就是: 若 $n \in N^*$, 则 $n+1 = n^+$ (后继数), $n^+ \in N^*$ 。

That is: if $n \in N^*$, then $n+1 = n^+$ (successor), $n^+ \in N^*$.

因此, 对于任意一个素数, 若以补集(即有序合数集合)元素个数来表示, 则得有序素数通项公式(4.1):

Therefore, for any prime number, if it is represented by the number of elements of the complement (that is, the set of ordered composite numbers), the formula (4.1) of ordered prime numbers is obtained:

$$P_m = m + \sum \text{Card}\{b_{r,m}\}$$

其中, Among them,

$$\sum \text{Card}(b_{r,m}) = \sum_{r,m=1}^{\infty} (\text{Card}\{b_{r,1}\} + \text{Card}\{b_{r,2}\} + \text{Card}\{b_{r,3}\} + \cdots + \text{Card}\{b_{r,m}\})$$

以上 $\sum \text{Card}(b_{r,m})$ 作者称之为有序素数数列的无穷特征级数。

The above $\sum \text{Card}(b_{r,m})$ author calls the sequence of ordered prime numbers infinite characteristic series.

举例说明如下。Examples are given below.

对于素数 1, 由于 $\{b_{0,1}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; 则有素数 $P_1 = 1 + \sum \{b_{0,1}\} = 1 + 0 = 1$;

For the prime number 1, since $\{b_{0,1}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; The prime number $P_1 = 1 + \sum \{b_{0,1}\} = 1 + 0 = 1$;

对于素数 2, 由于 $\{b_{0,2}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; 则有素数 $P_2 = 2 + \sum \{b_{0,2}\} = 2 + (0 + 0) = 2$;

For the prime number 2, since $\{b_{0,2}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; The prime number $P_2 = 2 + \sum \{b_{0,2}\} = 2 + (0 + 0) = 2$;

对于素数 3, 由于 $\{b_{0,3}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; 则有素数 $P_3 = 3 + \sum \{b_{0,3}\} = 3 + (0 + 0 + 0) = 3$;

For the prime number 3, since $\{b_{0,3}\} = \{\emptyset\}$, $\text{Card}\{\emptyset\} = 0$; The prime number $P_3 = 3 + \sum \{b_{0,3}\} = 3 + (0 + 0 + 0) = 3$;

对于素数 5, 由于 $\{b_{1,4}\} = \{4\}$, $\text{Card}\{4\} = 1$; 则有素数 $P_4 = 4 + \sum \{b_{1,4}\} = 4 + (0 + 0 + 0 + 1) = 5$;

For prime 5, since $\{b_{1,4}\} = \{4\}$, $\text{Card}\{4\} = 1$; The prime number $P_4 = 4 + \sum \{b_{1,4}\} = 4 + (0 + 0 + 0 + 1) = 5$;

对于素数 7, 由于 $\{b_{1,5}\} = \{6\}$, $\text{Card}\{6\} = 1$; 则有素数 $P_5 = 5 + \sum \{b_{1,5}\} = 5 + (0 + 0 + 0 + 1 + 1) = 7$;

For prime 7, since $\{b_{1,5}\} = \{6\}$, $\text{Card}\{6\} = 1$; The prime number $P_5 = 5 + \sum \{b_{1,5}\} = 5 + (0 + 0 + 0 + 1 + 1) = 7$;

对于素数 11, 由于 $\{b_{3,6}\} = \{8, 9, 10\}$, $\text{Card}\{8, 9, 10\} = 3$; 则有素数 $P_6 = 6 + \sum \{b_{3,6}\} = 6 + (0 + 0 + 0 + 1 + 1 + 3) = 11$;

For prime 11, since $\{b_{3,6}\} = \{8, 9, 10\}$, $\text{Card}\{8, 9, 10\} = 3$; The prime number $P_6 = 6 + \sum \{b_{3,6}\} = 6 + (0 + 0 + 0 + 1 + 1 + 3) = 11$;

对于素数 13, 由于 $\{b_{1,7}\} = \{12\}$, $\text{Card}\{12\} = 1$; 则有素数 $P_7 = 7 + \sum \{b_{1,7}\} = 7 + (0 + 0 + 0 + 1 + 1 + 3 + 1) = 13$;

For the prime number 13, since $\{b_{1,7}\} = \{12\}$, $\text{Card}\{12\} = 1$; The prime number $P_7 = 7 + \sum \{b_{1,7}\} = 7 + (0 + 0 + 0 + 1 + 1 + 3 + 1) = 13$;

...

依此类推, 对于任意素数 P_m , 若以集合表述, 则有:

And so on, for any prime number P_m , expressed as a set, there is:

$$P_m = m + \sum \text{Card}\{b_{r,m}\}$$

以上定理 1 成立。□

Theorem 1 above is true. □

若令 $S_m(P_m) = \sum \text{Card}\{b_{r,m}\}$, 上式也可以表示为:

If $S_m(P_m) = \sum \text{Card}\{b_{r,m}\}$, the above formula can also be expressed as:

$$P_m = m + S_m(P_m)$$

所以, 以上两种方法推论和证明结果都是一致的。

下面以定理 2，给出素数通项公式的复数几何证明方法 3。

复数几何证明方法 3.

定理 2. 有序素数的平行四边形分解。

任意一个有序素数都可以分解为它的一个序数 m 与它的一个前 m 项所有间隔合数的个数级数之和, 并且构成平行四边形 $AFBE$ 。或者表述为任意一个有序素数都可以分解为两个特定正整数之和, 并构成一个平行四边形。作者又称之为第二素数分布出现定理。

Figure 10: Schematic diagram of the parallelogram decomposition of any ordered prime number

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second prime number distribution occurrence theorem.

The above theorem 2 is sometimes referred to simply as the parallelogram decomposition theorem for prime numbers.

如图 10 所示。作一直角三角形 ABC ，设直角边长 AC 等于任意一个有序素数 P_m ；另一条直角边长 BC 等于 1。过 BC 中点 Q 作垂直平分线 L ，在垂直平分线上取一点 E ，并使线段 QE 等于有序素数 P_m 的排列序数 m 。然后连接 AE 和 EB ，再过 A 点作一平行于线段 EB 的直线 AF ，并与垂直平分 L 相交于点 F ，最后连接 FB ，便形成一个平行四边形 $AFBE$ 。其中， AB 和 EF 是平行四边形 $AFBE$ 的两条相互平分的对角线，且交点为 G 。因此，则有：

As shown in Figure 10. Make a constant Angle triangle ABC , and let the right Angle side length AC equal to any ordered prime number P_m ; The other right Angle, BC , is equal to 1. Take a point E on the vertical bisector L over the BC midpoint Q , and make the line segment QE equal to the ordinal m of the ordered prime P_m . Then, AE and EB are connected, and a straight AF parallel to line segment EB is formed through point A , and the vertical bisector L is intersected at point F , and finally FB is connected to form a parallelogram $AFBE$. Where AB and EF are the two bisecting diagonals of the parallelogram $AFBE$, and the intersection point is G . Therefore, there are:

$$AC = EQ + FQ \quad (4.11)$$

若令线段 $FQ = S_m$ ， S_m 为有序素数序列的前 m 项所有间隔合数的个数级数之和。

因为 $AC = P_m$ ， $EQ = m$ ，所以，即得任意有序素数的通项公式为：

If the line segment $FQ = S_m$ ， S_m is the number series sum of all the spacing composite numbers of the first m terms of the ordered prime sequence.

Since $AC = P_m$ and $EQ = m$, the general formula for any ordered prime number is

$$P_m = m + S_m \quad (4.12)$$

也就是说任意一个有序素数都可以分解为两个特定正整数之和，并且构成平行四边形 $AFBE$ 。如图 10 所示。

In other words, any ordered prime number can be decomposed into the sum of two specific positive integers, and form a parallelogram $AFBE$. As shown in Figure 10.

证明。 根据以上定理 2 命题，因为，在直角 $\triangle ABC$ 中， $AC \perp BC$ ， $FQ \perp BC$ ，并且垂直线 FQ 与直角 $\triangle ABC$ 的斜线 AB 相交于 G 点。

Proof. According to the above Theorem 2 proposition, because, in the right Angle $\triangle ABC$, $AC \perp BC$, $FQ \perp BC$, and the vertical line FQ intersects the diagonal line AB of the right Angle $\triangle ABC$ at the G -point.

现分别过直角 $\triangle ABC$ 直角边 BC 上的垂直平分线上的两点 F 和 E ，作两条分别平行于 BC 的直线，与另一直角边 AC 相交于点 E' 和 F' ，则有

$E'E // CQ$ ， $F'F // CQ$ ，得 $QE = CE'$ ，直角 $\triangle E'CQ \cong$ 直角 $\triangle EQB$ (边、角、边)

所以， $\angle CQE' = \angle QBE$ (两个三角形全等对应角相等)，可得， $E'Q // EB$ (同位角相等两条直线相互平行)。

Now pass the two points F and E on the vertical bisector on the right Angle $\triangle ABC$ side BC respectively to make two straight lines parallel to BC respectively, and intersect with the other Angle side AC at points E' and F' , then there is

$E'E//CQ, F'F//CQ, QE = CE', \text{right Angle } \triangle E'CQ \cong \text{Right Angle } \triangle EQB$ (side, Angle, side)
Therefore, $\angle CQE' = \angle QBE$ (two triangles with equal corresponding angles are equal), we can get, $E'Q//EB$ (the corresponding angles are equal and the two lines are parallel to each other).

又因为 $AF//EB$ (依据命题可知), 可得, 直角 $\triangle EQB \cong$ 直角 $\triangle AFF'$ 。

And because $AF//EB$ (according to the proposition), can be obtained, right Angle $\triangle EQB$ right Angle $\triangle AFF'$.

所以, $AF = EB$ 。又因为 $\angle EBA = \angle BAF$ (两条直线平行同位角相等)。

So $AF = EB$. And because $\angle EBA$ is equal to $\angle BAF$.

因此, $\triangle EBA \cong \triangle BAF$ (边, 角, 边)。

Thus, $\triangle EBA \cong \triangle BAF$ (side, Angle, side).

同理可得, $\triangle FEA \cong \triangle FEB$ (边, 角, 边), $\triangle AGE \cong \triangle FGB$ (角, 边, 角)。

By the same token, $\triangle FEA \cong \triangle FEB$ (side, Angle, side), $\triangle AGE \cong \triangle FGB$ (Angle, side, Angle).

所以, 我们可得四边形 $AFBE$ 是平行四边形。其中, AB 和 EF 是平行四边形 $AFBE$ 的两条相互平分的对角线, 且交点为 G 。

So, we have quadrilateral $AFBE$ is a parallelogram. Where AB and EF are the two bisecting diagonals of the parallelogram $AFBE$, and the intersection point is G .

其次, 根据勾股定理, 在直角 $\triangle ABC$ 中, 则有:

Secondly, according to the Pythagorean theorem, in the right Angle $\triangle ABC$, there is:

$$AB^2 = BC^2 + AC^2 \quad (4.13)$$

又因为, And because of

$$AC = E'A + E'C \quad (4.14)$$

将式(4.14)代入式(4.13), 得

Substitute equation (4.14) into equation (4.13) to get

$$AB^2 = E'A^2 + 2 E'A \cdot E'C + E'C^2 + BC^2 \quad (4.15)$$

又因为, And because of

$$FE = FQ - EQ \quad (4.16)$$

将式(4.16)两边平方, 得

Square both sides of the equation (4.16) to get

$$FE^2 = FQ^2 - 2FQ \cdot EQ + EQ^2 \quad (4.17)$$

将式(4.15)+式(4.17), 得

Add formula (4.15)+ formula (4.17) to get

$$AB^2 + FE^2 = E'A^2 + FQ^2 + E'C^2 + EQ^2 + BC^2 + 2E'A \cdot E'C - 2FQ \cdot EQ \quad (4.18)$$

又因为, And because of

$$E'A = FQ, E'C = EQ, BQ = QC$$

$$BC^2 = 2BQ^2 + 2QC^2$$

将式(4.18)式, 整理为:

The formula (4.18) is arranged as:

$$AB^2 + FE^2 = EA^2 + FQ^2 + E'C^2 + EQ^2 + 2BQ^2 + 2QC^2 \quad (4.19)$$

当 $FQ = E'A$, $E'C = EQ$ 时, 则有:

At that time, $FQ = E'A$, $E'C = EQ$ there was:

$$AB^2 + FE^2 = 2(EA^2 + QC^2) + 2(EQ^2 + BQ^2) \quad (4.20)$$

在直角 $\triangle AE'E$ 和直角 $\triangle AF'F$ 中, 根据勾股定理, 即得: aabb

In the right angles $\triangle AE'E$ and $\triangle AF'F$, according to the Pythagorean theorem,

$$AB^2 + FE^2 = 2EA^2 + 2EB^2 \quad (4.21)$$

is obtained

或者当 $AAE'A = FQ$, $EQ = E'C$ 时, 在直角 $\triangle FQB$ 和直角 $\triangle QE'C$ 中, 根据勾股定理, 则有:

Or at that time, in the right angles $\triangle FQB$ and $\triangle QE'C$, according to the Pythagorean theorem, there is

$$AB^2 + FE^2 = 2(FQ^2 + BQ^2) + 2(E'C^2 + QC^2) \quad (4.22)$$

即, That is,

$$AB^2 + FE^2 = 2FB^2 + 2E'Q^2$$

又因为 $E'Q = AF$, 所以, 得:

And because $E'Q = AF$, we get:

$$AB^2 + FE^2 = 2FB^2 + 2AF^2 \quad (4.23)$$

因此, 平行四边形 $AFBE$ 的两条对角线的平方和等于四条边平方和。

Thus, the sum of the squares of the two diagonals of a parallelogram $AFBE$ is equal to the sum of the squares of the four sides.

显然, 依据以上推理, 我们可以得到, 以下推论如下。

Obviously, from the above reasoning, we can conclude that the following inferences are as follows.

推论 1. 当任意一个平行四边形其中一条对角线长度确定时, 另一条对角线的长度不确定时, 可构成的平行四边形有无限多个。

Corollary1. When the length of one diagonal of any parallelogram is determined, the length of another diagonal is uncertain, there are infinite parallelograms that can be formed.

再次, 下面证明任意一个有序素数都可以分解为它的一个序数 m 与它的一个前 m 项所有间隔合数的个数级数之和 S_m , 并且构成平行四边形 $AFBE$ 。或者表述为任意一个有序素数 P_m 都可以分解为两个特定正整数之和, 并且构成平行四边形。

Again, it is shown below that any ordered prime number can be decomposed into the sum of the number series S_m of one of its ordinals m and all the interval composite numbers of one of its preceding m terms, and form the parallelogram $AFBE$. Or it can be expressed that any ordered prime number P_m can be decomposed into the sum of two specific positive integers and form a parallelogram.

如图 10 所示。设 $AC = P_m$, $EQ = m$, $FQ = S_m$ 。其中, P_m 、 S_m 和 m 均为正整数。即 P_m 、 S_m 、 $m \in N^*$ 。在直角 $\triangle ABC$ 中, 根据勾股定理, 得

As shown in Figure 10. Let $AC = P_m$, $EQ = m$, $FQ = S_m$. In the command, P_m , S_m , and m are positive integers. That is, P_m , S_m and $m \in N^*$. In the right Angle $\triangle ABC$, according to the

Pythagorean theorem, we get

$$AB^2 = P_m^2 + 1 \quad (4.24)$$

因为 $FE = S_m - m$, 两边同时平方, 得

Because $FE = S_m - m$, if you square both sides, you get

$$FE^2 = (S_m - m)^2 = S_m^2 - 2S_m m + m^2 \quad (4.25)$$

$$FB^2 = S_m^2 + \frac{1}{4} \quad (4.26)$$

$$AF^2 = m^2 + \frac{1}{4} \quad (4.27)$$

将以上四式, 即式(4.24)、式(4.25)、式(4.26)和式(4.27)分别代入式(4.23), 得:

By substituting the above four formulas, namely formula (4.24), formula (4.25), formula (4.26) and formula (4.27) respectively into formula (4.23), we get:

$$P_m^2 + 1 + S_m^2 - 2S_m m + m^2 = 2(S_m^2 + \frac{1}{4}) + 2(m^2 + \frac{1}{4})$$

整理, 得 Tidy up, get

$$P_m^2 = S_m^2 + 2S_m m + m^2$$

即, That is,

$$P_m^2 = (S_m + m)^2 \quad (4.28)$$

所以, 得 So,

$$P_m = S_m + m \quad (4.29)$$

因为, S_m 、 m 均为正整数。依据以上推理可知, P_m 也必然为正整数。

Because S_m 、 m is a positive integer. According to the above reasoning, P_m must also be a positive integer.

也就是说任意一个有序素数都可以分解为两个特定正整数之和, 并且构成平行四边形。如图 10 所示。所以定理 2 成立。□

That is to say, any ordered prime number can be decomposed into the sum of two specific positive integers and form a parallelogram. As shown in Figure 10. So theorem 2 is true. □

因此, 根据以上定理 2 可知, 任意一个有序素数都可以分解为它的一个序数 m 与它的一个前 m 项所有间隔合数的个数级数之和 S_m , 并且构成平行四边形 $AFBE$ 。或者任意一个有序素数 P_m 都可以分解为两个特定正整数之和, 并且构成平行四边形 $AFBE$ 。并且决定任意一个有序素数分解的 E 、 F 和 G 三个非平凡点都在平行四边形 $AFBE$ 较短的对角线上。也就是在 CB 等于 1 的 $1/2$ 的垂直线上。

Therefore, according to the above theorem 2, it can be seen that any ordered prime number can be decomposed into S_m , the sum of the number series of one of its ordinals m and all the spacing composite numbers of one of its preceding m terms, and form the parallel-shaped $AFBE$. Or any ordered prime P_m can be decomposed into the sum of two specific positive integers and form the parallelogram $AFBE$. It is also determined that the three nontrivial points E , F and G of any ordered prime decomposition are all on the shorter diagonal of the parallelogram $AFBE$. It's on the vertical line where CB is equal to $1/2$ of 1.

根据以上定理 2, 如果我们进一步把数的范围扩展到复数范围, 在复平面里, 我们也可将任意一个素数表示为两个特定的正整数之和。即

In the same way, according to the above law 2, in the complex plane, we can also represent any prime number as the sum of two specific positive integers. That's

$$z = z_1 + z_2$$

即根据复数相加的几何意义可知，三个复数向量构成一个平行四边形。以上关系式的推理证明如下所述。

That is, according to the geometric meaning of adding complex numbers, three complex vectors form a parallelogram. The reasoning proof of the above relation is described below.

将式(4.29)两边同时乘上虚数单位 i ，并且加上单位自然数 1 ，则有：

Multiply both sides of the equation (4.29) by the imaginary unit i and add the unit natural number 1 , and we get:

$$1 + iP_m = 1 + i(S_m + m)$$

$$1 + iP_m = (1/2 + im) + (1/2 + iS_m) \quad (4.30)$$

若令 If the

$$z = 1 + iP_m$$

$$z_1 = 1/2 + im$$

$$z_2 = 1/2 + iS_m$$

并且分别代入式(4.30)，可得：

And plug in the formula (4.30) respectively to get:

$$z = z_1 + z_2 \quad (4.31)$$

即三个复数向量构成一个平行四边形。

Three complex vectors form a parallelogram.

其中，

$$z = 1 + iP_m$$

$$z_1 = 1/2 + im$$

$$z_2 = 1/2 + iS_m$$

以上式中， m 为有序素数的序数， P_m 为有序素数数列的通项。 $S_m = S_m(P_m)$ 为有序素数数列的前 m 项所有间隔合数个数的特征无穷级数函数。 $m \in N^*$ 。

In the above formula, m is the ordinal number of the ordered prime number, and P_m is the general term of the sequence of ordered prime numbers. $S_m = S_m(P_m)$ is a characteristic infinite series function of all the spacing composite numbers of the first m terms of an ordered prime sequence. $m, S_m \in N^*$.

所以，根据推理过程和定理 1，可知以上证明成立。也就是说任意一个有序素数都可以分解为一个序数 m 与一个前 m 项合数的个数级数和 S_m ，并且构成一个平行四边形 AFBE。同时，确定任意一个素数数分解为两个特定正整数之和的三个相关的非平凡点 G、E 和 F 都在 CB 等于 1 的 1/2 的垂直线上。

Therefore, according to the reasoning process and theorem 1, we can see that the above proof is true. That is to say, any ordered prime number can be decomposed into an ordinal m and a composite number of m terms and S_m , and form a parallelogram AFBE. At the same time, it is determined that the three related nontrivial points G, E, and F, which decompose any prime number into the sum of two specific positive integers, are all on the vertical line where CB is equal to 1/2 of 1.

定理 3. 素数出现分布的原理规律

Theorem 3. Distribution law of prime numbers

所有素数都是分布在非零自然数密度比率函数等于 1 的直线上。用数学公式表示为：
All prime numbers are distributed on a line with a density ratio of non-zero natural numbers equal to 1. It is expressed by the mathematical formula:

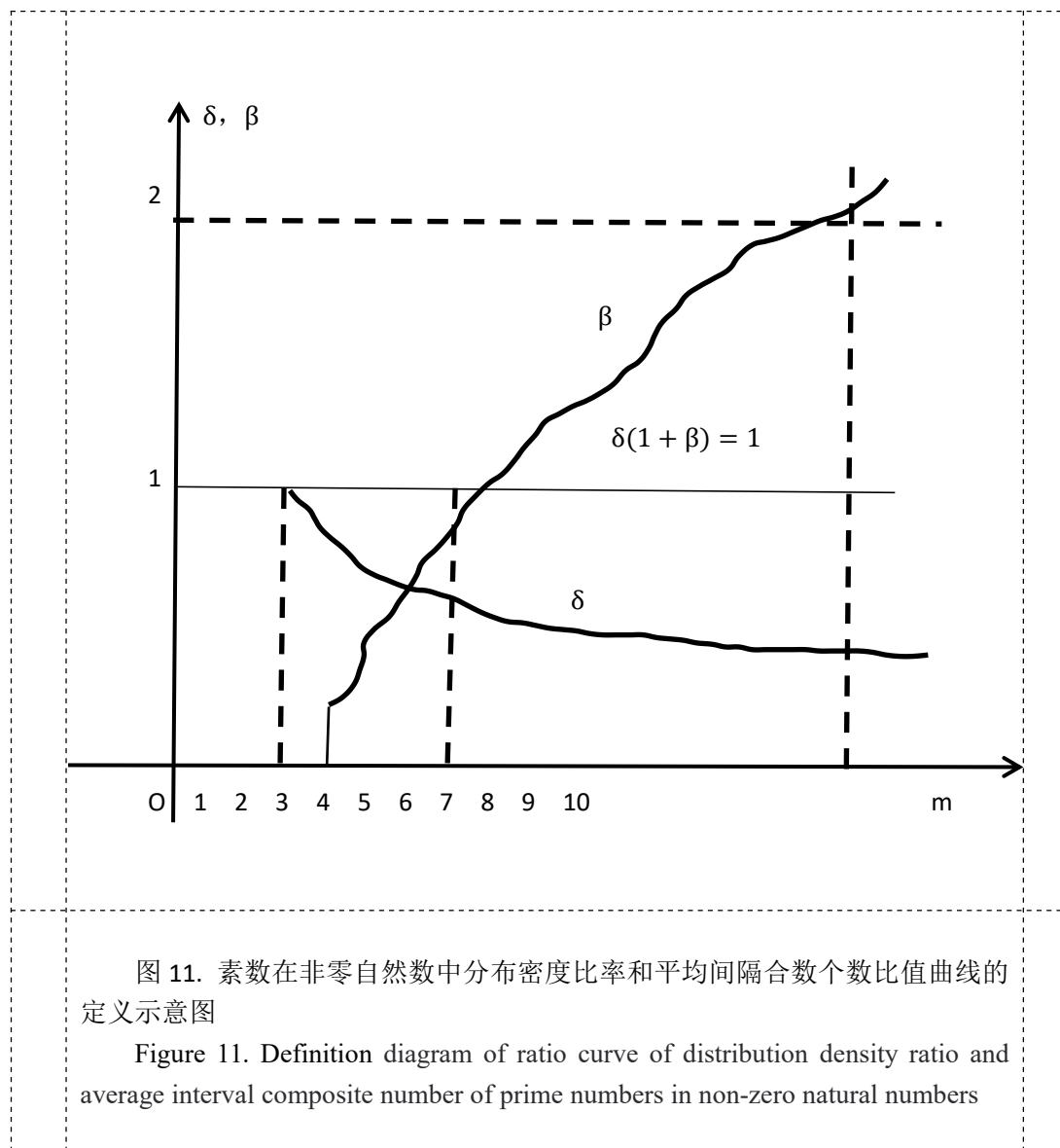
$$\delta(1 + \beta) = 1 \quad (4.32)$$

若令 Let

$$f(\delta, \beta) = \delta(1 + \beta) = 1,$$

作者则称 $f(\delta, \beta)$ 为有序素数的分布密度比率函数。

The authors call $f(\delta, \beta)$ the distribution density ratio function of ordered prime numbers.



其中, Among them,

$$\delta = \frac{m}{p_m}, \quad \beta = \frac{S_m(p_m)}{m}$$

如图 11 所示。Is shown in Figure 11.

为了揭示素数在非零自然数中的分布原理规律，这里我们首先给出以下 2 个定义。

In order to reveal the distribution principle of prime numbers in non-zero natural numbers, we first give the following two definitions.

定义 1. 在素数序列 $\{P_m\}$ 中，即从素数 1 开始，到第 m 个素数 p_m 的所有素数个数 m 与素数 p_m 之比 (这时这个素数的大小正好就是非零自然数的大小)：

Definition 1. In the sequence of prime numbers $\{P_m\}$, that is, the ratio of all prime numbers m to prime p_m starting from prime 1 to the m th prime p_m (in which case the size of the prime number is exactly the size of the non-zero natural number) :

$$\delta = \frac{m}{p_m} \quad (4.33)$$

以上比率，笔者称之为素数的非零自然数分布密度比率，用 δ 表示。

根据以上定理 1，可得的第 m 个素数通项公式 (4.1)：

The above ratio is called the ratio of distribution density of non-zero natural numbers of prime numbers and is expressed by δ .

According to the above theorem 1, the general term formula of the m th prime number (4.1) can be obtained:

$$P_m = m + S_m(P_m)$$

则有，There is

$$P_m \gg m$$

因此，素数 P_m 远远大于素数序数 m 。

Thus, the prime P_m is much larger than the prime ordinal m .

即，That is,

$$\lim_{m \rightarrow \infty} \delta = \lim_{m \rightarrow \infty} \frac{m}{P_m} \rightarrow 0 \text{ (无穷小, 但不等于 0)}$$

(无穷小, 但不等于 0 Infinitesimal, but not equal to zero)

也就是说，素数在非零自然数中的分布密度比率是越来越小，是一个永远不等于零的无穷小量。如图 11 和表 5 所示。

In other words, the ratio of the distribution density of prime numbers in non-zero natural numbers is getting smaller and smaller, and it is an infinite small quantity that never equals zero. Figure 11 and Table 5 show this.

定义 2. 任意一个素数 p_m 之前，所有合数个数之和 $S_m(P_m)$ 与素数个数 m 的比值为：

Definition 2. Before any prime p_m , the ratio of the sum of all composite numbers $S_m(P_m)$ to the prime number m is:

$$\beta = \frac{S_m(P_m)}{m} \quad (4.34)$$

以上比值，笔者称之为某素数的合数个数级数 $\sum m(P_m)$ 与素数个数 m 的比值，简称素数间隔合数的个数比值，用 β 表示。它与非零自然数分布密度比率 δ 共同决定有序素数出现的规律。

如图 11 所示。

The above ratio is called the ratio of the composite number series $S_m(P_m)$ of a prime number to the prime number m , referred to as the ratio of the composite number between prime numbers, expressed by β . Together with the distribution density ratio of non-zero natural numbers δ , it determines the regularity of the occurrence of ordered prime numbers. As shown in Figure 11.

根据以上定理 1，第 m 个素数通项公式(4.1):

According to the above theorem 1, the general term of the m -th prime number (4.1) is

$$P_m = m + S_m(P_m)$$

则有 There is

$$S_m(P_m) \gg m$$

即，That is,

$$\lim_{m \rightarrow \infty} \beta = \lim_{m \rightarrow \infty} \frac{S_m(P_m)}{m} \rightarrow \infty \text{ (无穷大量)}$$

(无穷大量 infinite amount)

也就是说，素数在非零自然数中的间隔合数个数的比值 β ，是越来越大的，是一个无穷大量。如图 11 和表 5 所示。

That is to say, the ratio β of the number of interval composite numbers of prime numbers in non-zero natural numbers is getting larger and larger, and is an infinite number. Figure 11 and Table 5 show this.

证明. 根据以上定理 1，第 m 个素数通项公式(4.1):

Proof of theorem 3. According to theorem 1 above, formula (4.1) for the m -th prime number:

$$P_m = m + S_m(P_m)$$

将式(4.1)，两边同时除 P_m ，则有

Divide both sides of equation (4.1) at the same time, then there is

$$1 = \frac{m}{P_m} + \frac{S_m(P_m)}{P_m} \quad (4.35)$$

$$1 = \frac{m}{P_m} + \frac{S_m(P_m)}{P_m} \cdot \frac{m}{m}$$

得 get

$$1 = \frac{m}{P_m} + \frac{S_m(P_m)}{m} \cdot \frac{m}{P_m} \quad (4.36)$$

将 Plug in

$$\delta = \frac{m}{p_m} \text{ and } \beta = \frac{S_m(P_m)}{m}$$

代入式(4.36)，可得式(4.37)

Substituting in the formula (4.36) gives the formula (4.37)

$$1 = \delta + \beta \cdot \delta \quad (4.37)$$

整理，可得式(4.32)

After sorting, the formula (4.32) can be obtained

$$\delta(1 + \beta) = 1$$

令 Let

$$f(\delta, \beta) = \delta(1 + \beta) = 1。$$

即 That is,

$$f(\delta, \beta) = 1$$

其中：f(δ, β)作者称之为有序非零自然数密度比率函数。

Among them: f(δ, β) the author calls it the ordered non-zero natural number density ratio function.

也就是说，所有素数总是分布在非零自然数密度比率函数等于 1 的直线上。所以，定理 3 成立。□

That is, all prime numbers are always distributed on a line where the non-zero natural number density ratio function is equal to 1. So theorem 3 is true. □

因此，作者又称之为第三素数分布出现定理。

Therefore, the author also calls it the occurrence theorem of the third prime number distribution.

根据以上定理 3，显然可以得到以下推论：

From theorem 3 above, it is obvious that the following inferences can be drawn:

推论 1. 素数在非零自然数序列中的分布总是越来越稀少的。

根据以上定理 3，所得的数学关系式，可得：

Corollary 1. The distribution of prime numbers in the sequence of non-zero natural numbers is always getting rarer.

According to the above theorem 3, the mathematical relation obtained can be obtained:

$$\delta = \frac{1}{\beta + 1} \quad (4.38)$$

依据以上关系式，可得到以下结论：

According to the above relationship, the following conclusions can be drawn:

1、当β=0 时，即素数间隔合数个数为零时，如素数序列中最初 1、2、3 个素数，其出现的密度比率为 1，即 100%。

1, When β=0, that is, when the number of prime interval composite number is zero, such as the first 1, 2, 3 prime numbers in the prime sequence, the density ratio of its occurrence is 1, that is, 100%.

2、当β→∞时，则δ→0，表明素数出现的密度比率会越来越小，几乎是合数，但永远不会等于零。表明素数有无穷多个。

2, when β→∞, δ→0, indicates that the density ratio of prime numbers will become smaller and smaller, almost composite, but never equal to zero. It shows that there are infinitely many prime numbers.

由于β = $\frac{S_m(P_m)}{m}$ ，为无穷大量，即当β→∞时，

Since β = $\frac{S_m(P_m)}{m}$ is infinitely large, that is, when β→∞

$$\delta = \lim_{\beta \rightarrow \infty} \frac{1}{\beta + 1} = 0 \quad (4.39)$$

3、对于给定任意大的 β 值，素数总会出现。但是 β 值越大， δ 密度比率就越小。即素数之间的距离越远，素数出现的密度比率就越小。

3. Given an arbitrarily large β value, a prime number will always appear. However, the larger the β value, the smaller the δ density ratio. That is, the greater the distance between primes, the smaller the density ratio of primes.

推论 2. 根据以上定理 3，若素数分布规律的非零自然数分布密度比率为 δ ，某素数的合数个数之和 $S_m(P_m)$ 与素数个数 m 的比值为 β ，则有

Corollary 2. According to the above theorem 3, if the distribution density ratio of non-zero natural numbers of the distribution law of prime numbers is δ , and the ratio of the sum of composite numbers of a prime number $S_m(P_m)$ to the number of prime numbers m is β , then there is

$$\beta = \frac{1 - \delta}{\delta} \quad (4.40)$$

由于 $\delta = \frac{m}{p_m}$ ，为无穷小量，即当 $\delta \rightarrow 0$ 时

Because $\delta = \frac{m}{p_m}$, is infinitely small, that is, when $\delta \rightarrow 0$

$$\beta = \lim_{\delta \rightarrow 0} \frac{1 - \delta}{\delta} = \lim_{\delta \rightarrow 0} \left(\frac{1}{\delta} - 1 \right) = \infty \quad (4.41)$$

所以，当素数的分布密度比率趋于 0 时，两个相邻素数之间的合数个数趋于无限多。

Therefore, when the distribution density ratio of prime numbers tends to 0, the number of composite numbers between two adjacent prime numbers tends to be infinite.

推论 3. 定理 3 的几何意义。即随着素数个数 m 增多，两个素数间隔合数个数的比值 β 曲线，从第 4 个素数开始，呈现逐渐向上递增趋势，直至无穷大。而每个素数在非零自然数中的分布比率 δ 曲线，则是从第 3 个素数 3 开始，呈现逐渐向下递减趋势，直至无穷小。依据定理 3 可知，两者始终满足数学关系式(4.32):

Corollary 3. Geometric meaning of Theorem 3.

That is, as the number of prime numbers m increases, the ratio β curve of the number of composite numbers separated by two prime numbers gradually increases from the fourth prime number until it reaches infinity. The δ curve of the distribution ratio of each prime number in non-zero natural numbers begins with the third prime number, 3, and gradually decreases until it is infinitesimal. According to theorem 3, they always satisfy the mathematical relation (4.32):

$$f(\delta, \beta) = \delta(1 + \beta) = 1$$

如图 11 所示。即：

As shown in Figure 11. To wit:

当 $\delta=0$ 时，则 $\beta=\infty$;

When $\delta = 0$, $\beta = \infty$;

当 $\beta=0$ 时，即素数间隔合数个数为零时，如素数序列中最初 1、2、3 个素数，其出现的密度比率为 1，即 100%。

When $\beta=0$, that is, when the number of prime interval composite numbers is zero, such as the

first 1, 2, and 3 prime numbers in the prime sequence, the density ratio of the occurrence is 1, that is, 100%.

素数在非零自然数中的分布密度比率是越来越小，是趋于零的。但是一个永远不等于零的无穷小量。

The distribution density ratio of prime numbers in non-zero natural numbers is getting smaller and smaller and tends to zero. But an infinitely small quantity that can never equal zero.

推论 4. 所有素数总是分布在非零自然数密度比率函数等于 1 的直线上，并且总是满足数学关系式是(4.32):

Corollary 4. All prime numbers are always distributed on a line where the non-zero natural number density ratio function is equal to 1. And the mathematical relation that always satisfies is (4.32):

$$f(\delta, \beta) = \delta(1 + \beta) = 1$$

其中， Among them,

$$\delta = \frac{m}{p_m}, \quad \beta = \frac{S_m(P_m)}{m}$$

若将 If

$$\delta\beta = \frac{m}{p_m} \times \frac{S_m(P_m)}{m}$$

即， That's

$$\delta\beta = \frac{S_m(P_m)}{p_m} \quad (4.42)$$

所以，有 So, there's

$$p_m = \frac{S_m(P_m)}{\delta\beta} \quad (4.43)$$

或者 Or

$$p_m = \frac{S_m(P_m)}{1 - \delta} \quad (4.44)$$

或者 Or

$$p_m = (1 + \frac{1}{\beta})S_m(P_m) \quad (4.45)$$

以上式(4.43)、式(4.44)和式(4.45)，可用于任意正整数的素数性质判别。因此，作者又称它们为任意正整数的素数性质判别数学公式。

The above equations (4.43), (4.44) and (4.45) can be used to determine the prime property of any positive integer. Therefore, they are also called the mathematical formulas for discriminating the prime properties of any positive integer.

其中： Among them:

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

式中： n 为无穷级数项数， K_m 为两个相邻素数间隔合数的个数， m 为素数序数； $n > m$ ； $n, m \in \mathbb{N}^*$ 。 $S_m(P_m)$ 作者称为素数有序数列的特征无穷级数函数。它是决定每个素数在非零自

然数序列分布出现规律一个重要函数变量。

In the formula, n is the number of infinite series terms, K_m is the number of composite numbers between two adjacent prime numbers, m is the ordinal number of prime numbers; $n > m$; $n, m \in \mathbb{N}^*$. A $S_m(P_m)$ characteristic infinite series function called a prime ordered sequence by the authors. It is an important function variable that determines the distribution of every prime number in the non-zero sequence of natural numbers.

也就是说，任意正整数若是素数，则由有序素数分布密度比率 δ 和有序素数间隔合数的个数级数和 $S_m(P_m)$ 两个变量决定的，并满足以上数学关系公式。

In other words, if any positive integer is a prime number, it is determined by the two variables of the ordered prime distribution density ratio δ and the number series and $S_m(P_m)$ of the ordered prime interval combination, and satisfies the above mathematical relationship formula.

定理 4 任意 2 个不连续素数的间隔合数个数等于两个素数的合数总个数之差。

Theorem 4. The number of separated fractions of any two discontinuous primes is equal to the difference of the total fractions of the two primes.

证明：假设 P_m 为素数序列中第 m 个素数； P_{m+j} 素数序列中第 $m+j$ 个素数。在 P_m 和 P_{m+j} 之间相隔有 j 个素数，素数 $P_m < P_{m+j}$ ； $m > j$ ； $m, j \in \mathbb{N}^*$

Proof. P_m is assumed to be the m th prime number in the prime sequence. The $m+j$ prime number in the P_{m+j} prime sequence. There are j prime numbers between P_m and P_{m+j} , prime $P_m < P_{m+j}$ ； $m > j$ ； $m, j \in \mathbb{N}^*$

依据定理 1，则有(4.1)：

According to theorem 1, there is (4.1)：

$$\begin{aligned} P_m &= m + S_m(P_m) \\ P_{m+j} &= (m+j) + S_{m+j}(P_{m+j}) \end{aligned} \quad (4.46)$$

将(4.46)减(4.1)，得

Subtract (4.46) from (4.1) and get

$$\begin{aligned} P_{m+j} - P_m &= [(m+j) - m] + [S_{m+j}(P_{m+j}) - S_m(P_m)] \\ P_{m+j} - P_m &= j + [S_{m+j}(P_{m+j}) - S_m(P_m)] \quad (m > j; m, j \in \mathbb{N}^*) \end{aligned} \quad (4.47)$$

当 $m \geq 4$ 时，则得

When $m \geq 4$, then

$$\begin{aligned} P_{m+j} - P_m &= (j-j) + [\sum(K_{m+j}+1) - \sum(K_m+1)] \\ P_{m+j} - P_m &= [\sum(K_m+1) + \sum(K_j+1) - \sum(K_m+1)] \end{aligned} \quad (4.48)$$

即，that is,

$$P_{m+j} - P_m = \sum_{j=1}^{\infty} (K_j + 1) \quad (4.49)$$

$$P_{m+j} - P_m = j + \sum_{j=1}^{\infty} K_j \quad (4.50)$$

式中: $m \geq 4; m > j; m, j \in N^*$. here: $m \geq 4; m > j; m, j \in N^*$.

P_m 素数序列中第 m 个素数; P_{m+j} 素数序列中第 $m+j$ 个素数。 $m > j; m, j \in N^*$

The m -th prime number in the P_m prime sequence; The $m+j$ prime number in the P_{m+j} prime sequence. $m > j; m, j \in N^*$

$K_j = 1$, 或 3, 或 5, 或 7, 或 9, ..., 或 $2n-1$, ..., $(n, j \in N^*)$, 均为奇数。所以, 任意 2 个素数间隔合数个数等于两个素数的合数个数总和之差(不包括素数个数在内)。定理 4 成立。证毕。□

$K_j = 1, \text{or } 3, \text{or } 5, \text{or } 7, \text{or } 9, \dots, \text{or } 2n-1, \dots, (n, j \in N^*)$, both are odd. Therefore, the number of composite numbers between any two primes is equal to the difference between the sum of the composite numbers of the two primes (excluding the number of primes). Theorem 4 is true. □

According to theorem 4 above, we can draw the following inferences:

定理 5 任意一个大素数可表示成一个小素数($P \neq 2$)与一个偶数之和。

Theorem 5. Any large prime number can be expressed as the sum of a small prime number ($P \neq 2$) and an even number.

证明: 根据以上定理 1, 将式(4.1), 移项可得:

Proof. According to theorem(4.1) above, transpose the equation (4.49) to obtain:

$$P_{m+j} = P_m + \sum_{j=1}^{\infty} (K_j + 1) \quad (4.51)$$

其中: $P \neq 2, P_{m+j} > P_m; m \geq 4; m, j \in N^*$. 由于 K_j 为奇数, 则有:

Where: $P \neq 2, P_{m+j} > P_m; m \geq 4; m, j \in N^*$. Since K_j is odd, there is:

$$\sum_{j=1}^{\infty} (K_j + 1)$$

必为偶数。即得一个大素数可表示成一个小素数($P \neq 2$)与一个偶数之和。

The value must be an even number. A large prime number can be expressed as the sum of a small prime number ($P \neq 2$) and an even number.

定理 6. 所有非零自然数构成的集合是所有合数集合和所有素数集合构成的并集。作者称之为素数与自然数的构成关系定理。即,

Theorem 6. The set of all non-zero natural numbers is the union of all composite numbers and all prime numbers. The author calls it the constitutive relation theorem of prime numbers and natural numbers. That is,

$$N^* = \{b_{r,m}\} \cup \{P_m\}$$

其中：第 m 个素数元素为 P_m ，第 m 个素数之前的所有合数元素为 $b_{r,m}$ ，下标 r 为第 m 个素数元素之前的合数序数，或者是合数个数。 N^* 为正整数， $r, m \in N^*$ 。

Where: the m -th prime element is P_m , all composite elements before the m -th prime number are $b_{r,m}$, and the subscript r is the composite ordinal number before the m -th prime element, or the composite number. $r, m \in N^*$.

除最初 1、2 和 3 三个连续素数外，在非零自然数序列中，不存在连续没有合数间隔的两个素数。所有非零自然数是由“合数个数为奇数+1 个素数”形成的结构单位构成的。也就是说单位素数 1 和单位偶素数 2 以及单位奇素数 3 之间是不存在合数间隔的。偶素数 2 是所有非零自然数中唯一的一个。

Except for the first three consecutive primes, 1, 2, and 3, there are no two consecutive primes without composite intervals in a sequence of non-zero natural numbers. All non-zero natural numbers are made up of structural units formed by the number of composite numbers being odd +1 prime numbers. That is to say, there is no composite interval between the unit prime 1 and the unit even prime 2 and the unit odd prime 3. The even prime number 2 is the only one of all non-zero natural numbers.

证明. 假设在素数序列中，存在连续 3 个没有合数间隔的素数，依据非零自然数秩序公理，即从第 1 个单位奇素数 1 开始，按“偶数—奇数—偶数—奇数……”依次排序，直至无穷无尽的原理规律。那么连续相连的三个素数当中，其中必然会有一个素数是偶数。因为素数集合 P 是奇数集合 B 的子集，即 $P \subseteq B$ ，除偶素数 2 外，与所有奇素数均为奇数相互矛盾。所以，除最初 1、2 和 3 三个素数外，再也没有连续 3 个素数没有合数间隔的素数存在。偶素数 2 是正整数集中独一无二的。

Proof. Suppose that in the sequence of prime numbers, there are three consecutive prime numbers with no composite interval, according to the axiom of the order of non-zero natural numbers, that is, starting from the first unit odd prime number 1, press "even - odd - even - odd..." Sort in sequence until the rules of principle are endless. Then of the three consecutive prime numbers, one of them must be even. Because the set of prime numbers P is a subset of the odd set B , that is, $AABB$, it contradicts that all odd prime numbers are odd except for the even prime number 2. So, apart from the first three primes, 1, 2, and 3, there are no longer three consecutive primes without composite intervals. The even prime number 2 is unique in the set of positive integers.

全体非零自然数就是这样的由合数与素数组成的结构链节，一节一节地无穷无尽地链接起来，形成一个有规律有秩序的相关联系和相互制约的统一整体。

All non-zero natural numbers are such a structural chain composed of composite numbers and prime numbers, which are endlessly linked one by one to form a unified whole of regular and orderly correlation and mutual constraints.

为了更好地论述非零自然数结构构成原理，这里首先定义素数 1、2、3 三个素数没有间隔合数为空集 \emptyset ，合数个数为 0。并且假设在所有非零自然数有序的集合中，任意一个有序素数为 P_m ，其之前的有序合数，或者说在素数 P_{m-1} 和 P_m 之间间隔的有序合数为 $b_{r,m}$ 。 $P_{r,m}$ 的下标 r 表示有序合数的序数或是有序合数的个数， m 表示有序素数的序数。因此，所有非零自然数构成有序的集合为 U ，即有

In order to better discuss the principle of the structure of non-zero natural numbers, we first define the three prime numbers 1, 2 and 3 as the empty set \emptyset without interval composite number, and the composite number is 0. And suppose that in the ordered set of all non-zero natural numbers, any ordered prime number is P_m , and the ordered composite number before it, or the ordered composite

number separated by the prime number P_{m-1} and P_m , is $b_{r,m}$. The subscript r of $b_{r,m}$ represents the ordinal number or number of ordinal composite numbers, and m represents the ordinal number of ordered prime numbers. Therefore, the ordered set of all non-zero natural numbers is U , that is,

$$U = N^*$$

也就是: That is:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots, n, \dots\} = N^*$$

若将以上有序非零自然数集合表示为由无穷多个任意一个素数和若干个合数构成的有序真子集。则以上有序非零自然数集合可表示为:

If the above set of ordered non-zero natural numbers is expressed as an ordered proper subset composed of infinitely many any prime numbers and some composite numbers. Then the above set of ordered non-zero natural numbers can be expressed as:

$$\{\emptyset, 1\} \cup \{\emptyset, 2\} \cup \{\emptyset, 3\} \cup \{4, 5\} \cup \{6, 7\} \cup \dots \cup \{b_{1,m}, b_{2,m}, b_{r,m}, \dots, P_m\} = N^*$$

更一般地, 我们可以将所有非零自然数构成有序的集合为:

More generally, we can form the ordered set of all non-zero natural numbers as:

$$\{b_{1,1}, P_1\} \cup \{b_{1,2}, P_2\} \cup \{b_{1,3}, P_3\} \cup \{b_{1,4}, P_4\} \cup \dots \cup \{b_{1,m}, b_{2,m}, b_{3,m}, \dots, P_{r,m}\} = N^*$$

依据以上非零自然数构成有序的集合, 可得:

According to the above non-zero natural numbers to form an ordered set, we can get:

由素数 $P_1 = 1$ 及其间隔的合数构成的子集为: $\{b_{1,1}, P_1\} = \{\emptyset, 1\}$;

由素数 $P_2 = 2$ 及其间隔的合数构成的子集为: $\{b_{1,2}, P_2\} = \{\emptyset, 2\}$;

由素数 $P_3 = 3$ 及其间隔的合数构成的子集为: $\{b_{1,3}, P_3\} = \{\emptyset, 3\}$;

由素数 $P_4 = 5$ 及其间隔的合数构成的子集为: $\{b_{1,4}, P_4\} = \{4, 5\}$;

由素数 $P_5 = 7$ 及其间隔的合数构成的子集为: $\{b_{1,5}, P_5\} = \{6, 7\}$;

由素数 $P_6 = 11$ 及其间隔的合数构成的子集为:

$$\{b_{1,6}, b_{2,6}, b_{3,6}, P_6\} = \{8, 9, 10, 11\};$$

由素数 $P_7 = 13$ 及其间隔的合数构成的子集为: $\{b_{1,7}, P_7\} = \{12, 13\}$;

.....

The subsets of prime $P_1 = 1$ and its composite numbers are: $\{b_{1,1}, P_1\} = \{\emptyset, 1\}$;

The subset consisting of prime $P_2 = 2$ and its composite numbers is: $\{b_{1,2}, P_2\} = \{\emptyset, 2\}$;

The subsets of prime $P_3 = 3$ and its composite numbers are: $\{b_{1,3}, P_3\} = \{\emptyset, 3\}$;

The subsets of prime $P_4 = 5$ and its composite numbers are: $\{b_{1,4}, P_4\} = \{4, 5\}$;

The subset of the prime number $P_5 = 7$ and its composite numbers at intervals is:

$$\{b_{1,5}, P_5\} = \{6, 7\};$$

The subset consisting of the prime number $P_6 = 11$ and the composite numbers of its intervals is: $\{b_{1,6}, b_{2,6}, b_{3,6}, P_6\} = \{8, 9, 10, 11\}$;

The subsets of prime $P_7 = 13$ and its composite numbers are: $\{b_{1,7}, P_7\} = \{12, 13\}$;

...

由素数 $P_m \in P$ (P 为素数集合) 及其间隔的合数构成的子集为:

$$\{b_{1,m}, b_{2,m}, b_{r,m}, \dots, P_m\}$$

The subset consisting of the prime number $P_m \in P$ (where P is the set of prime numbers) and the composite numbers at their intervals is: $\{b_{1,m}, b_{2,m}, b_{r,m}, \dots, P_m\}$

我们也可以将所有非零自然数构成有序的集合表示为由一个有序合数集合和一个有序素数

集合两个子集构成如下:

We can also express the ordered set of all non-zero natural numbers as a set of ordered composite numbers and a set of ordered primes consisting of two subsets as follows:

$$\{b_{r,1}, b_{r,2}, b_{r,3}, \dots, P_{r,m}\} \cup \{P_1, P_2, P_3, \dots, P_m\} = N^*$$

即, That is,

$$N^* = \{b_{r,m}\} \cup \{P_m\}$$

其中: 第 m 个素数元素为 P_m , 依次为 $P_1, P_2, P_3, \dots, P_m$; 第 m 个素数之前的所有合数元素为 $b_{r,m}$, 下标 r 为第 m 个素数元素之前的合数序数, 依次为 $b_{1,m}, b_{2,m}, b_{3,m}, \dots, P_{r,m}$. N^* 为正整数, $r, m \in N^*$. 以上定理 6 成立. \square

Where: the MTH prime element is P_m , followed by $P_1, P_2, P_3, \dots, P_m$; All composite elements before the m -th prime are $b_{r,m}$, and the subscript r is the composite ordinal number before the m -th prime element, in turn $b_{1,m}, b_{2,m}, b_{3,m}, \dots, P_{r,m}$. $r, m \in N^*$. The above theorem 6 holds. \square

本文研究结果表明, 素数是构成非零自然数的基本构件。犹如构成所有物质的化学元素一样, 素数是构成元素的原子核, 合数是构成环绕原子核运转的电子。所有非零自然数都是以素数为基本构架, 除最初 1, 2, 3 三个非零自然数外, 其他非零自然数均在两个素数之间插入奇数个数的合数, 而形成由“若干合数的奇数个数加一个素数”为非零自然数生成的结构单位链节。每个自然数结构单位, 若用数学式子表示, 则为:

The results of this paper show that prime numbers are the basic building blocks of non-zero natural numbers. Like the chemical elements that make up all matter, prime numbers are the atomic nuclei that make up the elements, and composite numbers are the electrons that orbit the nuclei. All non-zero natural numbers are based on prime numbers as the basic framework, except for the first three non-zero natural numbers, 1,2,3, other non-zero natural numbers are inserted between two prime numbers of odd numbers of composite numbers, and form a structural unit chain generated by "odd numbers of composite numbers (CN) plus a prime number" as non-zero natural numbers. Each structural unit of natural numbers, if expressed in mathematical formula, is:

$$(2n-1)CN + P_m$$

$$(2n-1)\text{个合数} + P_m$$

其中, 当 $m \geq 4$ 时, 每个有序素数之前的合数个数分别为:

$$K_m = \{1, 1, 3, 1, 3, \dots, \}$$

Where, when $m \geq 4$, the number of composite numbers before each ordered prime number is:

$$K_m = \{1, 1, 3, 1, 3, \dots, \}$$

定理 7. 有序正整数的素性判定

Theorem 7. Prime property determination of positive integers

首先, 所有素数的个位数必有 1、或 3、或 5、或 7、或 9。其中, 以 5 为个位数的素数只有 5 一个。除 5 外, 以 5 为个位的自然数均为合数。注意: 素数均为奇数, 但奇数不一定是素数。

First, all prime numbers must have 1, or 3, or 5, or 7, or 9. Among them, there is only 5 prime numbers with 5 digits as units. With the exception of 5, all natural numbers in the units place are composite numbers. Note: Prime numbers are odd, but odd numbers are not always prime

numbers.

数的进位制有许多种。但是，我们最常用、最熟悉的是十进制。在十进制计数方法中共有十个不同的数字符号 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 而且由低位向高位是“逢十进一”的。同一个数所在的位数相差一位，其值就有十倍之差。在十进制中的数 1 就是一，10 就是十，100 就是一百。又 10 是 1 的十倍，而 100 是 10 的十倍。在十进制中由 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 这十个数字符号，加上小数点等就可以构成一个数。

There are many kinds of decimal bases. However, the most commonly used and familiar is the decimal system. In the decimal counting method, there are ten different numeric symbols 0,1,2,3,4,5,6,7,8,9, and from low to high is "every ten". If the number of digits in the same number is different by one digit, its value will be different by a factor of ten. In decimal notation, 1 is one, 10 is ten, and 100 is one hundred. Ten is ten times one, and 100 is ten times ten. In the decimal system from 0,1,2,3,4,5,6,7,8,9, the ten numeric symbols, plus the decimal point, can form a number.

证明. 因为，在十进制中任意一个正整数 N 都能够写成

Proof. Because, in decimal, any positive integer N can be written as

$$N = a_n \times 10^n + a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + \dots + a_2 \times 10^2 + a_1 \times 10^1 + a_0$$

其中 $0 \leq a_i \leq 9$, 而 i 是 1 到 n 中的任一个非零自然数。

Where $0 \leq a_i \leq 9$, and i is any non-zero natural number from 1 to n .

因此，正整数 N ，可以看成由 $n+1$ 个元素组成的一个整数集合 $\{N\}$ 。假设 q 为十进制数码的任意一个整数，即 $q=1, 3, 5, 7$ 和 9 五个奇数，或者是 $2, 4, 6, 8$ 和 0 偶数(包括 0 数码)。

Therefore, the positive integer N can be seen as a set of integers $\{N\}$ consisting of $n+1$ elements. Suppose q is any integer of the decimal digits, that is, $q = 1, 3, 5, 7$, and 9 , five odd numbers, or $2, 4, 6, 8$, and 0 even numbers (including 0 digits).

$$q | \{a_n \times 10^n, a_{n-1} \times 10^{n-1}, a_{n-2} \times 10^{n-2}, \dots, a_2 \times 10^2, a_1 \times 10^1, a_0\}$$

当整数 N 的个位数码 a_0 为 $2, 4, 6, 8$ 和 0 时，均可被 $q=2$ 整除。所以，当整数个位数码为偶数数码时(包括 0 数码)，则整数必为合数。

When the units digit a_0 of the integer N is $2, 4, 6, 8$, and 0 , it is divisible by $q = 2$. Therefore, when the integer units digit is an even number (including the 0 digit), the integer must be a composite number.

当整数 N 的个位数码 a_0 为 5 时，除个位数码 5 外，其他从十位开始，直至 $n+1$ 位整数，可被 $q=5$ 整除。所以，当整数个位数码为数码 5 时，除 5 是素数外，其他均可被 5 整除。因此凡是个位含有 5 数码的整数，除 5 外，其他必为合数。

When the units digit a_0 of the integer N is 5 , except for the units digit 5 , the rest start from the tens digit and go up to the $n+1$ digit integer, which is divisible by $q = 5$. Therefore, when the integer units digit is the digit 5 , except that 5 is a prime number, other numbers can be integer 5 . Therefore, all integers containing 5 digits, except 5 , must be composite numbers.

当整数 N 的个位数码 a_0 为 $1, 3, 7$ 和 9 时，所有整数 N 必为奇数。由于素数集合是奇数集合的子集，所以凡素数的个位数码必是 $1, 3, 7$ 和 9 。但是整数 N 个位数码为奇数的整数却不一定是素数。因为素数集是奇数的子集，奇数包含素数。即素数包含于奇数之中。

When the units digit a_0 of integer N is $1, 3, 7$, and 9 , all integers N must be odd. Since the set

of prime numbers is a subset of the set of odd numbers, the units digits of any prime number must be 1,3,7, and 9. However, an integer N with odd digits is not necessarily a prime number. Since the set of prime numbers is a subset of odd numbers, odd numbers contain prime numbers. That is, prime numbers are included in odd numbers.

除偶素数 2 外，偶数均是合数。但是合数不一定是偶数，奇数也存在合数。以上定理 7 的逆定理是不成立的。

Even numbers are composite numbers except for the even prime number 2. But composite numbers don't have to be even, odd numbers also exist composite numbers. The converse of theorem 7 above is not true.

其次，依据定理 3 可知，所有素数总是分布在非零自然数密度比率函数等于 1 的直线上。即任意一个素数出现总是满足数学关系式是(4.1)：

Secondly, according to theorem 3, all prime numbers are always distributed on a line where the density ratio function of non-zero natural numbers is equal to 1. That is, any occurrence of a prime number always satisfies the mathematical relation (4.1):

$$P_m = m + S_m$$

若存在正整数 a_m 为素数，并依据定理 2 进行平行四边形分解，则必有：

If there is a positive integer a_m that is prime, and a parallelogram decomposition is performed according to theorem 2, then there must be:

$$a_m = m + S_m$$

移项，即得 Transfer, you get

$$S_m = a_m - m \quad (4.52)$$

将以下式 Will the following formula

$$S_m = S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

代入上式(4.52)，得 Plug in the above formula (4.52) to get

$$\sum_{n=m}^{n \rightarrow \infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + \cdots + K_m + \cdots = a_m - m \quad (4.53)$$

$$S_m = S_m(P_m) = a_m - m$$

$$S_m(P_m) = a_m - m$$

或 Or

$$S_m = a_m - m$$

以上式(4.52)和式(4.53)，作者均称为任意正整数的素数性质判别数学公式。

The above equations (4.52) and (4.53) are called the mathematical formulas for discriminating the prime properties of any positive integer.

也就是说，任意正整数若是素数，则由有序素数的序数 m 和有序素数间隔合数的个数级数和 $S_m(P_m)$ 两个变量决定的，并满足以上数学关系公式。

In other words, if any positive integer is a prime number, it is determined by the two variables of the ordinal m of the ordered prime number and the number series of the ordered prime interval composite number and $S_m(P_m)$, and satisfies the above mathematical relationship formula.

其中: Among them:

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

式中: n 为无穷级数项数, K_m 为两个相邻素数间隔合数的个数, m 为素数序数; $n > m$; $n, m \in \mathbb{N}^*$ 。 $S_m(P_m)$ 作者称为素数有序数列的特征无穷级数函数。它是决定每个素数在非零自然数序列分布出现规律一个重要函数变量。

In the formula, n is the number of infinite series terms, K_m is the number of composite numbers between two adjacent prime numbers, m is the ordinal number of prime numbers; $n > m$; $n, m \in \mathbb{N}^*$. A $S_m(P_m)$ characteristic infinite series function called a prime ordered sequence by the authors. It is an important function variable that determines the occurrence of the distribution of every prime number in a sequence of non-zero natural numbers.

也就是当以上 n 项无穷级数取 m 项时, 以上等式成立。所以 a_m 是素数 P_m

In other words, when the infinite series with n terms takes m terms, the above equation holds. So a_m is the prime number P_m

下面举例说明如何进行正整数的素性判定。

The following is an example of how to determine the primality of positive integers.

例如 1, 正整数 97, 依据定理 2 进行平行四边形分解, 即得

For example, 1, the positive integer 97, according to theorem 2 of the parallelogram decomposition, thus

$$97 = 26 + 71$$

将正整数 97 减序数 26, 得 71, 并存在

Subtract the positive integer 97 from the ordinal number 26 to get 71 and exist

$$\sum_{n=1}^{n \rightarrow 26, \infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + \cdots + K_{26} = 71$$

所以, 正整数 97 是素数。So, the positive integer 97 is prime.

例如 2, 正整数 9857, 依据定理 2 进行平行四边形分解, 即得

For example, 2, the positive integer 9857, performs a parallelogram decomposition according to theorem 2

$$9857 = 1217 + 8640$$

将正整数 9857 减序数 1217, 得 8640, 并存在

Subtract the positive integer 9857 from the ordinal number 1217 to get 8640 and exist

$$\sum_{n=1}^{n \rightarrow 1217, \infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + \cdots + K_{1217} = 8640$$

所以, 正整数 9857 是素数。So, the positive integer 9857 is prime.

例如 3, 正整数 9951, 依据定理 2 进行平行四边形分解, 即得

For example, 3, the positive integer 9951, according to theorem 2 of the parallelogram decomposition, thus

$$9951 = 1228 + 8723$$

将正整数 9951 减序数 1228, 得 8723, 不存在

Subtract the positive integer 9951 from the ordinal number 1228, to 8723, which does not exist

$$\sum_{n=1}^{n \rightarrow 1228, \infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + \cdots + K_{1228} = 8721 \neq 8723$$

并且 8723 也不等于下一个素数的合数个数级数 $S_{1229} = 8738$ 。即 $8723 \neq 8738$ 。

所以, 正整数 9951 不是素数。

And 8723 is not equal to the next prime number of the composite number series $S_{1229} = 8738$. $8723 \neq 8738$. So, the positive integer 9951 is not prime.

命题 8. 在有序素数序列中, 相邻两个素数所间隔的合数个数级数函数。求其数学通项表达计算公式。

Proposition 8. A composite number series function separated by two adjacent prime numbers in an ordered sequence of prime numbers. The expression formula of general terms of mathematics is obtained.

根据命题 8, 我们已知由所有非零自然数构成的有序非零自然数序列全集合为:

According to proposition 8, we know that the complete set of ordered non-zero natural number sequences consisting of all non-zero natural numbers is:

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots n \dots\} = N^*$$

其中, 构成有序素数序列集合为:

Among them, the sequence set of ordered prime numbers is:

$$P = \{1, 2, 3, 5, 7, \dots, P_m, \dots, \infty\}$$

构成的有序素数的合数, 即有序非零自然数序列集合的补集为:

The composition of the ordered prime numbers, that is, the complement of the sequence set of ordered non-zero natural numbers, is:

$$B = \{0, 0, 0, 1, 1, 3, \dots, K_m, \dots, \infty\}$$

即有序素数序列所间隔的合数个数级数和为:

That is, the sum of composite number series separated by an ordered sequence of prime numbers is:

$$S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

求以上合数个数级数函数的数学通项表达计算公式。

To obtain the mathematical general term expression calculation formula of the above composite number series function.

为了进一步加深理解, 我们这里首先从全体正整数分析考虑解决问题的方法。

In order to further deepen the understanding, we first consider the method of solving the problem from the analysis of all positive integers.

首先, 假设在所有非零自然数有序集合 U 中, 有正整数集合 A 元素, 按一定规律或者

规则排序, a_m 为有序正整数数列的第 m 通项, 则有序正整数数列 $\{a_m\}$ 中, 任意一通项 a_m 等于有序正整数数列的序数 m 加上有序正整数数列集合的补集 $B=\{b_{r,m}\}$ 元素的个数级数。即式(3.1):

First of all, suppose that in the ordered set of all non-zero natural numbers U , there is a positive integer set a element, sorted according to certain laws or rules, a_m is the MTH general term of the ordered positive integer series, then in the ordered positive integer series $\{a_m\}$, any one item a_m is equal to the ordinal number m of the ordered integer series plus the complement $B=\{b_{r,m}\}$ element of the ordered positive integer series. I.e. (3.1) :

$$a_m = m + \sum \text{Card}\{CuA\}$$

其中, 作者称 $\sum \text{Card}\{CuA\}$ 为任意有序正整数数列的特征级数函数。它是有序正整数集合 A 的补集元素个数的前 m 项级数之和。它或者是一个有限级数, 或者是一个无限级数。即, Among them, the authors claim that $\sum \text{Card}\{CuA\}$ is the characteristic series function of any ordered series of positive integers. It is the sum of the first m -term series of the number of complement elements of the ordered set of positive integers A . It's either a finite series or an infinite series. That is,

$$\sum \text{Card}\{CuA\} = \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\}$$

式中: $b_{r,m} < a_m$; $b_{r,m}$ 下标 r 、 m 为第 m 个补集正整数元素个数 r 。 $r, m \in \mathbb{N}^*$ 。

Where: $b_{r,m} < a_m$; $b_{r,m}$ subscript r and m are the number of positive integer elements r of the m complement. $r, m \in \mathbb{N}^*$.

其次, 提出命题 8。根据以上定理 1, 可得第 m 个有序正整数数列通项公式: 即任意有序正整数数列通项公式是有序正整数数列的序数 m 加上有序正整数数列集合的补集 $B=\{b_m\}$ 元素的个数级数。用数学式可以表示为:

Secondly, proposition 8 is put forward. According to the above theorem 1, the general term formula of the m -th ordered positive integer sequence can be obtained: that is, the general term formula of any ordered integer sequence is the number series of the ordinal number m of the ordered integer sequence plus the complement $B=\{b_m\}$ element of the set of ordered integers. The mathematical formula can be expressed as:

$$a_m = m + \sum_{m=1}^m \text{card}\{CuA\}$$

其中, a_m 是 $b_{r,m}$ 的后继数。即,

Where a_m is the successor to $b_{r,m}$. That is,

$$\sum_{m=1}^m \text{card}\{CuA\} = \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\}$$

式中: $b_{r,m} < a_m$; $r, m \in \mathbb{N}^*$ 。因为在有序非零自然数集 U 中, $U=\mathbb{N}^*$, 有序正整数集 A 是所有有序非零自然数集 U 的子集合, 则有补集为 CuA 。 r 为第 m 个补集正整数元素个数。

又因为数列 $A=\{a_m\}=\{a_1, a_2, a_3, \cdots, a_m\}$, 依次从小到大排列的一个数列。则,

Where: $b_{r,m} < a_m$; $r, m \in \mathbb{N}^*$. Because in the set of ordered non-zero natural numbers U , $U = \mathbb{N}^*$, and the set of ordered positive integers A is a subset of all ordered non-zero natural numbers U , then there is a complement $C_U A$. r is the number of positive integer elements of the MTH complement. And because of the sequence $A = \{a_m\} = \{a_1, a_2, a_3, \dots, a_m\}$, a sequence of numbers in order from smallest to largest. Then,

$$\{a_1, a_2, a_3, \dots, a_m\} \cup \{C_U A\} = \mathbb{N}^*$$

$$\{a_1, a_2, a_3, \dots, a_m, \dots\} \cup \{b_{r,1}, b_{r,2}, b_{r,3}, \dots, b_{r,m}, \dots\} = \mathbb{N}^*$$

所以, So,

$$\{b_{r,1}, a_1, b_{r,2}, a_2, b_{r,3}, a_3, \dots, b_{r,m}, a_m, \dots, \infty\} = \mathbb{N}^*$$

$$\{\{b_{r,1}, a_1\} \cup \{b_{r,2}, a_2\} \cup \{b_{r,3}, a_3\}, \dots, \cup \{b_{r,m}, a_m\}, \dots, \infty\} = \mathbb{N}^*$$

由此可见, 由任意一个有序正整数 a_m 和它前面的补集整数元素, 形成的一个集合 $\{b_{r,m}, a_m\}$ 为有序正整数集合的第 m 个子集。即,

Thus, the set $\{b_{r,m}, a_m\}$ formed by any ordered positive integer a_m and its preceding complement integer elements is the MTH subset of the set of ordered positive integers. That is,

$$\{b_{r,m}, a_m\} \subseteq \mathbb{N}^* \text{ (为正整数)}$$

(a positive integer)

其中, 正整数 a_m 是 $b_{r,m}$ 的后继数。

Where, the positive integer a_m is the successor of $b_{r,m}$.

根据自然数的秩序公理, 因前数 + 1 = 后继数, 可得:

According to the order axiom of natural numbers, since the previous number + 1 = the successor number, we get:

$$\sum_{n=1}^{\infty} \text{Card}\{b_{r,m}, a_m\} = n \quad (n > m; n, m \in \mathbb{N}^*) \quad (4.54)$$

将式(5.54)展开, 即

Expand the formula (5.54), i.e.

$$1 + \{b_{r,1}\} = a_1$$

$$2 + \{b_{r,1} + b_{r,2}\} = a_2$$

$$3 + \{b_{r,1} + b_{r,2} + b_{r,3}\} = a_3$$

.....

$$m + \{b_{r,1} + b_{r,2} + b_{r,3} + \dots + b_{r,m} + \dots\} = a_m$$

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$$\sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \dots + b_{r,m}, \dots\} = \sum_{m=1}^m \text{card}\{C_U A\}$$

则有：There is:

$$m + \sum_{n=1}^m \text{card}\{CuA\} = a_m$$

即，That is,

$$a_m = m + \sum_{n=1}^m \text{card}\{CuA\}$$

根据以上通项公式，可见任意有序整数数列中，任意一通项 a_m ，都是由一个有序整数数列的序数 m 和一个有序数列集合的补集 $B = \{b_m\}$ 元素的个数级数之和确定的。由于有序整数数列的序数 m 是按次序规律出现的。所以，作者称

According to the above general term formula, it can be seen that in any ordered integer sequence, any term a_m is determined by the sum of the number series of the ordinal number m of an ordered integer sequence and the complement $B = \{b_m\}$ elements of an ordered integer set. Since the ordinal m of the sequence of ordered integers appears according to the order law. So, the author calls it

$$\sum_{m=1}^m \text{card}\{CuA\} = \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\}$$

为任意有序正整数数列的特征级数函数。它是有序正整数集合 A 补集元素个数的前 m 项级数之和。它有可能是一个常量，也可能是一个所谓“无规律”可循的无穷大量。这要依据有序正整数数列出现的规律或者规则而确定。

Is the characteristic series function of any ordered series of positive integers. It is the sum of the first m -term series of the number of complementary elements of the ordered set of positive integers A . It may be a constant, or it may be an infinite number of so-called "irregular" to follow. This is determined according to the law or rule of the occurrence of ordered positive integer series.

为了书写方便，我们将任意有序正整数数列的特征级数函数，记作：

For the convenience of writing, we will write the characteristic series function of any ordered series of positive integers as:

$$S_m(b_{r,m}) = \sum_{m=1}^m \text{card}\{CuA\} = \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\}$$

有时候，也简记作 $S_m = S_m(b_{r,m})$ 。

Sometimes, it's called $S_m = S_m(b_{r,m})$.

为什么说 $\Sigma S_m(b_{r,m})$ 是有序正整数数列的特征级数函数，下面举例说明如下。

Why it is said that $\Sigma S_m(b_{r,m})$ is a characteristic series function of an ordered series of positive integers, the following examples are explained as follows.

比如，对于非零自然数列，因为每个自然数之间没有正整数间隔，正整数集合的补集元素个数是空集 \emptyset ，定义集合元素个数为 0，当有序正整数数列的序数 m 依次取 1, 2, 3, ..., m 时，

For example, for a non-zero natural sequence, because there is no positive integer interval between each natural number, the number of complement elements of the positive integer set is the empty set \emptyset , and the number of elements of the set is defined as 0, when the Ordinal Numbers m of the

ordered positive integer sequence are 1,2,3,... , m,

$$\begin{aligned} S_m(b_{r,m}) &= \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\} \\ &= 0+0+0+\cdots+0+\cdots \\ &= 0 \end{aligned}$$

非零自然数列的特征级数函数等于 0。因此，任何时候非零自然数列的每一项都是有规律均匀出现的。

The characteristic series function of a non-zero natural series is equal to 0. Therefore, every item in the non-zero natural sequence occurs regularly and evenly at any time.

又比如，偶数数列 2, 4, 6, 8, 10...和奇数数列 1, 3, 5, 7, 9, 11, ...。它们都是按照任意两个偶数间隔一个奇数出现，或任意两个奇数间隔一个偶数出现。即，它们间隔的奇数或偶数个数前 m 项级数个数之和为：

For example, even numbers 2,4,6,8,10... And the odd numbers 1,3,5,7,9,11,... . They all occur as an odd number between any two even numbers, or as an even number between any two odd numbers. That is, the sum of the number of m-term series before the odd or even number of their intervals is:

对于偶数数列，则有：

For even numbers, there is:

$$\begin{aligned} S_m(b_{r,m}) &= \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\} \\ &= 1+1+1+\cdots \\ &= m \end{aligned}$$

对于奇数数列，则有：

For odd numbers, there is:

$$\begin{aligned} S_m(b_{r,m}) &= \sum_{m=1}^m \{b_{r,m} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\} \\ &= 0+1+1+1+\cdots \\ &= m-1 \end{aligned}$$

再比如，模 4 余 3 的数列为 3, 7, 11, 15, 19, 23, 27, ...。第 1 个正整数 3 外，它们其他任意两个正整数都是间隔 3 个正整数出现。即它们间隔正整数个数前 m 项级数个数之和为：

For example, the numbers of module 4 remaining 3 are listed as 3, 7, 11, 15, 19, 23, 27,... . Except for the first positive integer 3, any other two positive integers of them are three positive integers apart. That is, the sum of the number of M-term series before the number of positive integers between them is

$$S_m(b_{r,m})=2+3+3+3+\cdots+\cdots=3(m-1)+2=3m-1$$

因此，模 4 余 3 的数列为 3, 7, 11, 15, 19, 23, 27, ...。每一项都是有规律均匀出现的。Therefore, the numbers of modulo 4 beyond 3 are listed as 3, 7, 11, 15, 19, 23, 27,... . Each of these items appears regularly and evenly.

但是，对于两个相邻素数对间隔合数的个数，或者是将有序素数集合的补集(即有序合

数集合)的元素个数有序排列，所形成的有序素数数列的其特征级数函数，即为：

However, for the number of two adjacent prime numbers that are separated by composite numbers, or the number of elements of the complement set of ordered prime numbers (that is, the ordered composite number set), the characteristic series function of the sequence of ordered prime numbers formed is:

$$P_m = m + \sum_{n=1}^m \text{card}\{\text{CuP}\}$$

式中， P_m 为第 m 个素数， $\{\text{CuP}\}$ 为有序素数集合对于全体有序正整数集合的补集，也就有序合数集合 $\text{card}\{\text{CuP}\}$ 是补集元素个数(card 是英文基数的缩写)。 m 为所有素数从 1 开始从小到大排列序数， $m \in N^*$ 。其中：

In the formula, P_m is the m -th prime number, $\{\text{CuP}\}$ is the complement of the set of ordered primes for the set of all ordered positive integers, that is, the set of ordered composite numbers $\text{card}\{\text{CuP}\}$ is the number of complement elements (card is the abbreviation of English radix). m is the ordinal number for all primes from smallest to largest starting from 1, $m \in N^*$. Among them:

$$\begin{aligned} S_m(b_{r,m}) &= \sum_{m=1}^m \text{card}\{\text{CuP}\} = \sum_{m=1}^m \{b_{r,1} + b_{r,2} + b_{r,3} + \cdots + b_{r,m}, \cdots\} \\ &= 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty \end{aligned}$$

上式， P_m 是 $b_{r,m}$ 的后继数，即 $b_{r,m} < P_m$ 。下标 r 表示第 m 个素数 P_m 间隔合数个数。本文作者所称的命题 8，是指有序素数数列的特征级数函数。对于有序素数数列的特征级数函数，如何求解其数学通项表达式，即命题还需要数学界共同研究与探讨。

In the above formula, P_m is the successor to $b_{r,m}$, that is, $b_{r,m} < P_m$. The subscript r represents the number of composite numbers between the m -th prime number P_m . The proposition 8 referred to by the author of this paper refers to the characteristic series function of ordered prime numbers. For the characteristic series function of ordered prime number series, how to solve its general term expression formula, that is, the proposition, still needs to be studied and discussed by the mathematical community.

定理 9. 素数的定义分段方程

Theorem 9. Definition of prime number piecewise equation

所有有理素数在实平面内或者复平面内都是分布在实轴 x 等于 1，或者复数 z 的实部(Rez)等于 1 的直线上。

All rational primes in the real or complex plane are distributed on a line where the real axis x is equal to 1, or the real part (Rez) of the complex number z is equal to 1.

证明。首先，在实数平面内，给出素数的定义分段方程。

Proof. First, in the plane of real numbers, the definition piecewise equation of prime numbers is given.

在实数平面内，假设有一个动点 $M(x, y \mid x, y \in \mathbb{R})$ ，该动点 M 的 x 轴坐标值和 y 轴坐标值分别与 x 和 y 两条坐标轴的距离的乘积是常数 c ，则该点的轨迹曲线方程可以表示为：

In the real plane, suppose there is a moving point $M(x, y \mid x, y \in \mathbb{R})$, and the product of the X-axis coordinate value and Y-axis coordinate value of the moving point M and the distance between the

x and y coordinate axes is a constant c, then the trajectory curve equation of the point can be expressed as:

$$|x| \cdot |y| = c \ (c > 0, c \in \mathbf{R}). \quad (4.55)$$

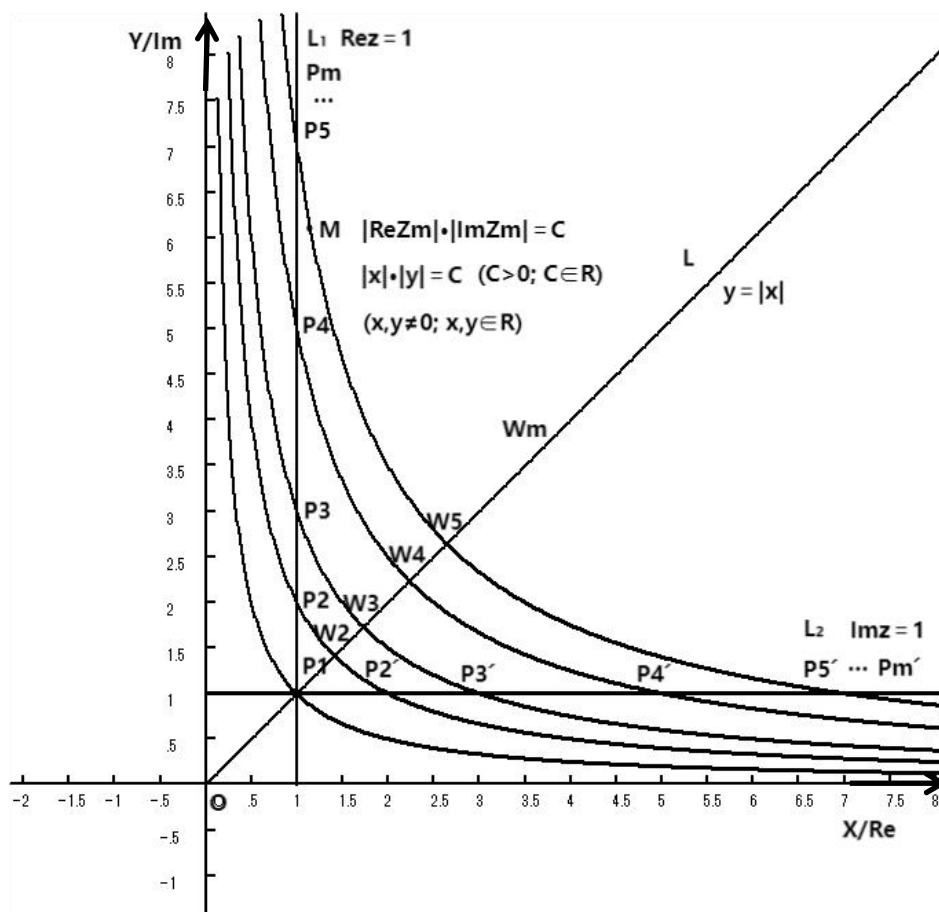


图 12. 素数的定义方程和分布示意图

Figure 12. Definition equation and distribution diagram of prime numbers

式中: x, y 为任意实数, 即 $x, y \in \mathbf{R}$.

Where: x, y is any real number, i.e. $x, y \in \mathbf{R}$.

根据命题, 现设 $M(x_m, y_m)$ 是实数方程曲线上的任意一动点, 则有动点 M 与 x 轴的距离为 $|y_m|$, 与 y 轴的距离为 $|x_m|$. 所以, $|x_m| \cdot |y_m| = c$, 即 $M(|x_m|, |y_m|)$ 是实数方程的解。

According to the proposition, let $M(x_m, y_m)$ be any point on the curve of the real equation, then the distance between the moving point M and the x axis is $|y_m|$, and the distance between the y axis is $|x_m|$. So, $|x_m| \cdot |y_m| = c$, that is, $M(|x_m|, |y_m|)$ is the solution of the real equation.

又设 $M(|x_m|, |y_m|)$ 是方程(4.55)的解, 可得:

Let $M(|x_m|, |y_m|)$ be the solution of equation (4.55), and we can get:

$$x_m y_m = \pm c \quad (4.56)$$

也就是 $|x_m| \cdot |y_m| = c$ 。而 $|x_m|, |y_m|$ 正是动点 M 到纵、横两条坐标轴的距离, 因此动点 M

到达这两条直线的距离的乘积是常数 c ，即动点 M 是方程(4.55)曲线上的任意动点。因此，以上动点 M 的轨迹，我们称之为等轴双曲线。它是关于 $|y| = |x|$ 对称的曲线。如图 12 所示。

Also known as $|x_m| \cdot |y_m| = c$. And $|x_m|$, $|y_m|$ is the distance between the moving point M and the vertical and horizontal coordinate axes, so the product of the distance between the moving point M and the two lines is a constant c , that is, the moving point M is any moving point on the curve of equation (4.55). Therefore, the trajectory of the above moving point M is called an equiaxial hyperbola. It is a symmetric curve about $|y| = |x|$. As shown in Figure 12. \square

根据素数在正整数范围内的定义，以上方程(4.55)或者(4.56)，当 $|x_m| = 1$ 或 $|y_m| = 1$ ($m \in N^*$)时，并且 c 分别取任意一个素数 p_m ($m \in N^*$)时，则得所有素数分布的分段曲线方程是：

According to the definition of prime numbers in the range of positive integers, the above equation (4.55) or (4.56), when $|x_m| = 1$ or $|y_m| = 1$ ($m \in N^*$), and c takes any prime number p_m ($m \in N^*$), then the piecewise curve equation of the distribution of all prime numbers is:

$$\left\{ \begin{array}{l} x_1 y_1 = 1 \quad (x_1, y_1 \in \mathbf{R}), \\ x_2 y_2 = 2 \quad (x_2, y_2 \in \mathbf{R}), \\ x_3 y_3 = 3 \quad (x_3, y_3 \in \mathbf{R}), \\ x_4 y_4 = 5 \quad (x_4, y_4 \in \mathbf{R}), \\ x_5 y_5 = 7 \quad (x_5, y_5 \in \mathbf{R}), \\ \vdots \\ x_m y_m = p_m \quad (x_m, y_m \in \mathbf{R}; m \in N^*). \end{array} \right. \quad (4.57)$$

依据以上素数系列方程(4.57)，所有素数分段方程通式也可以表示为：

According to the above prime number series equation (4.57), the general formula of all prime number piecewise equations can also be expressed as:

$$|x_m| \cdot |y_m| = p_m \quad (x_m, y_m \in \mathbf{R}; m \in N^*). \quad (4.58)$$

其中， x_m, y_m 为任意实数。

Where x_m, y_m is any real number.

当 $|x_m| = 1, y_m > 0$ 时，则 $y_m = p_m$ ($m \in N^*$)，其中： p_m 分别依次取 1, 2, 3, 5, 7, ..., p_m 所有素数。因此，在方程定义域内，根据正整数素数的定义，则所有素数均分布在等距离双曲线 $|x_m| \cdot |y_m| = c$ 与 $|x_m| = 1$ 的直线 L_1 相交的交点上。

When $|x_m| = 1, y_m > 0$, then $y_m = p_m$ ($m \in N^*$), where: p_m takes 1, 2, 3, 5, 7, ..., p_m all prime numbers. Therefore, in the domain of the equation, according to the definition of positive integer primes, all primes are distributed at the intersection of the equidistant hyperbola $|x_m| \cdot |y_m| = c$ and the line L_1 of $|x_m| = 1$.

当 $|y_m| = 1, x_m > 0$ 时，则 $x_m = p_m$ ($m \in N^*$)，其中： p_m 分别依次取 1, 2, 3, 5, 7, ..., p_m 所有素数。因此，在方程定义域内，根据正整数素数的定义，则所有素数均分布在等距离双曲线 $|x_m| \cdot |y_m| = c$ 与 $|y_m| = 1$ 的直线 L_2 相交的交点上。如图 12 所示。

When $|y_m| = 1, x_m > 0$, then $x_m = p_m$ ($m \in N^*$), where: p_m takes 1, 2, 3, 5, 7, ..., p_m all prime numbers. Therefore, in the domain of the equation, according to the definition of positive integer primes, all primes are distributed at the intersection of the equidistant hyperbola $|x_m| \cdot |y_m| = c$ and the line L_2 of $|y_m| = 1$. As shown in Figure 12.

由于素数的定义方程在实数范围之内是关于 $y=\pm x$ 直线对称的等轴双曲线。因此，这里只考虑实数平面坐标系中的第 I 象限情形，排除负素数。即 $x, y>0; x, y \in \mathbf{R}$ 。

Since the defining equation for prime numbers in the range of real numbers is an equiaxial hyperbola with respect to the linear symmetry of $y=\pm x$. Therefore, only the I quadrant in the real plane coordinate system is considered here, and negative primes are excluded. So $x, y>0; x, y \in \mathbf{R}$.

其次，在复数平面内，同理我们可以给出素数的定义分段方程。设在复平面内，有一个动点 $M: z_m=x_m+iy_m$ ，该动点 M 的实轴坐标值 x_m 和虚轴坐标值 y_m 分别与 x 和 y 两条坐标轴的距离的乘积是常数 c ，则该点的轨迹曲线方程可以表示为：

Secondly, in the complex plane, we can also give the definition of the prime number piecewise equation. Located in the complex plane, there is a moving point $M: z_m=x_m+iy_m$, and the product of the real axis coordinate value x_m and imaginary axis coordinate value y_m of the moving point M with the distance between x and y axes is a constant c , then the trajectory curve equation of the point can be expressed as:

$$|x_m| \cdot |y_m| = c \ (c>0, c \in \mathbf{R}). \quad (4.59)$$

式中： x_m, y_m 为任意实数，即 $x_m, y_m \in \mathbf{R}$ 。

Where: x_m, y_m is any real number, that is, $x_m, y_m \in \mathbf{R}$.

上式也可以等价地表示为：

The above equation can also be equivalently expressed as:

$$|\text{Re}z_m| \cdot |\text{Im}z_m| = c \ (c > 0, c \in \mathbf{R})$$

式中： $z_m=x_m+iy_m$ ； x_m, y_m 为任意实数，即 $x_m, y_m \in \mathbf{R}$ 。

Where: $z_m=x_m+iy_m$ ； x_m, y_m , is any real number, i.e. $x_m, y_m \in \mathbf{R}$..

同理，可推得，所有素数在复平面里的分段方程通式为：

By the same token, it can be deduced that the general formula of the piecewise equation for all prime numbers in the complex plane is:

$$|\text{Re}z_m| \cdot |\text{Im}z_m| = P_m \ (C \in \mathbf{R}; m \in \mathbf{N}^*). \quad (4.60)$$

根据素数在正整数范围之内的定义，以上方程(4.55)或者(4.56)，当 c 分别取任意一个素数 $p_m \ (m \in \mathbf{N}^*)$ 时，则得所有素数分布的分段曲线方程是：

According to the definition of prime numbers in the range of positive integers, the above equation (4.55) or (4.56), when c takes any prime number $p_m \ (m \in \mathbf{N}^*)$, then the piecewise curve equation of all prime numbers distribution is:

$$\left\{ \begin{array}{l} |\text{Re}z_1| \cdot |\text{Im}z_1| = 1, \\ |\text{Re}z_2| \cdot |\text{Im}z_2| = 2, \\ |\text{Re}z_3| \cdot |\text{Im}z_3| = 3, \\ |\text{Re}z_4| \cdot |\text{Im}z_4| = 5, \\ |\text{Re}z_5| \cdot |\text{Im}z_5| = 7, \\ \vdots \\ |\text{Re}z_m| \cdot |\text{Im}z_m| = p_m \ (m \in \mathbf{N}^*). \end{array} \right. \quad (4.61)$$

其中， $z_m=x_m+iy_m$ ； x_m, y_m 为任意实数。即 $x_m, y_m \in \mathbf{R}$ 。

Where $z_m=x_m+iy_m$ ； x_m, y_m is any real number. That's $x_m, y_m \in \mathbf{R}$.

当 $|\text{Re}z_m| = 1, |\text{Im}z_m| > 0$ 时，则 $|\text{Im}z_m| = p_m \ (m \in \mathbf{N}^*)$ ，其中： p_m 分别依次取 1, 2, 3,

5, 7, ..., p_m 所有素数。因此，在方程定义域内，根据正整数素数的定义，则所有素数均分布在等距离双曲线 $|\text{Rez}_m| \cdot |\text{Imz}_m| = c$ 与 $|\text{Rez}_m| = 1$ 的直线 L_1 相交的交点上。

When $|\text{Rez}_m| = 1, |\text{Imz}_m| > 0$, then $|\text{Imz}_m| = p_m (m \in N^*)$, where: p_m takes 1,2,3,5,7,... p_m all prime numbers. Therefore, in the domain of the equation, according to the definition of positive integer primes, all primes are distributed at the intersection of the equidistant hyperbola $|\text{Rez}_m| \cdot |\text{Imz}_m| = c$ and the line L_1 of $|\text{Rez}_m| = 1$.

当 $|\text{Imz}_m| = 1, |\text{Rez}_m| > 0$ 时，则 $|\text{Rez}_m| = p_m (m \in N^*)$ ，其中： p_m 分别依次取 1, 2, 3, 5, 7, ..., p_m 所有素数。因此，在方程定义域内，根据正整数素数的定义，则所有素数均分布在等距离双曲线 $|\text{Rez}_m| \cdot |\text{Imz}_m| = c$ 与 $|\text{Imz}_m| = 1$ 的直线 L_2 相交的交点上。如图 12 所示。因此，定理 9 成立。□

When $|\text{Imz}_m| = 1, |\text{Rez}_m| > 0$, $|\text{Rez}_m| = p_m (m \in N^*)$, where: p_m takes 1,2,3,5,7,... p_m All prime numbers. Therefore, in the domain of the equation, according to the definition of positive integer primes, all primes are distributed at the intersection of the equidistant hyperbola $|\text{Rez}_m| \cdot |\text{Imz}_m| = c$ and the line L_2 of $|\text{Imz}_m| = 1$. As shown in Figure 12. So theorem 9 is true. □

定理 10. 素数的正切定理。

Theorem 10. Tangent theorem for prime numbers.

在实平面内或者复平面内分布在实轴 x 等于 1，或者复数 z 的实部(Rez)等于 1 的直线上的素数。其相应线段对应角的正切函数等于该素数。

A prime number distributed in the real or complex plane on a line where the real axis x is equal to 1, or the real part (Rez) of the complex number z is equal to 1. The tangent function of the corresponding Angle of the corresponding line segment is equal to the prime number.

证明. 根据定理 10 命题作图。如图 10 所示。设在直角三角形 $\triangle ABC$ 中，直角边长 CA 为素数 P_m ，可以分解为排列序数为 $m (m \in N^*$ 为正整数) 的一个平行四边形；另一直角边长 BC 为 1， AC 所对应的锐角 $\angle ABC = \theta$ ，根据素数的平行四边形分解定理和正切函数的定义，则有：

Proof. Plot propositions according to Theorem 10. as shown in Figure 10. Set in the right triangle $\triangle ABC$, the right side length CA is the prime number P_m , which can be decomposed into a parallelogram with the ordinal number $m (m \in N^*$ is a positive integer); Another Angle side length BC is 1, the acute Angle $\angle ABC = \theta$ corresponding to AC , according to the parallelogram decomposition theorem of prime numbers and the definition of tangent function, there is:

$$\tan\theta = \frac{AC}{BC} = \frac{P_m}{1}$$

即 That's

$$P_m = \tan\theta \quad (4.62)$$

因此，素数所对应角的大小等于：

Therefore, the magnitude of the Angle corresponding to the prime number is equal to:

$$\theta = \arctan P_m \quad (4.63)$$

式中： $0 < \theta < \pi/2$ ； P_m 为有序素数。 $m \in N^*$ 为正整数。

Where: $0 < \theta < \pi/2$; P_m is an ordered prime number. $m \in N^*$ is a positive integer.

所以，素数在平行四边形分解中，等于素数所对应的角的正切三角函数值。定理 10 成立。□

Therefore, the prime number in the parallelogram decomposition is equal to the tangent trigonometric function of the Angle corresponding to the prime number. Theorem 10 is true. □

因此，有序素数 P_m 对应的复数，以指数形式可表示为：

Therefore, the complex number corresponding to the ordered prime P_m can be expressed in exponential form as:

$$z = re^{i\theta}$$

其中, $r = \sqrt{1 + p_m^2}$ 为复数 z 的模; 幅角 $\theta = \arctan P_m$, 这里

Where, $r = \sqrt{1 + p_m^2}$ is the module of the complex number z ; Angle $\theta = \arctan P_m$, here

$$\frac{\pi}{2} > \theta \geq \frac{\pi}{4}.$$

根据以上素数正切定理, 显然, 我们可以得到以下推论。

From the above tangent theorem of prime numbers, it is obvious that we can draw the following inferences.

推论 1. 如图 10 所示。在素数的平行四边形分解中, 有序素数序列的序数线段 EQ, 其对应角的正切函数等于 $2m$ 。

Corollary 1. See Figure 10. In the parallelogram decomposition of prime numbers, the ordinal line segment EQ of an ordered prime sequence has an Angle tangent function equal to $2m$.

证明. 根据素数的平行四边形的分解定理 2, 并且 $EQ=m$, $BQ=1/2$ 。因此, 在直角三角形 $\triangle BQE$ 中, 设 $\angle EBQ=\varphi$, 依据正切函数定义, 则有:

Proof. According to the decomposition theorem of the parallelogram of prime numbers 2, and $EQ = m$, $BQ = 1/2$. Therefore, in the right triangle $\triangle BQE$, let $\angle EBQ = \varphi$, according to the tangent function definition, there is:

$$\tan \varphi = \frac{EQ}{BQ} = \frac{m}{1/2} = 2m \quad (4.64)$$

即, That is,

$$\varphi = \arctan 2m \quad (4.65)$$

所以, 在素数的平行四边形分解中, 有序素数序列的序数 m , 其对应角的正切函数等于 $2m$ 。对应角大小等于 $\varphi = \arctan 2m$ 。推论 1 成立。□

Therefore, in the parallelogram decomposition of prime numbers, the ordinal m of the ordered prime sequence has an Angle tangent function equal to $2m$.

The corresponding Angle is equal to $\varphi = \arctan 2m$. Corollary 1 is true. □

因此, 有序素数数列的序数 m 对应的复数, 以指数形式可表示为:

Therefore, the complex number corresponding to the ordinal m of the ordered prime sequence can be expressed in exponential form as:

$$z_1 = r_1 e^{i\varphi}$$

其中, $r_1 = \sqrt{\frac{1}{4} + m^2}$ 为复数 z_1 的模; 幅角 $\varphi = \arctan 2m$, 这里

Where $r_1 = \sqrt{\frac{1}{4} + m^2}$ is the module of the complex number z_1 ; Angle $\varphi = \arctan 2m$,

$$\frac{\pi}{2} > \varphi \geq \frac{\pi}{4}.$$

Here.

推论 2. 如图 10 所示。在素数的平行四边形分解中, 有序素数序列的特征级数线段 FQ, 其对应角的正切函数等于 $2S_m$ 。

Corollary 2. Figure 10 shows. In the parallelogram decomposition of prime numbers, the tangent function of the corresponding Angle of the characteristic series segment FQ of the ordered prime

number sequence is equal to $2S_m$.

证明. 根据素数的平行四边形的分解定理 2, 并且 $FQ=S_m$, $BQ=1/2$ 。因此, 在直角三角形 $\triangle BQF$ 中, 设 $\angle FBQ=\omega$, 依据正切函数定义, 则有:

Proof. According to the resolution theorem of the parallelogram of prime numbers 2, and $FQ=S_m$, $BQ=1/2$. Therefore, in the right triangle $\triangle BQF$, let $\angle FBQ=\omega$, according to the tangent function definition, there is:

$$\tan\omega = \frac{FQ}{BQ} = \frac{S_m}{1/2} = 2S_m \quad (4.66)$$

即, Namely,

$$\omega = \arctan 2S_m \quad (4.67)$$

所以, 在素数的平行四边形分解中, 有序素数序列的特征级数 S_m , 其对应角的正切函数等于 $2S_m$ 。对应角大小等于 $\omega = \arctan 2S_m$ 。推论 2 成立。□

Therefore, in the parallelogram decomposition of prime numbers, the corresponding Angle tangent function of the characteristic series S_m of the ordered prime number sequence is equal to $2S_m$.

The corresponding Angle is equal to $\omega = \arctan 2S_m$. Corollary 2 is valid.

因此, 有序素数特征级数 S_m 对应的复数, 以指数形式可表示为:

Therefore, the complex number corresponding to the ordered prime series S_m can be expressed in exponential form as:

$$z_2 = r_2 e^{i\omega}$$

其中, $r_2 = \sqrt{\frac{1}{4} + S_m^2}$ 为复数 z_2 的模; 幅角 $\omega = \arctan 2S_m$, 这里

Where $r_2 = \sqrt{\frac{1}{4} + S_m^2}$ is a module of the complex number z_2 ; Angle $\omega = \arctan 2S_m$, this is

$$\frac{\pi}{2} > \omega \geq \frac{\pi}{4}.$$

显然, 依据定理 10, 我们可以得到。

Obviously, according to theorem 10, we can get.

推论 3. 任意有序素数的单位圆正切三角函数表达式。

Corollary 3. Trigonometric function expression of unit circle tangent for any ordered prime number. Formula (4.1)

$$P_m = m + S_m$$

for ordered prime numbers

可用单位圆正切三角函数表达式如下。

The unit circle tangent trigonometric function can be expressed as follows.

$$\tan\theta = \frac{1}{2}\tan\varphi + \frac{1}{2}\tan\omega$$

$$(\frac{\pi}{2} > \theta, \varphi, \omega \geq \frac{\pi}{4})$$

式中, In the formula,

$$P_m = \tan\theta$$

$$m = \frac{1}{2}\tan\varphi$$

$$S_m = \frac{1}{2} \tan \omega$$

其中, Among them,

$$S_m = S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \cdots + K_m + \cdots + \infty$$

式中: 当 $m \geq 4$ 时, $K_m = 1$, 或 3, 或 5, 或 7, 或 9, \cdots , 或 $2n - 1$, \cdots 。 ($n, m \in \mathbb{N}^*$), 均为奇数。

In the formula: when $m \geq 4$, $K_m = 1, \text{ or } 3, \text{ or } 5, \text{ or } 7, \text{ or } 9, \cdots, \text{ or } 2n - 1, \cdots$ ($n, m \in \mathbb{N}^*$), are odd.

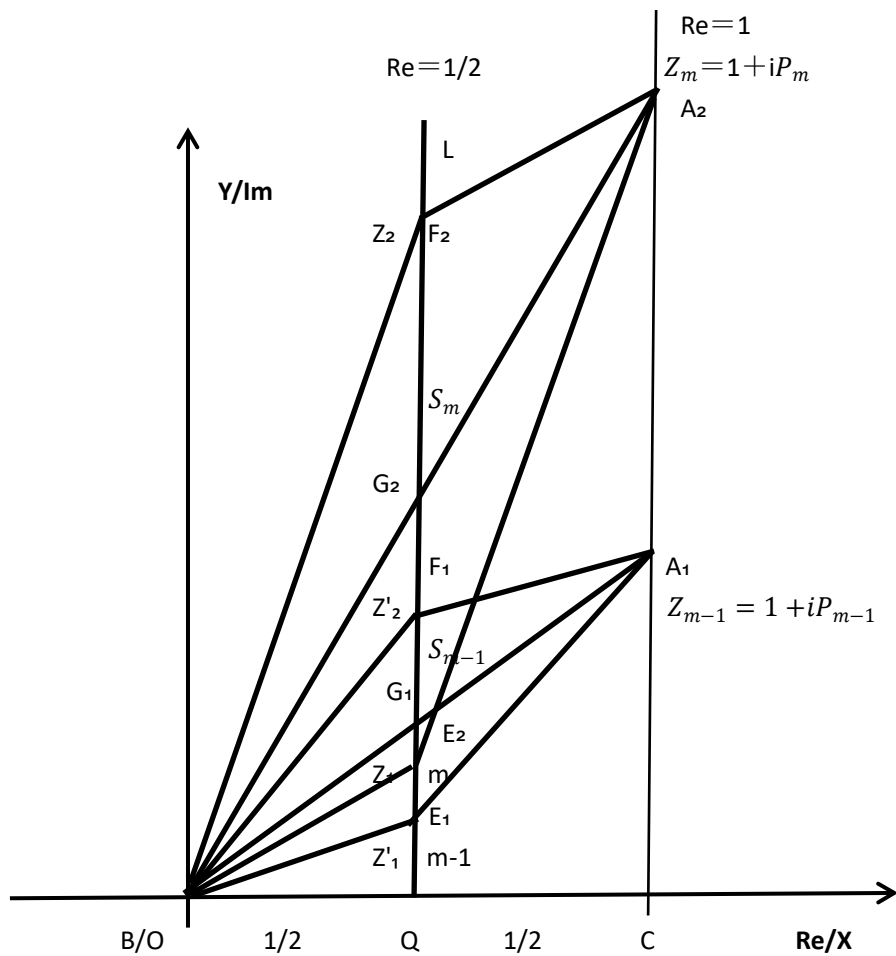


图 13. 两个相邻素数的几何意义示意图

Figure 13. Schematic diagram of the geometric meaning of two adjacent prime numbers

定理 11. 任意两个相邻素数对的几何特征及其意义。如图 13 所示。任意两个相邻素数对, 它们之间的序数之差总是等于 1; 它们的特征函数级数之差, 总是等于 1, 或者是 3, 或者

是 5, ..., 或者是 $2n-1$, 均为奇数。用数学式表示为:

Theorem 11. Geometric characteristics of any two pairs of adjacent prime numbers and their meanings. As shown in Figure 13. The ordinal difference between any two adjacent pairs of prime numbers is always equal to 1; The difference between their series of eigenfunctions is always equal to 1, or 3, or 5. Or $2n$ minus 1, which is odd. It is mathematically expressed as:

$$P_m - P_{m-1} = 1 + K_m$$

其中: 当 $m \geq 4$ 时, $K_m = 1$, 或 3, 或 5, 或 7, 或 9, ..., 或 $2n-1$, ...。 ($n, m \in N^*$), 均为奇数。

Where: when $m \geq 4$, $K_m = 1, \text{ or } 3, \text{ or } 5, \text{ or } 7, \text{ or } 9, \dots, 2n-1, \dots$. ($n, m \in N^*$), are odd.

证明. 首先证明, 在有序素数序列中, 任意相邻的两个素数的序数相差等于 1。根据定理 1 素数通项公式, 并进行素数的平行四边形分解, 如图 13 所示。

First, it is shown that in an ordered sequence of prime numbers, the ordinal difference between any two adjacent prime numbers is equal to 1. According to theorem 1, prime numbers pass the term formula, and the parallelogram decomposition of prime numbers is carried out, as shown in Figure 13.

因为 $E_2Q = m$, $E_1Q = m-1$, 则有

Because $E_2Q = m$, $E_1Q = m-1$, we have

$$E_2Q - E_1Q = m - (m-1) = 1$$

所以, 对于任意两个相邻素数对, 它们之间的序数之差总是等于 1。

So, for any two pairs of adjacent prime numbers, the difference between them is always equal to 1.

其次, 又根据定理 1 素数通项公式, 并进行素数的平行四边形分解, 则有:

Secondly, according to theorem 1 prime number general term formula, and the parallelogram decomposition of prime numbers, then:

$$P_m = m + S_m$$

其中, Among them,

$$S_m = S_m(P_m) = \sum_{n=1}^{\infty} K_m = 0 + 0 + 0 + 1 + 1 + 3 + 1 + 3 + \dots + K_m + \dots + \infty$$

式中: 当 $m \geq 4$ 时, $K_m = 1$, 或 3, 或 5, 或 7, 或 9, ..., 或 $2n-1$, ...。 ($n, m \in N^*$), 均为奇数。

In the formula: when $m \geq 4$, $K_m = 1, \text{ or } 3, \text{ or } 5, \text{ or } 7, \text{ or } 9, \dots, 2n-1, \dots$. ($n, m \in N^*$), are odd.

因为, Because,

$$F_2Q = \sum_{m=4}^{\infty} K_m, \quad F_1Q = \sum_{m=4}^{\infty} K_{m-1}$$

则有 There is

$$F_2Q - F_1Q = K_m$$

即, That is,

$$F_2F_1 = K_m$$

其中: 当 $m \geq 4$ 时, $K_m = 1$, 或 3, 或 5, 或 7, 或 9, ..., 或 $2n-1$, ...。 ($n, m \in N^*$), 均为奇数。如图 13 所示。

Where: when $m \geq 4$, $K_m = 1, \text{ or } 3, \text{ or } 5, \text{ or } 7, \text{ or } 9, \dots, 2n-1, \dots$. ($n, m \in N^*$), both are odd. As shown

in Figure 13.

因此，对于其他任意两个相邻素数对，它们之间的序数之差总是等于 1；它们的特征级数函数之差，等于 3，或者是 5， \dots ，或者是 $2n-1$ ，均为奇数。这就是任意两个相邻素数对或者 Polignac 猜想的几何特征及其意义，如图 13 所示。用数学式表示为：

Similarly, for any other pair of adjacent prime numbers, the ordinal difference between them is always equal to 1; The difference between their characteristic series functions is equal to 3, or 5... Or $2n$ minus 1, which is odd. This is the geometric feature of any two adjacent prime pairs or Polignac conjecture and its meaning, as shown in Figure 13. It is mathematically expressed as:

$$P_m - P_{m-1} = K_m + 1$$

式中： K_m 为有序素数无穷特征级数的通项，分别为 0, 0, 0, 1, 1, 3, 1, 3, \dots ， K_m , \dots , ∞ 。

Where: K_m is the general term of infinite characteristic series of ordered prime numbers, respectively 0,0,0,1,1,3,1,3,..., K_m , \dots , ∞

其中：当 $m \geq 4$ 时， $K_m = 1$ ，或 3，或 5，或 7，或 9， \dots ，或 $2n-1$ ， \dots 。（ $n, m \in N^*$ ），均为奇数。定理 11 成立。

Where: when $m \geq 4$, $K_m = 1$, or 3, or 5, or 7, or 9, \dots , $2n-1$, \dots . ($n, m \in N^*$), both are odd. Theorem 11 is true.

同理，当 $F_2 F_1 = E_2 E_1 = K_m = 1$ 时，In the same way, when $F_2 F_1 = E_2 E_1 = K_m = 1$,

$$P_m - P_{m-1} = 1 + 1$$

两个相邻素数对 P_m 和 P_{m-1} 即是孪生素数，等式中右边第一个“1”表示两个相邻素数的序数之差等于 1；第二个“1”表示两个相邻素数的特征级数函数之差等于 1。这就是孪生素数的几何特征及其意义。

Two adjacent primes for P_m and P_{m-1} are clonic primes, and the first "1" on the right of the equation means that the difference between the Ordinal Numbers of two adjacent primes is equal to 1; The second "1" means that the difference between the eigenseries functions of two adjacent prime numbers is equal to 1. This is the geometric character of twin prime numbers and its meaning.

有关更多有序素数在非零自然数的排序分布规律数据分析，详见表 4 所示。

For more data analysis of ordering distribution rules of ordered prime numbers in non-zero natural numbers, see Table 4.

(未完，待续 Unfinished, to be continued)

表 4. 非零自然数 10000 中的素数排序分布规律数据分析表 (素数 1230 个)

Table 4. Data analysis table of primes ordering distribution in non-zero natural number 10000 (1230 primes)

m	S_m	p_m	K_m	$\delta = \frac{m}{p_m}$	$\beta = \frac{S_m}{m}$	$\theta = \arctan P_m$ (π)
1	0	1	0	1	0	0.25
2	0	2	0	1	0	0.3524163823495...
3	0	3	0	1	0	0.3975836176504...
4	1	5	1	0.8	0.25	0.4371670418109...
5	2	7	1	0.714285...	0.40	0.4548327646991...
6	5	11	3	0.545454...	8.833333...	0.4711420616236...
7	6	13	1	0.538461...	0.857142...	0.4755627480278...
8	9	17	3	0.470588...	1.125	0.4812974407587...
9	10	19	1	0.473684...	1.111111...	0.4832622916434...
10	13	23	3	0.434782...	1.30	0.4861691504333...
11	18	29	5	0.379310...	1.636363...	0.4890281443839...
12	19	31	1	0.387096...	1.583333...	0.4897354985222...
13	24	37	5	0.351351...	1.846153...	0.4913991238945...
14	27	41	3	0.341463...	1.928571...	0.4922378831816...
15	28	43	1	0.348837...	1.86666...	0.4925987785896...
16	31	47	3	0.340425...	1.9375	0.4932284709159...
17	36	53	5	0.320754...	2.117647...	0.4939948656296...
18	41	59	5	0.305084...	2.277777...	0.4946054337162...
19	42	61	1	0.311475...	2.210526...	0.4947822725236...
20	47	67	5	0.298507...	2.35	0.4952494589096...
21	50	71	3	0.295774...	2.380952...	0.4955170585827...
22	51	73	1	0.301369...	2.318181...	0.4956398633167...
23	56	79	5	0.291139...	2.434782...	0.4959709761163...
24	59	83	3	0.289156...	2.458333...	0.4961651266785...
25	64	89	5	0.280898...	2.56	0.4964236349212...
26	71	97	7	0.268041...	2.730769...	0.4967185710297...
27	74	101	3	0.267326...	2.740740...	0.4968485199452...
28	75	103	1	0.271844...	2.678571...	0.4969097098495...

29	78	107	3	0.271028...	2.689655...	0.4970252278579...
30	79	109	1	0.275229...	2.633333...	0.4970798077419...
31	82	113	3	0.274336...	2.645161...	0.4971831718838...
32	95	127	13	0.251968...	2.96875	0.4974936747400...
33	98	131	3	0.251908...	2.969696...	0.4975702007359...
34	103	137	5	0.248175...	3.029411...	0.4976766114361...
35	104	139	1	0.251798...	2.971428...	0.4977100403255...
36	113	149	9	0.241610...	3.138888...	0.4978637241134...
37	114	151	1	0.245033...	3.081081...	0.4978920183254...
38	119	157	5	0.242038...	3.131578...	0.4979725759126...
39	124	163	5	0.239263...	3.179487...	0.4980472031118...
40	127	167	3	0.239520...	3.175	0.4980939755582...
41	132	173	5	0.236994...	3.219512...	0.4981600789532...
42	137	179	5	0.234636...	3.261904...	0.4982217509789...
43	138	181	1	0.237569...	3.209302...	0.4982413997373...
44	147	191	9	0.230366...	3.340909...	0.4983334713205...
45	148	193	1	0.233160...	3.288888...	0.4983507407370...
46	151	197	3	0.233502...	3.282608...	0.4983842276536...
47	152	199	1	0.236180...	3.234042...	0.4984004662969...
48	163	211	11	0.227488...	3.395833...	0.4984914336350...
49	174	223	11	0.219730...	3.55102...	0.4985726109749...
50	177	227	3	0.220264...	3.54	0.4985977628761...
51	178	229	1	0.222707...	3.490196...	0.4986100093322...
52	181	233	3	0.223175...	3.480769...	0.4986338715373...
53	186	239	5	0.221757...	3.509433...	0.4986681672439...
54	187	241	1	0.224066...	3.462962...	0.4986792196705...
55	196	251	9	0.219123...	3.563636...	0.4987318398324...
56	201	257	5	0.217898...	3.589285...	0.4987614463822...
57	206	263	5	0.216730...	3.614035...	0.4987897020827...
58	211	269	5	0.215613...	3.637931...	0.4988166973238...
59	212	271	1	0.217712...	3.593320...	0.4988254301053...
60	217	277	5	0.216606...	3.616666...	0.4988508718290...
61	220	281	3	0.217081...	3.606557...	0.4988672293862...

62	221	283	1	0.219081...	3.564516...	0.4988752347654...
63	230	293	9	0.215017...	3.650793...	0.4989136223540...
64	243	307	13	0.208469...	3.796875...	0.4989631636468...
65	246	311	3	0.209003...	3.784615...	0.4989764990701...
66	247	313	1	0.210862...	3.742424...	0.4989830389675...
67	250	317	3	0.211356...	3.731343...	0.4989958711977...
68	263	331	13	0.205438...	3.867647...	0.4990383416382...
69	268	337	5	0.204747...	3.884057...	0.4990554630506...
70	277	347	9	0.201729...	3.957142...	0.4990826829827...
71	278	349	1	0.203438...	3.915492...	0.4990879397849...
72	281	353	3	0.203966...	3.902777...	0.4990982746892...
73	286	359	5	0.203342...	3.917808...	0.4991133452286...
74	293	367	7	0.201634...	3.959459...	0.4991326727563...
75	298	373	5	0.201072...	3.973333...	0.4991466243336...
76	303	379	5	0.200527...	3.986842...	0.4991601341754...
77	306	383	3	0.201044...	3.974025...	0.4991689055799...
78	311	389	5	0.200514...	3.987179...	0.4991817244601...
79	318	397	7	0.198992...	4.025316...	0.4991982135693...
80	321	401	3	0.199501...	4.0125	0.4992062114056...
81	328	409	7	0.198044...	4.049382...	0.4992217377704...
82	337	419	9	0.195704...	4.109756...	0.4992403119765...
83	338	421	1	0.197149...	4.072289...	0.4992439209321...
84	347	431	9	0.194895...	4.130952...	0.4992614633062...
85	348	433	1	0.196304...	4.094117...	0.4992648745490...
86	353	439	5	0.195899...	4.104651...	0.4992749217867...
87	356	443	3	0.196388...	4.091954...	0.4992814687459...
88	361	449	5	0.195991...	4.102272...	0.4992910704679...
89	368	457	7	0.194748...	4.134831...	0.4993034805729...
90	371	461	3	0.195227...	4.122222...	0.4993095241064...
91	372	463	1	0.196544...	4.087912...	0.4993125067144...
92	375	467	3	0.197002...	4.076086...	0.4993183952897...
93	386	479	11	0.194154...	4.150537...	0.4993354709316...
94	393	487	7	0.193018...	4.180851...	0.4993463871893...

95	396	491	3	0.193482...	4.168421...	0.4993517119224...
96	403	499	7	0.192384...	4.197916...	0.4993621052904...
97	406	503	3	0.192842...	4.185567...	0.4993671779983...
98	411	509	5	0.192534...	4.193877...	0.4993746375704...
99	422	521	11	0.190019...	4.262626...	0.4993890412758...
100	423	523	1	0.191204...	4.23	0.4993913776323...
101	440	541	17	0.186691...	4.356435...	0.4994116274978...
102	445	547	5	0.186471...	4.362745...	0.4994180812951...
103	454	557	9	0.184919...	4.407766...	0.4994285286459...
104	459	563	5	0.184724...	4.413461...	0.4994346189139...
105	464	569	5	0.184534...	4.419047...	0.4994405807408...
106	465	571	1	0.185639...	4.386792...	0.4994425401738...
107	470	577	5	0.185441...	4.392523...	0.4994483369714...
108	479	587	9	0.183986...	4.435185...	0.4994577349603...
109	484	593	5	0.183811...	4.440366...	0.4994632216113...
110	489	599	5	0.183639...	4.445454...	0.4994685983464...
111	490	601	1	0.184692...	4.414414...	0.4994703667347...
112	495	607	5	0.184514...	4.419642...	0.4994756019798...
113	500	613	5	0.184339...	4.415929...	0.4994807347409...
114	503	617	3	0.184764...	4.412280...	0.4994841011224...
115	504	619	1	0.185783...	4.382608...	0.4994857679979...
116	515	631	11	0.183835...	4.439655...	0.4994955473538...
117	524	641	9	0.182527...	4.47832	0.4995034171170...
118	525	643	1	0.183514...	4.449152...	0.4995049616958...
119	528	647	3	0.183925...	4.436974...	0.4995080222060...
120	533	653	5	0.183767...	4.441666...	0.4995125426686...
121	538	659	5	0.183611...	4.446280...	0.4995169808165...
122	539	661	1	0.184568...	4.418032...	0.4995184422944...
123	550	673	11	0.182763...	4.471544...	0.4995270287489...
124	553	677	3	0.183161...	4.459677...	0.4995298232574...
125	558	683	5	0.183016...	4.464	0.4995339536475...
126	565	691	7	0.182344...	4.484126...	0.4995393492561...
127	574	701	9	0.181169...	4.519685...	0.4995459205845...

128	581	709	7	0.180535...	4.539062...	0.4995510441817...
129	590	719	9	0.179415...	4.573643...	0.4995572883436...
130	597	727	7	0.178817...	4.592307...	0.4995621599925...
131	602	733	5	0.178717...	4.595419...	0.4995657439444...
132	607	739	5	0.178619...	4.598484...	0.4995692696997...
133	610	743	3	0.179004...	4.586466...	0.4995715885679...
134	617	751	7	0.178428...	4.604477...	0.4995761521996...
135	622	757	5	0.178335...	4.607407...	0.4995795116234...
136	625	761	3	0.178712...	4.595588...	0.4995817218095...
137	632	769	7	0.178153...	4.613138...	0.4995860732031...
138	635	773	3	0.178525...	4.601449...	0.4995882151246...
139	648	787	13	0.176620...	4.661870...	0.4995955403877...
140	657	797	9	0.175658...	4.692857...	0.4996006151579...
141	668	809	11	0.174289...	4.737588...	0.4996065392780...
142	669	811	1	0.175092...	4.711267...	0.4996075095870...
143	678	821	9	0.174177...	4.741258...	0.4996122902207...
144	679	823	1	0.174969...	4.715277...	0.4996132324063...
145	682	827	3	0.175332...	4.703448...	0.4996151031063...
146	683	829	1	0.176115...	4.678082...	0.4996160316866...
147	692	839	9	0.175208...	4.707482...	0.4996206081818...
148	705	853	13	0.173505...	4.763513...	0.4996268350054...
149	708	857	3	0.173862...	4.751677...	0.4996285767307...
150	709	859	1	0.174621...	4.726666...	0.4996294415106...
151	712	863	3	0.174971...	4.715231...	0.4996311590455...
152	725	877	13	0.173318...	4.769736...	0.4996370470373...
153	728	881	3	0.173666...	4.758169...	0.4996386949495...
154	729	883	1	0.174405...	4.733766...	0.4996395133067...
155	732	887	3	0.174746...	4.722580...	0.4996411389500...
156	751	907	19	0.171995...	4.814102...	0.4996490520868...
157	754	911	3	0.172338...	4.802547...	0.4996505930204...
158	761	919	7	0.171926...	4.816455...	0.4996536346457...
159	770	929	9	0.171151...	4.842767...	0.4996573630105...
160	777	937	7	0.170757...	4.85625	0.4996602884041...

161	780	941	3	0.171094...	4.844720...	0.4996617324480...
162	785	947	5	0.171066...	4.845679...	0.4996638756411...
163	790	953	5	0.171038...	4.846625...	0.4996659918474...
164	803	967	13	0.169596...	4.896341...	0.4996708275359...
165	806	971	3	0.169927...	4.884848...	0.4996721835492...
166	811	977	5	0.169907...	4.885542...	0.4996741967502...
167	816	983	5	0.169888...	4.886227...	0.4996761853749...
168	823	991	7	0.169525...	4.898809...	0.4996787994166...
169	828	997	5	0.169508...	4.899408...	0.4996807324178...
170	839	1009	11	0.168483...	4.935294...	0.4996845294529...
171	842	1013	3	0.168805...	4.923976...	0.4996857751403...
172	847	1019	5	0.168792...	4.924418...	0.4996876253346...
173	848	1021	1	0.169441...	4.901734...	0.4996882372336...
174	857	1031	9	0.168768...	4.925287...	0.4996912611189...
175	858	1033	1	0.169409...	4.902857...	0.4996918588705...
176	863	1039	5	0.169393...	4.903409...	0.4996936383177...
177	872	1049	9	0.168732...	4.926553...	0.4996965588276...
178	873	1051	1	0.169362...	4.904494...	0.4996971362605...
179	882	1061	9	0.168708...	4.927374...	0.4996999907710...
180	883	1063	1	0.169332...	4.905555...	0.4997005552283...
181	888	1069	5	0.169317...	4.906077...	0.4997022359276...
182	905	1087	17	0.167433...	4.972527...	0.4997071667006...
183	908	1091	3	0.167736...	4.961748...	0.4997082403326...
184	909	1093	1	0.168344...	4.940217...	0.4997087742018...
185	912	1097	3	0.168641...	4.929729...	0.4997098361002...
186	917	1103	5	0.168631...	4.930107...	0.4997114145068...
187	922	1109	5	0.168620...	4.930481...	0.4997129758341...
188	929	1117	7	0.168307...	4.941489...	0.4997150315119...
189	934	1123	5	0.168299...	4.941798...	0.4997165540498...
190	939	1129	5	0.168290...	4.942105...	0.4997180604048...
191	960	1151	21	0.165942...	5.026178...	0.4997234493430
192	961	1153	1	0.166522...	5.005208...	0.4997239290491...
193	970	1163	9	0.165950...	5.025906...	0.4997263028308...

194	977	1171	7	0.165670...	5.036082...	0.4997281726654...
195	986	1181	9	0.165114...	5.056410...	0.4997304743352...
196	991	1187	5	0.165122...	5.056122...	0.4997318367220...
197	996	1193	5	0.165129...	5.055837...	0.4997331854051...
198	1003	1201	7	0.164862...	5.065656...	0.4997349626872...
199	1014	1213	11	0.164056...	5.095477...	0.4997375846545...
200	1017	1217	3	0.164338...	5.085	0.4997384471532...
201	1022	1223	5	0.164349...	5.084577...	0.4997397303227...
202	1027	1229	5	0.164361...	5.084158...	0.4997410009634...
203	1028	1231	1	0.164906...	5.064039...	0.4997414217577...
204	1033	1237	5	0.164915...	5.063725...	0.4997426759766...
205	1044	1249	11	0.164131...	5.092682...	0.4997451482640...
206	1053	1259	9	0.163621...	5.111650...	0.4997471725025...
207	1070	1277	17	0.162098...	5.169082...	0.4997507362403...
208	1071	1279	1	0.162627...	5.149038...	0.4997511260192...
209	1074	1283	3	0.162899...	5.138755...	0.4997519019316...
210	1079	1289	5	0.162916...	5.138095...	0.4997530567708...
211	1080	1291	1	0.163439...	5.118483...	0.4997534393318...
212	1085	1297	5	0.163454...	5.117924...	0.4997545799359...
213	1088	1301	3	0.163720...	5.107981...	0.4997553344938...
214	1089	1303	1	0.164236...	5.088785...	0.4997557100355...
215	1092	1307	3	0.164498...	5.079069...	0.4997564576709...
216	1103	1319	11	0.163760...	5.106481...	0.4997586733698...
217	1104	1321	1	0.164269...	5.087557...	0.4997590387393...
218	1109	1327	5	0.164280...	5.087155...	0.4997601282396...
219	1142	1361	33	0.160911...	5.214611...	0.4997661206253...
220	1147	1367	5	0.160936...	5.213636...	0.4997671471621...
221	1152	1373	5	0.160961...	5.212669...	0.4997681647269...
222	1159	1381	7	0.160753...	5.220720...	0.4997695077258...
223	1176	1399	17	0.159399...	5.273542...	0.4997724733152...
224	1185	1409	9	0.158977...	5.290178...	0.4997740881243...
225	1198	1423	13	0.158116...	5.324444...	0.4997763107281...
226	1201	1427	3	0.158374...	5.314159...	0.4997769377476

227	1202	1429	1	0.158852...	5.295154...	0.4997772499410...
228	1205	1433	3	0.159106...	5.285087...	0.4997778717135...
229	1210	1439	5	0.159138...	5.283842...	0.4997787978909...
230	1217	1447	7	0.158949...	5.291304...	0.4997800208462...
231	1220	1451	3	0.159200...	5.281385...	0.4997806272668...
232	1221	1453	1	0.159669...	5.262931...	0.4997809292251...
233	1226	1459	5	0.159698...	5.261802...	0.4997818301327...
234	1237	1471	11	0.159075...	5.286324...	0.4997836098999...
235	1246	1481	9	0.158676...	5.302127...	0.4997850710075...
236	1247	1483	1	0.159136...	5.283898...	0.4997853608645...
237	1250	1487	3	0.159381...	5.274261...	0.4997859382392...
238	1251	1489	1	0.159838...	5.256302...	0.4997862257633...
239	1254	1493	3	0.160080...	5.246861...	0.4997867985006...
240	1259	1499	5	0.160106...	5.245833...	0.4997876518752...
241	1270	1511	11	0.159497...	5.269709...	0.4997893382927...
242	1281	1523	11	0.158896...	5.293388...	0.4997909981349...
243	1288	1531	7	0.158719...	5.300411...	0.4997920902410...
244	1299	1543	11	0.158133...	5.323770...	0.4997937071668...
245	1304	1549	5	0.158166...	5.322448...	0.4997945062350...
246	1307	1553	3	0.158403...	5.313008...	0.4997950355169...
247	1312	1559	5	0.158434...	5.311740...	0.4997958243473...
248	1319	1567	7	0.158264...	5.318548...	0.4997968667243...
249	1322	1571	3	0.158497...	5.309236...	0.4997973839317...
250	1329	1579	7	0.158328...	5.316	0.4997984104853...
251	1332	1583	3	0.158559...	5.306772...	0.4997989198712...
252	1345	1597	13	0.157795...	5.337301...	0.4998006826270...
253	1348	1601	3	0.158026...	5.328063...	0.4998011806091...
254	1353	1607	5	0.158058...	5.326771...	0.4998019229339...
255	1354	1609	1	0.158483...	5.309803...	0.4998021691453...
256	1357	1613	3	0.158710...	5.300781...	0.4998026597362...
257	1362	1619	5	0.158739...	5.299610...	0.4998033910773...
258	1363	1621	1	0.159161...	5.282945...	0.4998036336546...
259	1368	1627	5	0.159188...	5.281853...	0.4998043578081...

260	1377	1637	9	0.158827...	5.296153...	0.4998055529342...
261	1396	1657	19	0.157513...	5.348659...	0.4998078999109...
262	1401	1663	5	0.157546...	5.347328...	0.4998085929959...
263	1404	1667	3	0.157768...	5.338403...	0.4998090522807...
264	1405	1669	1	0.158178...	5.321969...	0.4998092810976...
265	1428	1693	23	0.156526...	5.388679...	0.4998119847317...
266	1431	1697	3	0.156747...	5.379699...	0.4998124279025...
267	1432	1699	1	0.157151...	5.363295...	0.4998126487054...
268	1441	1709	9	0.156816...	5.376865...	0.4998137449679...
269	1452	1721	11	0.156304...	5.397769...	0.4998150436662...
270	1453	1723	1	0.156703...	5.381481...	0.4998152583572...
271	1462	1733	9	0.156376...	5.394833...	0.4998163243791...
272	1469	1741	7	0.156232...	5.400735...	0.4998171683795...
273	1474	1747	5	0.156267...	5.399267...	0.4998177963071...
274	1479	1753	5	0.156303...	5.397810...	0.4998184199363...
275	1484	1759	5	0.156338...	5.396363...	0.4998190393110...
276	1501	1777	17	0.155317...	5.438405...	0.4998208723395...
277	1506	1783	5	0.155356...	5.436823...	0.4998214751246...
278	1509	1787	3	0.155567...	5.428057...	0.4998218747325...
279	1510	1789	1	0.155953...	5.412186...	0.4998220738663...
280	1521	1801	11	0.155469...	5.432142...	0.4998232593817...
281	1530	1811	9	0.155162...	5.444839...	0.4998242353098...
282	1541	1823	11	0.154690...	5.464539...	0.4998253922905...
283	1548	1831	7	0.154560...	5.469964...	0.4998261551859...
284	1563	1847	15	0.153762...	5.503521...	0.4998276611504...
285	1576	1861	13	0.153143...	5.529824...	0.4998289576273...
286	1581	1867	5	0.153186...	5.527972...	0.4998295073081...
287	1584	1871	3	0.153393...	5.519163...	0.4998298718033...
288	1585	1873	1	0.153764...	5.503472...	0.4998300534671...
289	1588	1877	3	0.153969...	5.494809...	0.4998304156334...
290	1589	1879	1	0.154337...	5.479310...	0.4998305961383...
291	1598	1889	9	0.154049...	5.491408...	0.4998314929293...

292	1609	1901	11	0.153603...	5.510273...	0.4998325566245...
293	1614	1907	5	0.153644...	5.508532...	0.4998330834520...
294	1619	1913	5	0.153685...	5.506802...	0.4998336069748...
295	1636	1931	17	0.152770...	5.545762...	0.4998351580229...
296	1637	1933	1	0.153129...	5.530405...	0.4998353285784...
297	1652	1949	15	0.152385...	5.562289...	0.4998366804216...
298	1653	1951	1	0.152742...	5.546979...	0.4998368478429...
299	1674	1973	21	0.151545...	5.598662...	0.4998386670760...
300	1679	1979	5	0.151591...	5.596666...	0.4998391562106...
301	1686	1987	7	0.151484...	5.601328...	0.4998398037950...
302	1691	1993	5	0.151530...	5.599337...	0.4998402860715...
303	1694	1997	3	0.151727...	5.590759...	0.4998406059791...
304	1695	1999	1	0.152076...	5.575657...	0.4998407654529...
305	1698	2003	3	0.152271...	5.567213...	0.4998410834449...
306	1705	2011	7	0.152163...	5.571895...	0.4998417156340...
307	1710	2017	5	0.152206...	5.570032...	0.4998421864848...
308	1719	2027	9	0.151948...	5.581168...	0.4998429650417...
309	1720	2029	1	0.152291...	5.566343...	0.4998431198322...
310	1729	2039	9	0.152035...	5.577419...	0.4998438892296...
311	1742	2053	13	0.151485...	5.601286...	0.4998449537939...
312	1751	2063	9	0.151236...	5.612179...	0.4998457053508...
313	1756	2069	5	0.151280...	5.610223...	0.4998461527977...
314	1767	2081	11	0.150888...	5.627388...	0.4998470399511...
315	1768	2083	1	0.151224...	5.612698...	0.4998471868162...
316	1771	2087	3	0.151413...	5.604430...	0.4998474797020...
317	1772	2089	1	0.151747...	5.589905...	0.4998476257243...
318	1781	2099	9	0.151500...	5.600628...	0.4998483516616...
319	1792	2111	11	0.151113...	5.617554...	0.4998492137080...
320	1793	2113	1	0.151443...	5.603125...	0.4998493564304...
321	1808	2129	15	0.150775...	5.632398...	0.4998504885567...
322	1809	2131	1	0.151102...	5.618012...	0.4998506288771...
323	1814	2137	5	0.151146...	5.616099...	0.4998510482625...
324	1817	2141	3	0.151331...	5.608024...	0.4998513265469...

325	1818	2143	1	0.151656...	5.593846...	0.4998514652995...
326	1827	2153	9	0.151416...	5.604294...	0.4998521551958...
327	1834	2161	7	0.151318...	5.608562...	0.4998527025157...
328	1851	2179	17	0.150527...	5.643292...	0.4998539192914...
329	1874	2203	23	0.149341...	5.696048...	0.4998555107288...
330	1877	2207	3	0.149524...	5.687878...	0.4998557726033...
331	1882	2213	5	0.149570...	5.685800...	0.4998561636400...
332	1889	2221	7	0.149482...	5.689759...	0.4998566817358...
333	1904	2237	15	0.148860...	5.717717...	0.4998577068104...
334	1905	2239	1	0.149173...	5.703592...	0.4998578339146...
335	1908	2243	3	0.149353...	5.695522...	0.4998580874431...
336	1915	2251	7	0.149266...	5.699404...	0.4998585917968...
337	1930	2267	15	0.148654...	5.727002...	0.4998595898255...
338	1931	2269	1	0.148964...	5.713017...	0.4998597135894...
339	1934	2273	3	0.149142...	5.705014...	0.4998599604638...
340	1941	2281	7	0.149057...	5.708823...	0.4998604516151...
341	1946	2287	5	0.149103...	5.706744...	0.4998608177237...
342	1951	2293	5	0.149149...	5.704678...	0.4998611819162...
343	1954	2297	3	0.149325...	5.696793...	0.4998614236542...
344	1965	2309	11	0.148982...	5.712209...	0.4998621438430...
345	1966	2311	1	0.149286...	5.698550...	0.4998622631474...
346	1987	2333	21	0.148306...	5.742774...	0.4998635619945...
347	1992	2339	5	0.148353...	5.740634...	0.4998639119851...
348	1993	2341	1	0.138654...	5.727011...	0.4998640282499...
349	1998	2347	5	0.148700...	5.724928...	0.4998643758555...
350	2001	2351	3	0.148872...	5.717142...	0.4998646066069...
351	2006	2357	5	0.148918...	5.715099...	0.4998649512655...
352	2019	2371	13	0.148460...	5.735795...	0.4998657486852...
353	2024	2377	5	0.148506...	5.733711...	0.4998660875610...
354	2027	2381	3	0.148677...	5.725988...	0.4998663125294...
355	2028	2383	1	0.148971...	5.712676...	0.4998664247303...
356	2033	2389	5	0.149016...	5.710674...	0.4998667602061...
357	2036	2393	3	0.149185...	5.703081...	0.4998669829219...

358	2041	2399	5	0.149228...	5.701117...	0.4998673156032...
359	2052	2411	11	0.148900...	5.715877...	0.4998679759983...
360	2057	2417	5	0.148944...	5.713888...	0.4998683037368...
361	2062	2423	5	0.148988...	5.711911...	0.4998686298522...
362	2075	2437	13	0.148543...	5.732044...	0.4998693845431...
363	2078	2441	3	0.148709...	5.724517...	0.4998695985791...
364	2083	2447	5	0.148753...	5.722527...	0.4998699183210...
365	2094	2459	11	0.148434...	5.736986...	0.4998705531237...
366	2101	2467	7	0.148358...	5.740437...	0.4998709728947...
367	2106	2473	5	0.148402...	5.738419...	0.4998712859406...
368	2109	2477	3	0.148566...	5.730978...	0.4998714937953...
369	2134	2503	25	0.147423...	5.783197...	0.4998728286579...
370	2151	2521	17	0.146767...	5.813513...	0.4998737366642...
371	2160	2531	9	0.146582...	5.822102...	0.4998742355315...
372	2167	2539	7	0.146514...	5.825268...	0.4998746317960...
373	2170	2543	3	0.146677...	5.817694...	0.4998748289934...
374	2175	2549	5	0.146724...	5.815508...	0.4998751236289...
375	2176	2551	1	0.147001...	5.802666...	0.4998752215327...
376	2181	2557	5	0.147047...	5.800531...	0.4998755143253...
377	2202	2579	21	0.146180...	5.840848...	0.4998765762426...
378	2213	2591	11	0.145889...	5.854497...	0.4998771478694...
379	2214	2593	1	0.146162...	5.841688...	0.4998772426261...
380	2229	2609	15	0.145649...	5.865789...	0.4998779954501...
381	2236	2617	7	0.145586...	5.868766...	0.4998783684101...
382	2239	2621	3	0.145745...	5.861256...	0.4998785540363...
383	2250	2633	11	0.145461...	5.874673...	0.4998791075309...
384	2263	2647	13	0.145069...	5.893229...	0.4998797469319...
385	2272	2657	9	0.144900...	5.901298...	0.4998801995215...
386	2273	2659	1	0.145167...	5.888601...	0.4998802896309...
387	2276	2663	3	0.145324...	5.881136...	0.4998804694437...
388	2283	2671	7	0.145263...	5.884020...	0.4998808274536...
389	2288	2677	5	0.145311...	5.881748...	0.4998810945568...
390	2293	2683	5	0.145359...	5.879487...	0.4998813604653...

391	2296	2687	3	0.145515...	5.872122...	0.4998815370779...
392	2297	2689	1	0.145779...	5.859693...	0.4998816251872...
393	2300	2693	3	0.145933...	5.852417...	0.4998818010131...
394	2305	2699	5	0.145979...	5.850253...	0.4998820637748...
395	2312	2707	7	0.145917...	5.853164...	0.4998824123118...
396	2315	2711	3	0.146071...	5.845959...	0.4998825858090...
397	2316	2713	1	0.146332...	5.833753...	0.4998826723657...
398	2321	2719	5	0.146377...	5.831658...	0.4998829312718...
399	2330	2729	9	0.146207...	5.839598...	0.4998833602521...
400	2331	2731	1	0.146466...	5.8275	0.4998834456711...
401	2340	2741	9	0.146296...	5.835411...	0.4998838708967...
402	2347	2749	7	0.146234...	5.838308...	0.4998842088497...
403	2350	2753	3	0.146385...	5.831265...	0.4998843770896...
404	2363	2767	13	0.146006...	5.849009...	0.4998849620989...
405	2372	2777	9	0.145840...	5.856790...	0.4998853763513...
406	2383	2789	11	0.145571...	5.869458...	0.4998858695329...
407	2384	2791	1	0.145825...	5.857493...	0.4998859513176...
408	2389	2797	5	0.145870...	5.855392...	0.4998861959697...
409	2392	2801	3	0.146019...	5.848410...	0.4998863584888...
410	2393	2803	1	0.146271...	5.836585...	0.4998864395744...
411	2408	2819	15	0.145796...	5.858880...	0.4998870841174...
412	2421	2833	13	0.145428...	5.876213...	0.4998876421203...
413	2424	2837	3	0.145576...	5.869249...	0.4998878005382...
414	2429	2843	5	0.145620...	5.867149...	0.4998880373292...
415	2436	2851	7	0.145562...	5.869879...	0.4998883515001...
416	2441	2857	5	0.145607...	5.867788...	0.4998885859736...
417	2444	2861	3	0.145753...	5.860911...	0.4998887417430...
418	2461	2879	17	0.145189...	5.887559...	0.4998894373485...
419	2468	2887	7	0.145133...	5.890214...	0.4998897437223...
420	2477	2897	9	0.144977...	5.897619...	0.4998901243101...
421	2482	2903	5	0.145022...	5.895486...	0.4998903514042...
422	2487	2909	5	0.145067...	5.893364...	0.4998905775614...
423	2494	2917	7	0.145011...	5.895981...	0.4998908776572...

424	2503	2927	9	0.144858...	5.903301...	0.4998912504701...
425	2514	2939	11	0.144607...	5.915294...	0.4998916944968...
426	2527	2953	13	0.144260...	5.931924...	0.4998922079668...
427	2530	2957	3	0.144403...	5.925058...	0.4998923537794...
428	2535	2963	5	0.144448...	5.922897...	0.4998925717603...
429	2540	2969	5	0.144493...	5.920745...	0.4998927888601...
430	2541	2971	1	0.144732...	5.909302...	0.4998928610319...
431	2568	2999	27	0.143714...	5.958236...	0.4998938613289...
432	2569	3001	1	0.143952...	5.946759...	0.4998939320645...
433	2578	3011	9	0.143806...	5.953810...	0.4998942843326...
434	2585	3019	7	0.143756...	5.956221...	0.4998945644668...
435	2588	3023	3	0.143896...	5.949425...	0.4998947039779...
436	2601	3037	13	0.143562...	5.965596...	0.4998951893728...
437	2604	3041	3	0.143702...	5.958810...	0.4998953272362...
438	2611	3049	7	0.143653...	5.961187...	0.4998956018777...
439	2622	3061	11	0.143417...	5.972665...	0.4998960111483...
440	2627	3067	5	0.143462...	5.970454...	0.4998962145826...
441	2638	3079	11	0.143228...	5.981859...	0.4998966190727...
442	2641	3083	3	0.143366...	5.975113...	0.4998967532030...
443	2646	3089	5	0.143412...	5.972911...	0.4998969537471...
444	2665	3109	19	0.142811...	6.002252...	0.4998976166371...
445	2674	3119	9	0.142673...	6.008988...	0.4998979448941...
446	2675	3121	1	0.142902...	5.997758...	0.4998980102930...
447	2690	3137	15	0.142492...	6.017897...	0.4998985304828...
448	2715	3163	25	0.141637...	6.060267...	0.4998993645666...
449	2718	3167	3	0.141774...	6.053452...	0.4998994916717...
450	2719	3169	1	0.142000...	6.042222...	0.4998995551039...
451	2730	3181	11	0.141779...	6.053215...	0.4998999340221...
452	2735	3187	5	0.141826...	6.050884...	0.4999001224111...
453	2738	3191	3	0.141961...	6.044150...	0.4999002476102...
454	2749	3203	11	0.141742...	6.055066...	0.4999006213313...
455	2754	3209	5	0.141788...	6.052747...	0.4999008071436...
456	2761	3217	7	0.141746...	6.054824...	0.4999010538153...

457	2764	3221	3	0.141881...	6.048140...	0.4999011766917...
458	2771	3229	7	0.141839...	6.050218...	0.4999014215311...
459	2792	3251	21	0.141187...	6.082788...	0.4999020886262...
460	2793	3253	1	0.141407...	6.071739...	0.4999021488238...
461	2796	3257	3	0.141541...	6.065075...	0.4999022689971...
462	2797	3259	1	0.141761...	6.054112...	0.4999023289732...
463	2808	3271	11	0.141546...	6.064794...	0.4999026872894...
464	2835	3299	27	0.140648...	6.109913...	0.4999035132232...
465	2836	3301	1	0.140866...	6.098924...	0.4999035716823...
466	2841	3307	5	0.140913...	6.096566...	0.4999037466354...
467	2846	3313	5	0.140959...	6.094218...	0.4999039209548...
468	2851	3319	5	0.141006...	6.091880...	0.4999040946440...
469	2854	3323	3	0.141137...	6.085387...	0.4999042100883...
470	2859	3329	5	0.141183...	6.082978...	0.4999043827435...
471	2860	3331	1	0.141398...	6.072186...	0.4999044401451...
472	2871	3343	11	0.141190...	6.082627...	0.4999047831658...
473	2874	3347	3	0.141320...	6.076109...	0.4999048969594...
474	2885	3359	11	0.141113...	6.086497...	0.4999052367142...
475	2886	3361	1	0.141326...	6.075789...	0.4999052931041...
476	2895	3371	9	0.141204...	6.081932...	0.4999055740501...
477	2896	3373	1	0.141417...	6.071278...	0.4999056300394...
478	2911	3389	15	0.141044...	6.089958...	0.4999060755748...
479	2912	3391	1	0.141256...	6.079331...	0.4999061309711...
480	2927	3407	15	0.140886...	6.097916...	0.4999065718001...
481	2932	3413	5	0.140931...	6.095634...	0.4999067360453...
482	2851	3433	19	0.140401...	6.122406...	0.4999072793832...
483	2966	3449	15	0.140040...	6.140786...	0.4999077095165...
484	2973	3457	7	0.140005...	6.142561...	0.4999079230901...
485	2976	3461	3	0.140132...	6.136082...	0.4999080295066...
486	2977	3463	1	0.140340...	6.125514...	0.4999080826227...
487	2980	3467	3	0.140467...	6.119096...	0.4999081886710...
488	2981	3469	1	0.140674...	6.108606...	0.4999082416035...
489	3002	3491	21	0.140074...	6.139059...	0.4999088198574...

490	3009	3499	7	0.140040...	6.140816...	0.4999090283288...
491	3020	3511	11	0.139846...	6.150712...	0.4999093392544...
492	3025	3517	5	0.139891...	6.148373...	0.4999094939216...
493	3034	3527	9	0.139778...	6.154158...	0.4999097505308...
494	3035	3529	1	0.139982...	6.143724...	0.4999098016781...
495	3038	3533	3	0.140107...	6.137373...	0.4999099037991...
496	3043	3539	5	0.140152...	6.135080...	0.4999100565476...
497	3044	3541	1	0.140355...	6.124748...	0.4999101073488...
498	3049	3547	5	0.140400...	6.122489...	0.4999102594085...
499	3058	3557	9	0.140286...	6.128256...	0.4999105117014...
500	3059	3559	1	0.140488...	6.118	0.4999105619899...
501	3070	3571	11	0.140296...	6.127744...	0.4999108625377...
502	3079	3581	9	0.140184...	6.133466...	0.4999111114554...
503	3080	3583	1	0.140385...	6.123260...	0.4999111610723...
504	3089	3593	9	0.140272...	6.128968...	0.4999114083278...
505	3102	3607	13	0.140005...	6.142574...	0.4999117521824...
506	3107	3613	5	0.140049...	6.140316...	0.4999118987328...
507	3110	3617	3	0.140171...	6.134122...	0.4999119961630...
508	3115	3623	5	0.140215...	6.131889...	0.4999121419050...
509	3122	3631	7	0.140181...	6.133595...	0.4999123354783...
510	3127	3637	5	0.140225...	6.131372...	0.4999124800994...
511	3132	3643	5	0.140269...	6.129158...	0.4999126242442...
512	3147	3659	15	0.139928...	6.146484...	0.4999130063191...
513	3158	3671	11	0.139743...	6.155945...	0.4999132906896...
514	3159	3673	1	0.139940...	6.145914...	0.4999133379040...
515	3162	3677	3	0.140059...	6.139805...	0.4999134321788...
516	3175	3691	13	0.139799...	6.153100...	0.4999137605314...
517	3180	3697	5	0.139843...	6.150870...	0.4999139004927...
518	3183	3701	3	0.139962...	6.144787...	0.4999139935481...
519	3190	3709	7	0.139929...	6.146435...	0.4999141790567...
520	3199	3719	9	0.139822...	6.151923...	0.4999144098202...
521	3206	3727	7	0.139790...	6.153550...	0.4999145935394...
522	3211	3733	5	0.139833...	6.151340...	0.4999147308120...

523	3216	3739	5	0.139876...	6.149139...	0.4999148676441...
524	3237	3761	21	0.139324...	6.177480...	0.4999153656265...
525	3242	3767	5	0.139368...	6.175238...	0.4999155004303...
526	3243	3769	1	0.139559...	6.165399...	0.4999155452696...
527	3252	3779	9	0.139454...	6.170777...	0.4999157687539...
528	3265	3793	13	0.139203...	6.183712...	0.4999160796523...
529	3268	3797	3	0.139320...	6.177693...	0.4999161680593...
530	3273	3803	5	0.139363...	6.175471...	0.4999163003211...
531	3290	3821	17	0.138968...	6.195856...	0.4999166946142...
532	3291	3823	1	0.139157...	6.186090...	0.4999167381954...
533	3300	3833	9	0.139055...	6.191369...	0.4999169554190...
534	3313	3847	13	0.138809...	6.204119...	0.4999172576437...
535	3316	3851	3	0.138924...	6.198130...	0.4999173435785...
536	3317	3853	1	0.139112...	6.188432...	0.4999173864835...
537	3326	3863	9	0.139011...	6.193668...	0.4999176003419...
538	3339	3877	13	0.138767...	6.206319...	0.4999178978903...
539	3342	3881	3	0.138881...	6.200371...	0.4999179825098...
540	3349	3889	7	0.138853...	6.201851...	0.4999181512267...
541	3366	3907	17	0.138469...	6.221811...	0.4999185283134...
542	3369	3911	3	0.138583...	6.215867...	0.4999186116391...
543	3374	3917	5	0.138626...	6.213627...	0.4999187363085...
544	3375	3919	1	0.138810...	6.204044...	0.4999187777802...
545	3378	3923	3	0.138924...	6.198165...	0.4999188605966...
546	3383	3929	5	0.138966...	6.195970...	0.4999189845051...
547	3384	3931	1	0.139150...	6.186471...	0.4999190257239...
548	3395	3943	11	0.138980...	6.195255...	0.4999192721584...
549	3398	3947	3	0.139092...	6.189435...	0.4999193539702...
550	3417	3967	19	0.138643...	6.212727...	0.4999198009877...
551	3438	3989	21	0.138129...	6.239564...	0.4999202030886...
552	3449	4001	11	0.137965...	6.248188...	0.4999204424195...
553	3450	4003	1	0.138146...	6.238698...	0.4999204821684...
554	3453	4007	3	0.138258...	6.232851...	0.4999205615473...
555	3458	4013	5	0.138300...	6.230630...	0.4999206803190...

556	3463	4019	5	0.138342...	6.228417...	0.4999207987360...
557	3464	4021	1	0.138522...	6.219030...	0.4999208381299...
558	3469	4027	5	0.138564...	6.216845...	0.4999209560765...
559	3490	4049	21	0.138058...	6.243291...	0.4999213855569...
560	3491	4051	1	0.138237...	6.233928...	0.4999214243693...
561	3496	4057	5	0.138279...	6.231729...	0.4999215405768...
562	3511	4073	15	0.137981...	6.247330...	0.4999218487896...
563	3516	4079	5	0.138024...	6.245115...	0.4999219637460...
564	3527	4091	11	0.137863...	6.253546...	0.4999221926473...
565	3528	4093	1	0.138040...	6.244247...	0.4999222306670...
566	3533	4099	5	0.138082...	6.242049...	0.4999223445035...
567	3544	4111	11	0.137922...	6.250440...	0.4999225711797...
568	3559	4127	15	0.137630...	6.265845...	0.4999228713641...
569	3560	4129	1	0.137805...	6.256590...	0.4999229087236...
570	3563	4133	3	0.137914...	6.250877...	0.4999229833341...
571	3568	4139	5	0.137956...	6.248686...	0.4999230949794...
572	3581	4153	13	0.137731...	6.260489...	0.4999233542306...
573	3584	4157	3	0.137839...	6.254799...	0.4999234279817...
574	3585	4159	1	0.138013...	6.245644...	0.4999234648040...
575	3602	4177	17	0.137658...	6.264347...	0.4999237946181...
576	3625	4201	23	0.137110...	6.293402...	0.4999242299737...
577	3634	4211	9	0.137022...	6.298093...	0.4999244099073...
578	3639	4217	5	0.137064...	6.295847...	0.4999245174578...
579	3640	4219	1	0.137236...	6.286701...	0.4999245532400...
580	3649	4229	9	0.137148...	6.291379...	0.4999247316433...
581	3650	4231	1	0.137319...	6.282271...	0.4999247672228...
582	3659	4241	9	0.137231...	6.286941...	0.4999249446167...
583	3660	4243	1	0.137402...	6.277873...	0.4999249799952...
584	3669	4253	9	0.137314...	6.282534...	0.4999251563883...
585	3674	4259	5	0.137356...	6.280341...	0.4999252618266...
586	3675	4261	1	0.137526...	6.271331...	0.4999252969067...
587	3684	4271	9	0.137438...	6.275979...	0.4999254718144...
588	3685	4273	1	0.137608...	6.267006...	0.4999255066977...

589	3694	4283	9	0.137520...	6.271646...	0.4999256806256...
590	3699	4289	5	0.137561...	6.269491...	0.4999257845930...
591	3706	4297	7	0.137537...	6.270727...	0.4999259227646...
592	3735	4327	29	0.136815...	6.309121...	0.4999264363576...
593	3744	4337	9	0.136730...	6.313659...	0.4999266059763...
594	3745	4339	1	0.136897...	6.304713...	0.4999266398062...
595	3754	4349	9	0.136813...	6.309243...	0.4999268084891...
596	3761	4357	7	0.136791...	6.310402...	0.4999269428779...
597	3766	4363	5	0.136832...	6.308207...	0.4999270433461...
598	3775	4373	9	0.136748...	6.312709...	0.4999272101805...
599	3792	4391	17	0.136415...	6.330550...	0.4999275085673...
600	3797	4397	5	0.136456...	6.328333...	0.4999276074867...
601	3808	4409	11	0.136312...	6.336106...	0.4999278045178...
602	3819	4421	11	0.136168...	6.343853...	0.4999280004793...
603	3820	4423	1	0.136332...	6.334991...	0.4999280330362...
604	3837	4441	17	0.136005...	6.352649...	0.4999283247284...
605	3842	4447	5	0.136046...	6.350413...	0.4999284214344...
606	3845	4451	3	0.136149...	6.344884...	0.4999284857603...
607	3850	4457	5	0.136190...	6.342668...	0.4999285820325...
608	3855	4463	5	0.136231...	6.340460...	0.4999286780459...
609	3872	4481	17	0.135907...	6.357963...	0.4999289645434...
610	3873	4483	1	0.136069...	6.349180...	0.4999289962344...
611	3882	4493	9	0.135989...	6.353518...	0.4999291542664...
612	3895	4507	13	0.135788...	6.364379...	0.4999293743330...
613	3900	4513	5	0.135829...	6.362153...	0.4999294682293...
614	3903	4517	3	0.135930...	6.356677...	0.4999295306882...
615	3904	4519	1	0.136092...	6.347967...	0.4999295618763...
616	3907	4523	3	0.136192...	6.342532...	0.4999296241695...
617	3930	4547	23	0.135693...	6.369529...	0.4999299956276...
618	3931	4549	1	0.135854...	6.360841...	0.4999300264055...
619	3942	4561	11	0.135715...	6.368336...	0.4999302105062...
620	3947	4567	5	0.135756...	6.366129...	0.4999303021937...
621	3962	4583	15	0.135500...	6.380032...	0.4999305455201...

622	3969	4591	7	0.135482...	6.381028...	0.4999306665473...
623	3974	4597	5	0.135523...	6.378812...	0.4999307570412...
624	3979	4603	5	0.135563...	6.376602...	0.4999308472993...
625	3996	4621	17	0.135252...	6.3936	0.4999311166671...
626	4011	4637	15	0.135001...	6.407348...	0.4999313543495...
627	4012	4639	1	0.135158...	6.398724...	0.4999313839445...
628	4015	4643	3	0.135257...	6.393312...	0.4999314430580...
629	4020	4649	5	0.135279...	6.391096...	0.4999315315376...
630	4021	4651	1	0.135454...	6.382539...	0.4999315609801...
631	4026	4657	5	0.135494...	6.380348...	0.4999316491558...
632	4031	4663	5	0.135535...	6.378164...	0.4999317371045...
633	4040	4673	9	0.135459...	6.382306...	0.4999318831839...
634	4045	4679	5	0.135499...	6.380126...	0.4999319705318...
635	4056	4691	11	0.135365...	6.387401...	0.4999321445573...
636	4067	4703	11	0.135232...	6.394654...	0.4999323176947...
637	4084	4721	17	0.134929...	6.411302...	0.4999325757505...
638	4085	4723	1	0.135083...	6.402821...	0.4999326043020...
639	4090	4729	5	0.135123...	6.400625...	0.4999326898114...
640	4093	4733	3	0.135220...	6.395312...	0.4999327466973...
641	4110	4751	17	0.134918...	6.411856...	0.4999330014983...
642	4117	4759	7	0.134902...	6.412772...	0.4999331141245...
643	4140	4783	23	0.134434...	6.438569...	0.4999334497425...
644	4143	4787	3	0.134531...	6.433229...	0.4999335053516...
645	4144	4789	1	0.134683...	6.424806...	0.4999335331214...
646	4147	4793	3	0.134779...	6.419504...	0.4999335885913...
647	4152	4799	5	0.134819...	6.417310...	0.4999336716229...
648	4253	4801	1	0.134971...	6.408950...	0.4999336992539...
649	4164	4813	11	0.134843...	6.416024...	0.4999338645581...
650	4167	4817	3	0.134938...	6.410769...	0.4999339194765...
651	4180	4831	13	0.134754...	6.420890...	0.4999341109746...
652	4209	4861	29	0.134128...	6.455521...	0.4999345176133...
653	4218	4871	9	0.134058...	6.459418...	0.4999346520464...
654	4223	4877	5	0.134098...	6.457186...	0.4999347324417...

655	4234	4889	11	0.133974...	6.464122...	0.4999348926402...
656	4247	4903	13	0.133795...	6.474085...	0.4999350785474...
657	4252	4909	5	0.133835...	6.471841...	0.4999351578973...
658	4261	4919	9	0.133767...	6.475683...	0.4999352897170...
659	4272	4931	11	0.133644...	6.482549...	0.4999354471949...
660	4273	4933	1	0.133792...	6.474242...	0.4999354733667...
661	4276	4937	3	0.133886...	6.468986...	0.4999355256467...
662	4281	4943	5	0.133926...	6.466767...	0.4999356039081...
663	4288	4951	7	0.133912...	6.467571...	0.4999357079616...
664	4293	4957	5	0.133951...	6.465361...	0.4999357857813...
665	4302	4967	9	0.133883...	6.469172...	0.4999359150630...
666	4303	4969	1	0.134030...	6.460960...	0.4999359408569...
667	4306	4973	3	0.134124...	6.455772...	0.4999359923824...
668	4319	4987	13	0.133948...	6.465568...	0.4999361720710...
669	4324	4993	5	0.133987...	6.463378...	0.4999362487718...
670	4329	4999	5	0.134026...	6.461194...	0.4999363252886...
671	4332	5003	3	0.134119...	6.456035...	0.4999363761978...
672	4337	5009	5	0.134158...	6.453869...	0.4999364524092...
673	4338	5011	1	0.134304...	6.445765...	0.4999364777725...
674	4347	5021	9	0.134236...	6.449554...	0.4999366042856...
675	4348	5023	1	0.134381...	6.441481...	0.4999366295277...
676	4363	5039	15	0.134153...	6.454142...	0.4999368307437...
677	4374	5051	11	0.134032...	6.460856...	0.4999369808192...
678	4381	5059	7	0.134018...	6.461651...	0.4999370804739...
679	4398	5077	17	0.133740...	6.477172...	0.4999373035489...
680	4401	5081	3	0.133831...	6.472058...	0.4999373529064...
681	4406	5087	5	0.133870...	6.469897...	0.4999374267973...
682	4417	5099	11	0.133751...	6.476539...	0.4999375740572...
683	4418	5101	1	0.133895...	6.468521...	0.4999375985332...
684	4423	5107	5	0.133933...	6.466374...	0.4999376718460...
685	4428	5113	5	0.133972...	6.464233...	0.4999377449868...
686	4433	5119	5	0.134010...	6.462099...	0.4999378179562...
687	4460	5147	27	0.133475...	6.491994...	0.4999381562303...

688	4465	5153	5	0.133514...	6.489825...	0.4999382282394...
689	4478	5167	13	0.133346...	6.499274...	0.4999383956101...
690	4481	5171	3	0.133436...	6.494202...	0.4999384432639...
691	4488	5179	7	0.133423...	6.494934...	0.4999385383506...
692	4497	5189	9	0.133359...	6.498554...	0.4999386567966...
693	4504	5197	7	0.133346...	6.499278...	0.4999387512252...
694	4515	5209	11	0.133230...	6.505763...	0.4999388923243...
695	4532	5227	17	0.132963...	6.520863...	0.4999391027583...
696	4535	5231	3	0.133052...	6.515804...	0.4999391493247...
697	4536	5233	1	0.133193...	6.507890...	0.4999391725812...
698	4539	5237	3	0.133282...	6.502865...	0.4999392190409...
699	4562	5261	23	0.132864...	6.526466...	0.4999394963158...
700	4573	5273	11	0.132751...	6.532857...	0.4999396340067...
701	4578	5279	5	0.132790...	6.530670...	0.4999397026174...
702	4579	5281	1	0.132929...	6.522792...	0.4999397254530...
703	4594	5297	15	0.132716...	6.534850...	0.4999399075170...
704	4599	5303	5	0.132755...	6.532670...	0.4999399755077...
705	4604	5309	5	0.132793...	6.530496...	0.4999400433448...
706	4617	5323	13	0.132631...	6.539660...	0.4999402010365...
707	4626	5333	9	0.132570...	6.543140...	0.4999403131666...
708	4639	5347	13	0.132410...	6.552259...	0.4999404694440...
709	4642	5351	3	0.132498...	6.547249...	0.4999405139445...
710	4671	5381	29	0.131945...	6.578873...	0.4999408455895...
711	4676	5387	5	0.131984...	6.576652...	0.4999409114753...
712	4681	5393	5	0.132022...	6.574438...	0.4999409772144...
713	4686	5399	5	0.132061...	6.572230...	0.4999410428074...
714	4693	5407	7	0.132051...	6.572829...	0.4999411300383...
715	4698	5413	5	0.132089...	6.570629...	0.4999411952923...
716	4701	5417	3	0.132176...	6.565642...	0.4999412387146...
717	4702	5419	1	0.132312...	6.557880...	0.4999412604018...
718	4713	5431	11	0.132204...	6.564066...	0.4999413901891...
719	4718	5437	5	0.132242...	6.561891...	0.4999414548680...
720	4721	5441	3	0.132328...	6.556944...	0.4999414979079...

721	4722	5443	1	0.132463...	6.549237...	0.4999415194042...
722	4727	5449	5	0.132501...	6.547091...	0.4999415837983...
723	4748	5471	21	0.132151...	6.567081...	0.4999418187017...
724	4753	5477	5	0.132189...	6.564917...	0.4999418824388...
725	4754	5479	1	0.132323...	6.557241...	0.4999419036534...
726	4757	5483	3	0.132409...	6.552341...	0.4999419460363...
727	4774	5501	17	0.132157...	6.566712...	0.4999421359966...
728	4775	5503	1	0.132291...	6.559065...	0.4999421570265...
729	4778	5507	3	0.132376...	6.554183...	0.4999421990407...
730	4789	5519	11	0.132270...	6.560273...	0.4999423247177...
731	4790	5521	1	0.132403...	6.552667...	0.4999423456108...
732	4795	5527	5	0.132440...	6.550546...	0.4999424081992...
733	4798	5531	3	0.132525...	6.545702...	0.4999424498494...
734	4823	5557	25	0.132085...	6.570844...	0.4999427191141...
735	4828	5563	5	0.132122...	6.568707...	0.4999427808947...
736	4833	5569	5	0.132160...	6.566576...	0.4999428425421...
737	4836	5573	3	0.132244...	6.561736...	0.4999428835667...
738	4843	5581	7	0.132234...	6.562330...	0.4999429654393...
739	4852	5591	9	0.132176...	6.565629...	0.4999430674507...
740	4883	5623	31	0.131602...	6.598648...	0.4999433914489...
741	4898	5639	15	0.131406...	6.609986...	0.4999435520690...
742	4899	5641	1	0.131536...	6.602425...	0.4999435720824...
743	4904	5647	5	0.131574...	6.600269...	0.4999436320377...
744	4907	5651	3	0.131658...	6.595430...	0.4999436719372...
745	4908	5653	1	0.131788...	6.587919...	0.4999436918657...
746	4911	5657	3	0.131872...	6.583109...	0.4999437316805...
747	4912	5659	1	0.132002...	6.575635...	0.4999437515669...
748	4921	5669	9	0.131945...	6.578877...	0.4999438507879...
749	4934	5683	13	0.131796...	6.587449...	0.4999439891108...
750	4939	5689	5	0.131833...	6.585333...	0.4999440481837...
751	4942	5693	3	0.131916...	6.580559...	0.4999440874964...
752	4949	5701	7	0.131906...	6.581117...	0.4999441659563...
753	4958	5711	9	0.131850...	6.584329...	0.4999442637221...

754	4963	5717	5	0.131887...	6.582228...	0.4999443222174...
755	4982	5737	19	0.131601...	6.598675...	0.4999445163181...
756	4985	5741	3	0.131684...	6.593915...	0.4999445549759...
757	4986	5743	1	0.131812...	6.586525...	0.4999445742847...
758	4991	5749	5	0.131849...	6.584432...	0.4999446321302...
759	5020	5779	29	0.131337...	6.613965...	0.4999449195564...
760	5023	5783	3	0.131419...	6.609210...	0.4999449576546...
761	5030	5791	7	0.131410...	6.609724...	0.4999450336931...
762	5039	5801	9	0.131356...	6.612860...	0.4999451284462...
763	5044	5807	5	0.131393...	6.610747...	0.4999451851415...
764	5049	5813	5	0.131429...	6.608638...	0.4999452417197...
765	5056	5821	7	0.131420...	6.609150...	0.4999453169759...
766	5061	5827	5	0.131457...	6.607049...	0.4999453732824...
767	5072	5839	11	0.131358...	6.612777...	0.4999454855483...
768	5075	5843	3	0.131439...	6.608072...	0.4999455228678...
769	5080	5849	5	0.131475...	6.605981...	0.4999455787513...
770	5081	5851	1	0.131601...	6.598701...	0.4999455973537...
771	5086	5857	5	0.131637...	6.596627...	0.4999456530846...
772	5089	5861	3	0.131718...	6.591968...	0.4999456901752...
773	5094	5867	5	0.131753...	6.589909...	0.4999457457161...
774	5095	5869	1	0.131879...	6.582687...	0.4999457642046...
775	5104	5879	9	0.131825...	6.585806...	0.4999458564580...
776	5105	5881	1	0.131950...	6.578608...	0.4999458748710...
777	5120	5897	15	0.131761...	6.589446...	0.4999460217257...
778	5125	5903	5	0.131797...	6.587403...	0.4999460765910...
779	5144	5923	19	0.131521...	6.603337...	0.4999462586724...
780	5147	5927	3	0.131601...	6.598717...	0.4999462949412...
781	5158	5939	11	0.131503...	6.604353...	0.4999464034545...
782	5171	5953	13	0.131362...	6.612531...	0.4999465295005...
783	5198	5981	27	0.130914...	6.638569...	0.4999467798222...
784	5203	5987	5	0.130950...	6.636479...	0.4999468331579...
785	5222	6007	19	0.130680...	6.652229...	0.4999470101742...
786	5225	6011	3	0.130760...	6.647582...	0.4999470454361...

787	5242	6029	17	0.130535...	6.660736...	0.4999472035356...
788	5249	6037	7	0.130528...	6.661167...	0.4999472734995...
789	5254	6043	5	0.130564...	6.659062...	0.4999473258508...
790	5257	6047	3	0.130643...	6.654430...	0.4999473606940...
791	5262	6053	5	0.130679...	6.652338...	0.4999474128724...
792	5275	6067	13	0.130542...	6.660353...	0.4999475342206...
793	5280	6073	5	0.130577...	6.658259...	0.4999475860557...
794	5285	6079	5	0.130613...	6.656171...	0.4999476377885...
795	5294	6089	9	0.130563...	6.659119...	0.4999477237833...
796	5295	6091	1	0.130684...	6.652010...	0.4999477409483...
797	5304	6101	9	0.130634...	6.654956...	0.4999478266049...
798	5315	6113	11	0.130541...	6.660401...	0.4999479290228...
799	5322	6121	7	0.130534...	6.660826...	0.4999479970783...
800	5331	6131	9	0.130484...	6.66375	0.4999480818980...
801	5332	6133	1	0.130604...	6.656679...	0.4999480988287...
802	5341	6143	9	0.130555...	6.659600...	0.4999481833170...
803	5348	6151	7	0.130547...	6.660024...	0.4999482507099...
804	5359	6163	11	0.130455...	6.665422...	0.4999483514711...
805	5368	6173	9	0.130406...	6.668332...	0.4999484351395...
806	5391	6197	23	0.130062...	6.688585...	0.4999486348421...
807	5392	6199	1	0.130182...	6.681536...	0.4999486514141...
808	5395	6203	3	0.130259...	6.676980...	0.4999486845262...
809	5402	6211	7	0.130252...	6.677379...	0.4999487506225...
810	5407	6217	5	0.130287...	6.675308...	0.4999488000830...
811	5410	6221	3	0.130364...	6.670776...	0.4999488330037...
812	5417	6229	7	0.130358...	6.671182...	0.4999488987183...
813	5434	6247	17	0.130142...	6.683886...	0.4999490459607...
814	5443	6257	9	0.130094...	6.686732...	0.4999491273959...
815	5448	6263	5	0.130129...	6.684662...	0.4999491761322...
816	5453	6269	5	0.130164...	6.682598...	0.4999492247753...
817	5454	6271	1	0.130282...	6.675642...	0.4999492409689...
818	5459	6277	5	0.130317...	6.673594...	0.4999492894880...
819	5468	6287	9	0.130268...	6.676434...	0.4999493701473...

820	5479	6299	11	0.130179...	6.681707...	0.4999494666004...
821	5480	6301	1	0.130296...	6.674786...	0.4999494826402...
822	5489	6311	9	0.130248...	6.677615...	0.4999495626868...
823	5494	6317	5	0.130283...	6.675577...	0.4999496105930...
824	5499	6323	5	0.130317...	6.673543...	0.4999496584084...
825	5504	6329	5	0.130352...	6.671515...	0.4999497061331...
826	5511	6337	7	0.130345...	6.671912...	0.4999497696254...
827	5516	6343	5	0.130379...	6.669891...	0.4999498171395...
828	5525	6353	9	0.130332...	6.672770...	0.4999498961304...
829	5530	6359	5	0.130366...	6.670687...	0.4999499434056...
830	5531	6361	1	0.130482...	6.663855...	0.4999499591442...
831	5536	6367	5	0.130516...	6.661853...	0.4999500063006...
832	5541	6373	5	0.130550...	6.659855...	0.4999500533683...
833	5546	6379	5	0.130584...	6.657863...	0.4999501003474...
834	5555	6389	9	0.130536...	6.660671...	0.4999501784499...
835	5562	6397	7	0.130529...	6.661077...	0.4999502407560...
836	5585	6421	23	0.130197...	6.680622...	0.4999504267429...
837	5590	6427	5	0.130231...	6.678614...	0.4999504730226...
838	5611	6449	21	0.129942...	6.695704...	0.4999506419780...
839	5612	6451	1	0.130057...	6.688915...	0.4999506572804...
840	5629	6469	17	0.129850...	6.701190...	0.4999507945766...
841	5632	6473	3	0.129924...	6.696789...	0.4999508249832...
842	5639	6481	7	0.129918...	6.697149...	0.4999508856837...
843	5648	6491	9	0.129872...	6.699881...	0.4999509613489...
844	5677	6521	29	0.129428...	6.726303...	0.4999511869523...
845	5684	6529	7	0.129422...	6.726627...	0.4999512467631...
846	5701	6547	17	0.129219...	6.738770...	0.4999513808028...
847	5704	6551	3	0.129293...	6.734356...	0.4999514104894...
848	5705	6553	1	0.129406...	6.727594...	0.4999514253191...
849	5714	6563	9	0.129361...	6.730270...	0.4999514993320...
850	5719	6569	5	0.129395...	6.728235...	0.4999515436316...
851	5720	6571	1	0.129508...	6.721504...	0.4999515583801...
852	5725	6577	5	0.129542...	6.719483...	0.4999516025720...

853	5728	6581	3	0.129615...	6.715123...	0.4999516319884...
854	5745	6599	17	0.129413...	6.727166...	0.4999517639212...
855	5752	6607	7	0.129408...	6.727485...	0.4999518223272...
856	5763	6619	11	0.129324...	6.732476...	0.4999519096715...
857	5780	6637	17	0.129124...	6.744457...	0.4999520400958...
858	5795	6653	15	0.128964...	6.754079...	0.4999521554360...
859	5800	6659	5	0.128998...	6.752037...	0.4999521985457...
860	5801	6661	1	0.129109...	6.745348...	0.4999522128983...
861	5812	6673	11	0.129027...	6.750290...	0.4999522988335...
862	5817	6679	5	0.129061...	6.748259...	0.4999523416853...
863	5826	6689	9	0.129017...	6.750869...	0.4999524129340...
864	5827	6691	1	0.129128...	6.744212...	0.4999524271583...
865	5836	6701	9	0.129085...	6.746820...	0.4999524981519...
866	5837	6703	1	0.129195...	6.740184...	0.4999525123252...
867	5842	6709	5	0.129229...	6.738177...	0.4999525547944...
868	5851	6719	9	0.129185...	6.740783...	0.4999526254079...
869	5864	6733	13	0.129065...	6.747986...	0.4999527239144...
870	5867	6737	3	0.129137...	6.743678...	0.4999527519839...
871	5890	6761	23	0.128827...	6.762342...	0.4999529197036...
872	5891	6763	1	0.128936...	6.755733...	0.4999529336265...
873	5906	6779	15	0.128780...	6.765177...	0.4999530447139...
874	5907	6781	1	0.128889...	6.758581...	0.4999530585630...
875	5916	6791	9	0.128847...	6.761142...	0.4999531276860...
876	5917	6793	1	0.128956...	6.754566...	0.4999531414862...
877	5926	6803	9	0.128913...	6.757126...	0.4999532103654...
878	5945	6823	19	0.128682...	6.771070...	0.4999533475181...
879	5948	6827	3	0.128753...	6.766780...	0.4999533748522...
880	5949	6829	1	0.128862...	6.760227...	0.4999533885072...
881	5952	6833	3	0.128933...	6.755959...	0.4999534157933...
882	5959	6841	7	0.128928...	6.756235...	0.4999534702698...
883	5974	6857	15	0.128773...	6.765571...	0.4999535788414...
884	5979	6863	5	0.128806...	6.763574...	0.4999536194253...
885	5984	6869	5	0.128839...	6.761581...	0.4999536599382...

886	5985	6871	1	0.128947...	6.755079...	0.4999536734268...
887	5996	6883	11	0.128868...	6.759864...	0.4999537541938...
888	6011	6899	15	0.128714...	6.769144...	0.4999538614460...
889	6018	6907	7	0.128710...	6.769403...	0.4999539148857...
890	6021	6911	3	0.128780...	6.765168...	0.4999539415592...
891	6026	6917	5	0.128813...	6.763187...	0.4999539815116...
892	6055	6947	29	0.128400...	6.788116...	0.4999541802383...
893	6056	6949	1	0.128507...	6.781634...	0.4999541934258...
894	6065	6959	9	0.128466...	6.784116...	0.4999542592493...
895	6066	6961	1	0.128573...	6.777653...	0.4999542723913...
896	6071	6967	5	0.128606...	6.775669...	0.4999543117720...
897	6074	6971	3	0.128675...	6.771460...	0.4999543379882...
898	6079	6977	5	0.128708...	6.769487...	0.4999543772561...
899	6084	6983	5	0.128741...	6.767519...	0.4999544164565...
900	6091	6991	7	0.128736...	6.767777...	0.4999544686190...
901	6096	6997	5	0.128769...	6.765815...	0.4999545076627...
902	6099	7001	3	0.128838...	6.761640...	0.4999545336546...
903	6110	7013	11	0.128760...	6.766334...	0.4999546114524...
904	6115	7019	5	0.128793...	6.764380...	0.4999546502515...
905	6122	7027	7	0.128788...	6.764640...	0.4999547018807...
906	6133	7039	11	0.128711...	6.769315...	0.4999547791044...
907	6136	7043	3	0.128780...	6.765159...	0.4999548047871...
908	6149	7057	13	0.128666...	6.772026...	0.4999548944474...
909	6160	7069	11	0.128589...	6.776677...	0.4999549710165...
910	6169	7079	9	0.128549...	6.775824...	0.4999550346257...
911	6192	7103	23	0.128255...	6.796926...	0.4999551865572...
912	6197	7109	5	0.128288...	6.794956...	0.4999552243797...
913	6208	7121	11	0.128212...	6.799561...	0.4999552998337...
914	6213	7127	5	0.128244...	6.797592...	0.4999553374653...
915	6214	7129	1	0.128348...	6.791256...	0.4999553499952...
916	6235	7151	21	0.128093...	6.806768...	0.4999554873606...
917	6242	7159	7	0.128090...	6.806979...	0.4999555371023...
918	6259	7177	17	0.127908...	6.818082...	0.4999556486158...

919	6268	7187	9	0.127869...	6.820457...	0.4999557103264...
920	6273	7193	5	0.127902...	6.818478...	0.4999557472703...
921	6286	7207	13	0.127792...	6.825190...	0.4999558332337...
922	6289	7211	3	0.127860...	6.821041...	0.4999558577334...
923	6290	7213	1	0.127963...	6.814734...	0.4999558699730...
924	6295	7219	5	0.127995...	6.812770...	0.4999559066513...
925	6304	7229	9	0.127956...	6.815135...	0.4999559676464...
926	6311	7237	7	0.127953...	6.815334...	0.4999560163211...
927	6316	7243	5	0.127985...	6.813376...	0.4999560527565...
928	6319	7247	3	0.128052...	6.809267...	0.4999560770133...
929	6324	7253	5	0.128084...	6.807319...	0.4999561133483...
930	6353	7283	29	0.127694...	6.831182...	0.4999562941254...
931	6366	7297	13	0.127586...	6.837808...	0.4999563779794...
932	6375	7307	9	0.127548...	6.840128...	0.4999564376783...
933	6376	7309	1	0.127650...	6.833869...	0.4999564495985...
934	6387	7321	11	0.127578...	6.838329...	0.4999565209828...
935	6396	7331	9	0.127540...	6.840641...	0.4999565802913...
936	6397	7333	1	0.127642...	6.834401...	0.4999565921336...
937	6412	7349	15	0.127500...	6.843116...	0.4999566866397...
938	6413	7351	1	0.127601...	6.836886...	0.4999566984241...
939	6430	7369	17	0.127425...	6.847710...	0.4999568041953...
940	6453	7393	23	0.127147...	6.864893...	0.4999569444225...
941	6470	7411	17	0.126973...	6.875664...	0.4999570489968...
942	6475	7417	5	0.127005...	6.873673...	0.4999570837421...
943	6490	7433	15	0.126866...	6.882290...	0.4999571761221...
944	6507	7451	17	0.126694...	6.893008...	0.4999572795753...
945	6512	7457	5	0.126726...	6.891005...	0.4999573139487...
946	6513	7459	1	0.126826...	6.884778...	0.4999573253942...
947	6530	7477	17	0.126655...	6.895459...	0.4999574281283...
948	6533	7481	3	0.126721...	6.891350...	0.4999574508910...
949	6538	7487	5	0.126753...	6.889357...	0.4999574849894...
950	6539	7489	1	0.126852...	6.883157...	0.4999574963433...
951	6548	7499	9	0.122681...	6.885383...	0.4999575530224...

952	6555	7507	7	0.126814...	6.885504...	0.4999575982570...
953	6564	7517	9	0.126779...	6.887722...	0.4999576546648...
954	6569	7523	5	0.126811...	6.885744...	0.4999576884375...
955	6574	7529	5	0.126842...	6.883769...	0.4999577221564...
956	6581	7537	7	0.126840...	6.883891...	0.4999577670314...
957	6584	7541	3	0.126906...	6.879832...	0.4999577894331...
958	6589	7547	5	0.126937...	6.877870...	0.4999578229913...
959	6590	7549	1	0.127036...	6.871741...	0.4999578341655...
960	6599	7559	9	0.127000...	6.873958...	0.4999578899478...
961	6600	7561	1	0.127099...	6.867845...	0.4999579010865...
962	6611	7573	11	0.127030...	6.872141...	0.4999579677955...
963	6614	7577	3	0.127095...	6.868120...	0.4999579899849...
964	6619	7583	5	0.127126...	6.866182...	0.4999580232250...
965	6624	7589	5	0.127157...	6.864248...	0.4999580564126...
966	6625	7591	1	0.127255...	6.858178...	0.4999580674635...
967	6636	7603	11	0.127186...	6.862461...	0.4999581336466...
968	6639	7607	3	0.127251...	6.858471...	0.4999581556613...
969	6652	7621	13	0.127148...	6.864809...	0.4999582325305...
970	6669	7639	17	0.126979...	6.875257...	0.4999583309485...
971	6672	7643	3	0.127044...	6.871266...	0.4999583527561...
972	6677	7649	5	0.127075...	6.869341...	0.4999583854249...
973	6696	7669	19	0.126874...	6.881808...	0.4999584939517...
974	6699	7673	3	0.126938...	6.877823...	0.4999585155891...
975	6706	7681	7	0.126936...	6.877948...	0.4999585587964...
976	6711	7687	5	0.126967...	6.876024...	0.4999585911429...
977	6714	7691	3	0.127031...	6.872057...	0.4999586126791...
978	6721	7699	7	0.127029...	6.872188...	0.4999586556845...
979	6724	7703	3	0.127093...	6.868232...	0.4999586771537...
980	6737	7717	13	0.126992...	6.874489...	0.4999587521207...
981	6742	7723	5	0.127023...	6.872579...	0.4999587841662...
982	6745	7727	3	0.127086...	6.868635...	0.4999588055022...
983	6758	7741	13	0.126986...	6.874872...	0.4999588800045...
984	6769	7753	11	0.126918...	6.879065...	0.4999589436496...

985	6772	7757	3	0.126982...	6.875126...	0.4999589648208...
986	6773	7759	1	0.127078...	6.869168...	0.4999589753983...
987	6802	7789	29	0.126717...	6.891590...	0.4999591334080...
988	6805	7793	3	0.126780...	6.887651...	0.4999591543841...
989	6828	7817	23	0.126519...	6.904954...	0.4999592797896...
990	6833	7823	5	0.126549...	6.902020...	0.4999593110207...
991	6838	7829	5	0.126580...	6.900100...	0.4999593422040...
992	6849	7841	11	0.126514...	6.904233...	0.4999594044274...
993	6860	7853	11	0.126448...	6.908358...	0.4999594664606...
994	6873	7867	13	0.126350...	6.914486...	0.4999595385935...
995	6878	7873	5	0.126381...	6.912562...	0.4999595694291...
996	6881	7877	3	0.126444...	6.908634...	0.4999595899600...
997	6882	7879	1	0.126538...	6.902708...	0.4999596002177...
998	6885	7883	3	0.126601...	6.898797...	0.4999596207174...
999	6902	7901	17	0.126439...	6.908908...	0.4999597127092...
1000	6907	7907	5	0.126470...	6.907	0.4999597432800...
1001	6918	7919	11	0.126404...	6.911088...	0.4999598042828...
1002	6925	7927	7	0.126403...	6.911177...	0.4999598448486...
1003	6930	7933	5	0.126433...	6.909272...	0.4999598752194...
1004	6933	7937	3	0.126496...	6.905378...	0.4999598954410...
1005	6944	7949	11	0.126430...	6.909452...	0.4999599559838...
1006	6945	7951	1	0.126524...	6.903578...	0.4999599660565...
1007	6956	7963	11	0.126459...	6.907646...	0.4999600263864...
1008	6985	7993	29	0.126110...	6.929563...	0.4999601764188...
1009	7000	8009	15	0.125983...	6.937561...	0.4999602559764...
1010	7001	8011	1	0.126076...	6.931683...	0.4999602658988...
1011	7006	8017	5	0.126107...	6.929772...	0.4999602956362...
1012	7027	8039	21	0.125886...	6.943675...	0.4999604042935...
1013	7040	8053	13	0.125791...	6.949654...	0.4999604731299...
1014	7045	8059	5	0.125822...	6.947731...	0.4999605025580...
1015	7054	8069	9	0.125790...	6.944975...	0.4999605515076...
1016	7065	8081	11	0.125727...	6.953740...	0.4999606100872...
1017	7070	8087	5	0.125757...	6.951819...	0.4999606393119...

1018	7071	8089	1	0.125849...	6.945972...	0.4999606490438...
1019	7074	8093	3	0.125911...	6.942100...	0.4999606684931...
1020	7081	8101	7	0.125910...	6.942156...	0.4999607073343...
1021	7090	8111	9	0.125878...	6.944172...	0.4999697557780...
1022	7095	8117	5	0.125908...	6.942270...	0.4999607847869...
1023	7100	8123	5	0.125938...	6.940371...	0.4999608137529...
1024	7123	8147	23	0.125690...	6.956054...	0.4999609291905...
1025	7136	8161	13	0.125597...	6.961951...	0.4999609962155...
1026	7141	8167	5	0.125627...	6.960038...	0.4999610248702...
1027	7144	8171	3	0.125688...	6.956183...	0.4999610439499...
1028	7151	8179	7	0.125687...	6.956225...	0.4999610820534...
1029	7162	8191	11	0.125625...	6.960155...	0.4999611390691...
1030	7179	8209	17	0.125472...	6.969902...	0.4999612242801...
1031	7188	8219	9	0.125441...	6.971871...	0.4999612714582...
1032	7189	8221	1	0.125532...	6.966085...	0.4999612808801...
1033	7198	8231	9	0.125501...	6.968054...	0.4999613279207...
1034	7199	8233	1	0.125592...	6.962282...	0.4999613373151...
1035	7202	8237	3	0.125652...	6.958454...	0.4999613560902...
1036	7207	8243	5	0.125682...	6.956563...	0.4999613842187...
1037	7226	8263	19	0.125499...	6.968177...	0.4999614776855...
1038	7231	8269	5	0.125529...	6.966281...	0.4999615056373...
1039	7234	8273	3	0.125589...	6.962463...	0.4999615242494...
1040	7247	8287	13	0.125497...	6.968269...	0.4999615892500...
1041	7250	8291	3	0.125557...	6.964457...	0.4999616077813...
1042	7251	8293	1	0.125648...	6.958733...	0.4999616170403...
1043	7254	8297	3	0.125708...	6.954937...	0.4999616355448...
1044	7267	8311	13	0.125616...	6.960727...	0.4999617001702...
1045	7272	8317	5	0.125646...	6.958851...	0.4999617278003...
1046	7283	8329	11	0.125585...	6.962715...	0.4999617829409...
1047	7306	8353	23	0.125344...	6.978032...	0.4999618927469...
1048	7315	8363	9	0.125313...	6.979961...	0.4999619383134...
1049	7320	8369	5	0.125343...	6.978074...	0.4999619656010...
1050	7327	8377	7	0.125343...	6.978095...	0.4999620019237...

1051	7336	8387	9	0.125312...	6.980019...	0.4999620472296...
1052	7337	8389	1	0.125402...	6.974334...	0.4999620562779...
1053	7366	8419	29	0.125074...	6.995251...	0.4999621914853...
1054	7369	8423	3	0.125133...	6.991461...	0.4999622094402...
1055	7374	8429	5	0.125163...	6.989573...	0.4999622363406...
1056	7375	8431	1	0.125252...	6.983901...	0.4999622452989...
1057	7386	8443	11	0.125192...	6.987701...	0.4999622989595...
1058	7389	8447	3	0.125251...	6.983931...	0.4999623168125...
1059	7402	8461	13	0.125162...	6.989612...	0.4999623791650...
1060	7407	8467	5	0.125191...	6.987735...	0.4999624058244...
1061	7440	8501	33	0.124808...	7.012252...	0.4999625561834...
1062	7451	8513	11	0.124750...	7.016007...	0.4999626089645...
1063	7458	8521	7	0.124750...	7.015992...	0.4999626440693...
1064	7463	8527	5	0.124780...	7.014097...	0.4999626703547...
1065	7472	8537	9	0.124751...	7.015962...	0.4999627140816...
1066	7473	8539	1	0.124838...	7.010318...	0.4999627228147...
1067	7476	8543	3	0.124897...	7.006560...	0.4999627402686...
1068	7495	8563	19	0.124722...	7.017790...	0.4999628272936...
1069	7504	8573	9	0.124693...	7.019644...	0.4999628706538...
1070	7511	8581	7	0.124694...	7.019626...	0.4999629052692...
1071	7526	8597	15	0.124578...	7.027077...	0.4999629743067...
1072	7527	8599	1	0.124665...	7.021455...	0.4999629829183...
1073	7536	8609	9	0.124637...	7.023299...	0.4999630259165...
1074	7549	8623	13	0.124550...	7.028864...	0.4999630859463...
1075	7552	8627	3	0.124608...	7.025116...	0.4999631030619...
1076	7553	8629	1	0.124695...	7.019516...	0.4999631116137...
1077	7564	8641	11	0.124638...	7.023212...	0.4999631628417...
1078	7569	8647	5	0.124667...	7.021335...	0.4999631884023...
1079	7584	8663	15	0.124552...	7.028730...	0.4999632563909...
1080	7589	8669	5	0.124581...	7.026851...	0.4999632818220...
1081	7596	8677	7	0.124582...	7.026827...	0.4999633156753...
1082	7599	8681	3	0.124640...	7.023105...	0.4999633325786...
1083	7606	8689	7	0.124640...	7.023084...	0.4999633663384...

1084	7609	8693	3	0.124698...	7.019372...	0.4999633831951...
1085	7614	8699	5	0.124726...	7.017511...	0.4999634084509...
1086	7621	8707	7	0.124727...	7.017495...	0.4999634420713...
1087	7626	8713	5	0.124756...	7.015639...	0.4999634672460...
1088	7631	8719	5	0.124784...	7.013786...	0.4999634923861...
1089	7642	8731	11	0.124727...	7.017447...	0.4999635425627...
1090	7647	8737	5	0.124756...	7.015596...	0.4999635675993...
1091	7650	8741	3	0.124814...	7.011915...	0.4999635842712...
1092	7655	8747	5	0.124842...	7.01007	0.4999636092506...
1093	7660	8753	5	0.124871...	7.008234...	0.4999636341957...
1094	7667	8761	7	0.124871...	7.008226...	0.4999636674027...
1095	7684	8779	17	0.124729...	7.017351...	0.4999637418971...
1096	7687	8783	3	0.124786...	7.013686...	0.4999637584100...
1097	7706	8803	19	0.124616...	7.024612...	0.4999638407491...
1098	7709	8807	3	0.124673...	7.020947...	0.4999638571721...
1099	7720	8819	11	0.124617...	7.024567...	0.4999639063516...
1100	7721	8821	1	0.124702...	7.019090...	0.4999639145352...
1101	7730	8831	9	0.124674...	7.020890...	0.4999639553974...
1102	7735	8837	5	0.124702...	7.019056...	0.4999639798704...
1103	7736	8839	1	0.124787...	7.013599...	0.4999639880207...
1104	7745	8849	9	0.124759...	7.015398...	0.4999640287168...
1105	7756	8861	11	0.124703...	7.019004...	0.4999640774308...
1106	7757	8863	1	0.124788...	7.013562...	0.4999640855370...
1107	7760	8867	3	0.124844...	7.009936...	0.4999641017384...
1108	7779	8887	19	0.124676...	7.020758...	0.4999641825267...
1109	7784	8893	5	0.124704...	7.018935...	0.4999642066923...
1110	7813	8923	29	0.124397...	7.038773...	0.4999643270329...
1111	7818	8929	5	0.124426...	7.036903...	0.4999643510040...
1112	7821	8933	3	0.124482...	7.033273...	0.4999643669668...
1113	7828	8941	7	0.124482...	7.033243...	0.4999643988496...
1114	7837	8951	9	0.124455...	7.035008...	0.4999644386230...
1115	7848	8963	11	0.124400...	7.038565...	0.4999644862339...
1116	7853	8969	5	0.124428...	7.036738...	0.4999645099916...

1117	7854	8971	1	0.124512...	7.031333...	0.4999645179038...
1118	7881	8999	27	0.124236...	7.049194...	0.4999646283048...
1119	7882	9001	1	0.124319...	7.043789...	0.4999646361643...
1120	7887	9007	5	0.124347...	7.041964...	0.4999646597218...
1121	7890	9011	3	0.124403...	7.038358...	0.4999646754095...
1122	7891	9013	1	0.124486...	7.032976...	0.4999646832480...
1123	7906	9029	15	0.124377...	7.040071...	0.4999647458317...
1124	7917	9041	11	0.124322...	7.043594...	0.4999647926241...
1125	7918	9043	1	0.124405...	7.038222...	0.4999648004108...
1126	7923	9049	5	0.124433...	7.036412...	0.4999648237501...
1127	7932	9059	9	0.124406...	7.038154...	0.4999648625803...
1128	7939	9067	7	0.124407...	7.038120...	0.4999648935827...
1129	7962	9091	23	0.124188...	7.052258...	0.4999649862627...
1130	7973	9103	11	0.124134...	7.055752...	0.4999650324195...
1131	7978	9109	5	0.124162...	7.053934...	0.4999650554523...
1132	7995	9127	17	0.124027...	7.062720...	0.4999651243689...
1133	8000	9133	5	0.124055...	7.060900...	0.4999651472807...
1134	8003	9137	3	0.124110...	7.057319...	0.4999651625385...
1135	8016	9151	13	0.124030...	7.062555...	0.4999652158359...
1136	8021	9157	5	0.124058...	7.060739...	0.4999652386278...
1137	8024	9161	3	0.124113...	7.057167...	0.4999652538058...
1138	8035	9173	11	0.124059...	7.060632...	0.4999652992603...
1139	8042	9181	7	0.124060...	7.060579...	0.4999653294973...
1140	8047	9187	5	0.124088...	7.058771...	0.4999653521405...
1141	8058	9199	11	0.124035...	7.062226...	0.4999653973383...
1142	8061	9203	3	0.124089...	7.058669...	0.4999654123780...
1143	8066	9209	5	0.124117...	7.056867...	0.4999654349131...
1144	8077	9221	11	0.124064...	7.060314...	0.4999654798953...
1145	8082	9227	5	0.124092...	7.058515...	0.4999655023425...
1146	8093	9239	11	0.124039...	7.061954...	0.4999655471495...
1147	8094	9241	1	0.124120...	7.056669...	0.4999655546061...
1148	8109	9257	15	0.124014...	7.063588...	0.4999656141422...
1149	8128	9277	19	0.123854...	7.073977...	0.4999656882736...

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1151	8132	9283	1	0.123990...	7.065160...	0.4999657104508...
1152	8141	9293	9	0.123964...	7.066840...	0.4999657473490...
1153	8158	9311	17	0.123832...	7.075455...	0.4999658135662...
1154	8165	9319	7	0.123833...	7.075389...	0.4999658429139...
1155	8168	9323	3	0.123887...	7.071861...	0.4999658575689...
1156	8181	9337	13	0.123808...	7.076989...	0.4999659087624...
1157	8184	9341	3	0.123862...	7.073465...	0.4999659233609...
1158	8185	9343	1	0.123943...	7.068221...	0.4999659306555...
1159	8190	9349	5	0.123970...	7.066436...	0.4999659525205...
1160	8211	9371	21	0.123786...	7.078448...	0.4999660324527...
1161	8216	9377	5	0.123813...	7.076658...	0.4999660541873...
1162	8229	9391	13	0.123735...	7.081755...	0.4999661047934...
1163	8234	9397	5	0.123762...	7.079965...	0.4999661264355...
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1165	8248	9413	9	0.123765...	7.079828...	0.4999661840130...
1166	8253	9419	5	0.123792...	7.078044...	0.4999662055542...
1167	8254	9421	1	0.123872...	7.072836...	0.4999662127284...
1168	8263	9431	9	0.123846...	7.074486...	0.4999662485542...
1169	8264	9433	1	0.123926...	7.069289...	0.4999662557102...
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1172	8289	9461	21	0.123876...	7.072525...	0.4999663555771...
1173	8290	9463	1	0.123956...	7.067348...	0.4999663626878...
1174	8293	9467	3	0.124009...	7.063884...	0.4999663769002...
1175	8298	9473	5	0.124036...	7.062127...	0.4999663981964...
1176	8303	9479	5	0.124063...	7.060374...	0.4999664194656...
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1178	8319	9497	5	0.124039...	7.061969...	0.4999664831120...
1179	8332	9511	13	0.123961...	7.067005...	0.4999665324482...
1180	8341	9521	9	0.123936...	7.068644...	0.4999665675995...
1181	8352	9533	11	0.123885...	7.071972...	0.4999666096837...
1182	8357	9539	5	0.123912...	7.070219...	0.4999666306861...

1183	8364	9547	7	0.123913...	7.070160...	0.4999666586482...
1184	8367	9551	3	0.123966...	7.066722...	0.4999666726117...
1185	8402	9587	35	0.123604...	7.090295...	0.4999667977589...
1186	8415	9601	13	0.123528...	7.095278...	0.4999668461738...
1187	8426	9613	11	0.123478...	7.098567...	0.4999668875600...
1188	8431	9619	5	0.123505...	7.096801...	0.4999669082144...
1189	8434	9623	3	0.123558...	7.093355...	0.4999669219697...
1190	8439	9629	5	0.123585...	7.091596...	0.4999669425812...
1191	8440	9631	1	0.123663...	7.086481...	0.4999669494460...
1192	8451	9643	11	0.123612...	7.089765...	0.4999669905750...
1193	8456	9649	5	0.123639...	7.088013...	0.4999670111011...
1194	8467	9661	11	0.123589...	7.091289...	0.4999670520769...
1195	8482	9677	15	0.123488...	7.097907...	0.4999671065531...
1196	8483	9679	1	0.123566...	7.092809...	0.4999671133500...
1197	8492	9689	9	0.123542...	7.094402...	0.4999671472922...
1198	8499	9697	7	0.123543...	7.094323...	0.4999671743956...
1199	8520	9719	21	0.123366...	7.105921...	0.4999672486999...
1200	8521	9721	1	0.123444...	7.100833...	0.4999672554382...
1201	8532	9733	11	0.123394...	7.104079...	0.4999672958096...
1202	8537	9739	5	0.123421...	7.102329...	0.4999673159579...
1203	8540	9743	3	0.123473...	7.098919...	0.4999673293764...
1204	8545	9749	5	0.123499...	7.097176...	0.4999673494835...
1205	8562	9767	17	0.123374...	7.105394...	0.4999674096564...
1206	8563	9769	1	0.123451...	7.100331...	0.4999674163286...
1207	8574	9781	11	0.123402...	7.103562...	0.4999674563045...
1208	8579	9787	5	0.123429...	7.101821...	0.4999674762557...
1209	8582	9791	3	0.123480...	7.098428...	0.4999674895429...
1210	8593	9803	11	0.123431...	7.101652...	0.4999675293394...
1211	8600	9811	7	0.123432...	7.101568...	0.4999675558164...
1212	8605	9817	5	0.123459...	7.099834...	0.4999675756458...
1213	8616	9829	11	0.123410...	7.103050...	0.4999676152319...
1214	8619	9833	3	0.123461...	7.099670...	0.4999676284058...
1215	8624	9839	5	0.123488...	7.097942...	0.4999676481566...

1216	8635	9851	11	0.123439...	7.101151...	0.4999676875560...
1217	8640	9857	5	0.123465...	7.099424...	0.4999677072248...
1218	8641	9859	1	0.123541...	7.094417...	0.4999677137757...
1219	8652	9871	11	0.123493...	7.097621...	0.4999677530255...
1220	8663	9883	11	0.123444...	7.100819...	0.4999677921799...
1221	8666	9887	3	0.123495...	7.097461...	0.4999678052103...
1222	8679	9901	13	0.123421...	7.102291...	0.4999678507337...
1223	8684	9907	5	0.123448...	7.100572...	0.4999678702043...
1224	8699	9923	15	0.123349...	7.107026...	0.4999679220109...
1225	8704	9929	5	0.123375...	7.105306...	0.4999679413953...
1226	8705	9931	1	0.123451...	7.100326...	0.4999679478516...
1227	8714	9941	9	0.123428...	7.101874...	0.4999679800940...
1228	8721	9949	7	0.123429...	7.101791...	0.4999680058412...
1229	8738	9967	17	0.123306...	7.109845...	0.4999680636214...
1230	8743	9973	5	0.123332...	7.108130...	0.4999680828351...
...
∞	∞	∞	∞	0	∞	0.5

表中: P_m 为素数; m 是素数的序数; S_m 为合数总计; K_m 是合数个数。表中的数据不能四舍五入;否则会影响计算结果。

In the table: P_m is prime; m is the ordinal of a prime number; S_m is composite total; K_m is a composite number. Do not round off the data in the table; otherwise, the calculation result will be affected.

备注: 表中素数包括自然数 1。本表原始素数数据来源参考美国克莱数学研究所网站。

网址: <https://www.claymath.org> 数据链接 Data link: <https://t5k.org/lists/small/millions/>

Note: The prime numbers in the table include the natural number 1. The original prime data source in this table refer to the Clay Mathematics Institute website. Website: <https://www.claymath.org>

Data link: <https://t5k.org/lists/small/millions/>

说明: 例如 1, 查表 4, 由于第 1, 2, 3 个素数没有合数间隔, 因此合数个数均为 0; 根据定理 1 素数通项公式(4.1):

Note: For example, 1, look at Table 4, because the first 1, 2, 3 prime numbers have no composite interval, so the composite number is 0; According to theorem 1 prime number general term formula (4.1) :

$$P_m = m + S_m(P_m)$$

则有: There are: $P_1 = 1 + \sum 0 = 1$; $P_2 = 2 + \sum 0 = 2$; $P_3 = 3 + \sum 0 = 3$.

例如 2, 查表 4, 第 4 个素数间隔合数个数是 1, 由于前 3 个素数间隔合数个数为 0, 则

其合数个数之和为 $\Sigma 1$ 。所以，第 4 个素数：

For example, 2, look at Table 4, the number of the fourth prime interval is 1, because the number of the first three prime interval is 0, then the sum of the number of the composite number is $\Sigma 1$. So, the fourth prime number:

$$P_4 = 4 + \Sigma 1 = 5$$

例如 3，查表 4，第 61 个素数的素数间隔合数个数之和为 $\Sigma 220$ ；根据定理 1 素数通项公式(4.1)，则有：

For example, 3, see Table 4, the sum of the number of prime interval composites of the 61st prime number is $\Sigma 220$; According to theorem 1 prime number general term formula (4.1), then:

$$P_{61} = 61 + \Sigma 220 = 281$$